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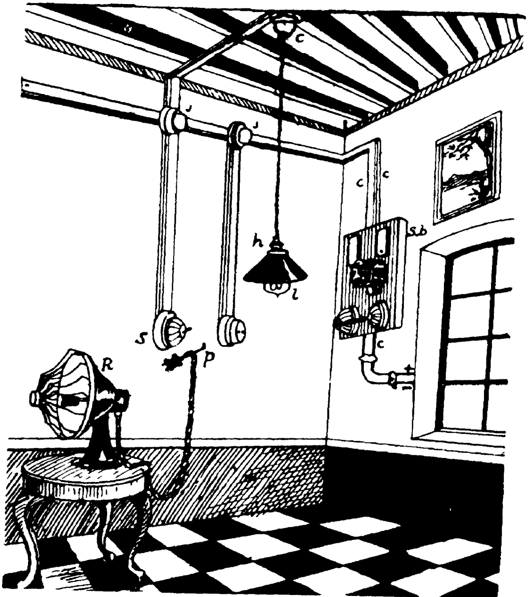
A TEXT-BOOK OF
PRACTICAL PHYSICS

FOR INTERMEDIATE CLASSES



Science in the service of home and humanity

(A TRIUMPH OF PHYSICS)

A TEXT-BOOK OF
PRACTICAL PHYSICS
FOR INTERMEDIATE CLASSES

BY

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SECOND EDITION

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PREFACE

THE efforts of Principal A. S. Hemmy, B.A., M.Sc., as a member of the University Inspection Committee, combined with the devotion of teachers and examiners to the ideals of precision, efficiency and thoroughness, have resulted, during the last twenty years, in the establishment of elementary physics laboratories, and systematic experimenting in Intermediate Classes, all over the Panjab. The good old days when candidates for the F.A. and F.Sc. placed the specific gravity stool, beaker and all, in the balance pan, and offered to the examiner wonderful values after equally wonderful observations, are gone, never to return. The writer of this book shared in the ceremonies as an examiner at, perhaps, the very first really practical test arranged by the University in connection with Intermediate Physics, and the difference between 1909 and 1926 is certainly vast. The average Panjabi student is now an enthusiastic experimenter, is willing to take pains, and his work is is often as good as the apparatus will allow. On this splendid structure the architects may very well congratulate themselves.

This book is the outcome of a feeling shared by the author with many teachers who meet annually for examination work, that the average experimenter pursues the high ideals, referred to above, with more vigour than judgment, and that much honest and patient work on his part is often sadly discounted by his (*i*) trying to achieve no more than piecemeal or *microscopic accuracy*, (*ii*) ignorance or imperfect realisation of the notion of all-round or *telescopic accuracy*, (*iii*) feeble grasp of principles, and inability to distinguish between an important precaution and an unimportant nicety, with the result that he either fails altogether or succeeds but partially, and, in addition, feels inclined to attribute the failure irrelevantly to some minute flaw in the apparatus. It is also felt that, while at work, *the student spends so much of his mental energy in the observance of details that he ceases to* THINK, *merely because he has so much to* DO.

That this feeling is not groundless, is borne out by the following experiences with which every teacher is familiar :

(1) The student's grasp of the law of signs is defective and the uprights very shaky, but the index correction is religiously measured, with the *net* result that the focal length of an *evidently* strong convex lens is *put down without a qualm* as 4080 cm. !

(3) The specific gravity of an *oil* is, through some miscalculation, put down as 8·7, but the candidate, through sheer want of a sense of proportion, attributes the discrepancy to the absence of milligram weights from the box supplied.

(3) After dropping the hot solid into the *accurately* weighed calorimeter, the student waits ten to fifteen minutes for "the steady temperature," obtains ·05 as the specific heat of copper, and in the end looks suspiciously at the balance pointer which is, unfortunately, slightly bent.

These instances are by no means exaggerated, and are apt to remind one of Holme's striking statement: "Insanity is the logic of an *accurate* mind *overtasked*." They indicate that there is room for improvement in the direction of a better grasp of the *essentials* as distinguished from the *incidentals* of practical work, and in the creation of a critical faculty, *even though this may leave somewhat less time for actual experimenting*. Unless so supplemented, the student's enthusiasm for piecemeal exactness is liable to create a mentality that is averse to independent thinking, so necessary for original research.

This book is offered to the numerous earnest teachers and students of physics in the Panjab and elsewhere, as a systematic attempt to create (even though on a small scale) in the young experimenter the habits of calm reasoning and sound criticism. The author has sought to do this by

(1) Combining with the emphasis on accuracy an emphasis on estimating the amount of harm that a certain error is likely to cause.

(2) Questions (brief answers to which are to be *recorded* in the note-book) designed to make the student think intelligently, and to elicit from him information as to the relative merits of various methods and appliances.

(3) Special exercises on estimating accuracy in percentage and significant figures, and locating the weak spots of methods and apparatus.

While the book covers the Intermediate syllabi of the Panjab and Delhi Universities, the author has not confined himself, or invariably adhered to, the "regulation" experiments. Some easy experiments that ought to be in the school course, have been omitted, and others (nominally B.Sc.) that the Intermediate student can profitably perform, describeed.

The measurement of depth and inside diameters (a subject hitherto ignored) with inside callipers, etc., as also the (practically) important subject of measurements on charts and maps has been introduced. An experiment on the construction of a vernier by the student is included. The experiments on acceleration and the 3rd law of Newton. although not capable of extreme accuracy, will be found useful aids in the grasp of fundamental notions. Similarly, no apology is, perhaps, necessary for the introduction of simple experiments on pulleys and the wheel and axle, and the work done against friction, as these elucidate the great principle of conservation of energy. Changes are suggested in connection with the usual apparatus for specific and latent heat determination. The optical lantern has been added to the list of optical instruments the student is required to construct in the laboratory. Exercises on wiring and commutators are another feature. Voltmeters and ammeters have been described earlier than the tangent galvanometer, which, on account of the multiplicity of its coils and difficulties connected with setting, should, the author believes, be used only *after* Ohm's law has been studied with the help of the simpler moving coil instruments.

As no two teachers will agree as to what exactly a class should do, a book containing neither more nor less than what may be needed in all cases is an impossibility. Experiments described in small type will, perhaps, not be performed by many, and undoubtedly, some teachers will miss other experiments that are not included. It is expected, nevertheless, that the book will meet, to a great extent, all usual needs.

Experiments that *overlap,* or are very nearly *repetitions* of others, are given the same number as the principal of the group, with a letter affixed. This may help in designing a course of work. For instance, a student who has performed experiment 23, need perform experiments 23*a* to 23*d* only if he has spare time at his disposal. These subsidiary experiments are generally printed in small type, and the directions accompanying them are also suitably curtailed.

Efforts have been made to keep the special pitfalls of the student in view, and it is hoped that the directions given may help to decrease the number of failures in connection with the spherometer, application of zero corrections, specific and latent heats, and the setting of magnetometers and tangent galvanometers. Graphical methods have often been used, and the straight line graph given special attention.

The author believes that teachers owe it to the student it to train him in the use of labour saving devices, *viz.* tables of logarithms, reciprocals squares, etc. The use of logarithms is calculated to make records neater, and to reduce the temptation to "cooking.' Arithmetical processes carried to eight or ten figures become automatically impossible with four figure tables. The first two exercises are devoted to reading such tables, and using them for multiplication, division, etc.

Plans and tables for entering data systematically are given throughout, and specimens of calculations with logarithms (without the use of loose slips) occasionally supplied. The data used are mostly based on actual experiments by students, and thus convey a correct idea of the possibilities of the method illustrated.

Tables of ordinary four-figure logarithms, antilogarithms and reciprocals are supplied. Other mathematical and physical tables actually needed for the calculation of results have been selected with care and suitably curtailed, so as not to exceed unduly the demands of intermediate work, as it is imperative for the beginner to realise that killing flies with siege-guns is positively *unscientific.* Such tables (density (*d*) and specific volume (*v*) of water, log (*d*), log (*v*), sines, tangents, log *sin*, log *tan*) are provided with explanatory notes and illustrations from practical work calculated to convince the student of their labour saving character.

The large majority of the diagrams and illustrations bave been specially prepared for this book. Others have been reproduced from catalogues of apparatus, the items so selected having been in use for years in the Prince of Wales College laboratory. The names of firms concerned appear clearly on such illustrations, and the author further thankfully acknowledges their contribution to the utility of the book. Special thanks are due to Messrs. W. G. Pye and Co. of Cambridge and Messrs. F.E. Becker and Son, London for the prompt despatch of blocks for a large number of illustrations.

The author shall be grateful to teachers who send him suggestions for improvement, or notes of errors in facts, figures or presswork.

S. R. SURI.

PRINCE OF WALES COLLEGE,
1st September, 1926.

—o—

The book has been recommended by the Punjab University as suitable for use by Intermediate classes.

1st February, 1927. S. R. SURI.

SPECIAL PREFACE

TO

PART VIII

(CURRENT ELECTRICITY)

IT may, perhaps, be conceded that in this age of "electricity for everything," the student of physics should evince keen interest in practical work with currents, and that the teaching of current electricity should be regarded as of special importance. On the other hand, it is, unfortunately, a fact, that of all experiments offered, the average Panjab Intermediate candidate avoids those on current electricity the most, and will often cheerfully prefer a risky latent heat determination on the basis of a reduced maximum of marks, to a simple measurement of E. M. F., etc.

The reason is not far to seek. The candidate's training in this section starts with, and is often confined to, the hurried performance of somewhat elaborate experiments with an elaborate instrument, the tangent galvanometer, a form, of all others, the *least* suited to the needs of a beginner. The numerous adjustments and readings necessary with this instrument are hard to carry out, and although, perhaps, a valuable discipline in themselves, always divert the attention of the student from what is the immediate object of study, the working of Ohm's law, and its applications to the measurement of electromotive force and resistance, etc. The student's grasp of the proper functions of the different coils of the instrument, generally, remains very inadequate. Suitable deflections with given resistances, etc., are often not obtainable with the proper coils, and at the same time the student has been repeatedly told that a deflection of 45° is *the* thing to be aimed at. In this conflict, the interests of accuracy, as understood by the student, are always held paramount, with the result that the low resistance coil is frequently chosen for E. M. F. work and the high resistance coil for comparison of resistances, because these give (*"accurate"*!) deflections of nearly 45°, an advantage which the commoner alternative arrangements have been found to lack. When to all this are added the elusive mysteries of commutators, two-way switches and other indispensable accessories (which always have a local atmosphere about them, and are never the same as the ones the candidate is accustomed to), it may be admitted that the path selected for the student's training in current electricity is somewhat thorny, and his anxiety to avoid in the examination hall such a formidable collection of pitfalls, something which a sympathetic teacher should readily understand.

The author has, therefore, thought it advisable to split up the course into two parts,

(*i*) *Qualitative*, *i.e.*, connecting apparatus and tracing the current to check the connections.

(*ii*) *Quantitative*, *i.e.*, measurement of electromotive force, resistances, etc.

A considerable amount of space has been given in the earlier chapters to a study of circuits, wiring diagrams, bells, lamps, switches and commutators, with the object of familiarising the student with the usual types of connection, and the commoner instruments necessary for practical work in voltaic electricity. This grounding in the fundamentals of work with currents having been secured, elaborate quantitative work can be taken up with confidence, and the loss of time due to failure thus reduced to a minimum,

Quantitative experiments have been described in the following order of descending practical importance and increasing difficulty:

(*i*) *Rapid* measurements with the voltmeter

and ammeter (*everyday* methods).
- (*ii*) *Accurate* measurements by null (or *research*) methods.
- (*iii*) Measurements (neither rapid nor accurate) with the tangent galvanometer.

It is often forgotten that *the proper function of a tangent galvanometer is to measure a current in absolute units*, or to calibrate an ammeter. For mere *comparisons* of resistance and electromotive force, the labour involved in using the tangent galvanometer is scarcely worthwhile, moving coil pointer galvanometers being equally accurate, and much more convenient. Experiments on the measurement of current in absolute units and the calibration of an ammeter have, therefore, been given a prominent place. The *high* and *low* resistance coils of the tangent galvanometer have been named, respectively, the *voltmeter* coil and the *ammeter* coil. This should help the student (already trained in the use of the voltmeter and the ammeter) in selecting the proper coils for comparison of E.M.F.s, as distinguished from comparison of resistances.

The author will feel amply repaid, if the exercises given in this book help any section of students to an increased interest in practical work with currents.

S. R. SURI.

PRINCE OF WALES COLLEGE,
1st January, 1927.

PREFACE TO
THE SECOND EDITION

The book has been revised, some paragraphs re-written, and improved plans and tables for recording observations supplied in many cases. Some of the illustrations have been replaced by better ones, while some others that were necessary (*e. g.* Fortin's barometer) but absent in the first edition, added. These changes, it is hoped, will enhance the utility of the book.

The author feels indebted to Professor D. R. Hooja, M. Sc., and M. F. Haqe, Esq. M. Sc., of the Prince of Wales College physics department, as also to Professor A. R. Soni, M. Sc., of the D. A. V. College, Jullundur, for helpful criticism and valuable suggestions.

S. R. SURI

PRINCE OF WALES COLLEGE,
1st October, 1928.

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PART I

ESSENTIALS OF PRACTICAL WORK

CHAPTER I

TO THE EXPERIMENTER

The True Scientist.—The true scientist has no prejudices, and will never accept or reject a proposition without testing it by reason, observation or experiment. Science has given man the control of steam, air and electricity but an even greater gain is the *habit of experimenting*, and the belief that the solid facts of science and philosophy are discovered more easily by studying nature direct than by reasoning unaided by positive first hand knowledge. The laws of falling bodies, for instance, which centuries of empty argument had failed to define, were established by Galileo by actual experiment in an hour. The true scientist studies *stern facts*, and in interpreting the Book of Nature, his way is to *see* and *think*, but to keep his own pet notions carefully in the background. When something unusual happens, he does not readily *believe* or *disbelieve*, but starts an experimental research on the subject with a mind open to conviction. Such investigations always result in increased information and a better understanding of the laws of nature.

Avoid Prejudice.—The work in a junior physics laboratory is generally of the nature of verification of well-known relations or laws, for example, the principle of Archimedes or the formula for the pendulum. If a student obtains figures which seem to verify these, he does not in any way *add* to human knowledge. *The importance of the work lies in its giving him an opportunity of studying nature honestly and conscientiously, and as long as this is not done, good results are only a coincidence.* On the other hand, work properly done, without bias, may *not* lead to good results on account of the imperfection of apparatus, and yet is of more educational value. **The success of an experiment lies, almost entirely, in its being performed in the best style.** If the experiment is properly performed, the results are bound to be fair. In fact, the chief aim of work in a junior laboratory is to train the student in the habits of honesty, patience and calm reasoning, and thus enable him always, no matter what his actual occupation in after life, to think and act with the precision and accuracy of a *scienist.*

Fine *work* is more important than fine *results.* Students who are unduly anxious to obtain fine results often invent excuses for changing the entries in their note books. They are generally slow to see their own mistakes, and like to throw the responsibility for stupid failures on small defects in the apparatus. For instance, a student who gets the value ·007 for the specific heat of iron through some absurd mistake, and begins to think that the failure is due to the balance pointer being slightly bent, is *prejudiced. This kind of thinking is against the spirit of true science.* The apparatus *may* be crude, but always "do *your* (honest) best and leave the rest" to the examiner, or the teacher.

A Student's Duty to the Laboratory.—While you owe it to yourself to be honest in experimenting, you have a duty to discharge as regards apparatus and laboratory fittings. Some students, unfortunately, are of opinion, that as they pay fees, they have the right to deface and break apparatus or even to apply it to their own use. The argument is faulty and hence *unscientific*, as the fees paid by the student are always much less than the actual cost of his education. Wanton breakage of apparatus raises this cost still more, and increased fees may become necessary.

Another class of students perform experiments at home with apparatus obtained from the laboratory without permission. This method of obtaining a training also cannot be recommended, as it is *irregular*, and hence *against the spirit of true science or true sport.*

All students who think aright (like *scientists*) *help* in the management of the laboratory in which they work. To *damage* it is like suicide.

General Rules.—The following general rules should be observed :—

(1) Report, *voluntarily*, all breakage or loss of apparatus *Do not wait to be detected.*

(2) Handle small weights, nuts and screws with the greatest care, and look upon their loss as a calamity to be avoided at all costs.

(3) Do not allow water, mercury, common salt or other chemicals to come in contact with metal parts of apparatus. Apparatus so soiled should be wiped clean with a wet cloth, dried, and smeared with vaseline.

(4) Apparatus should not be unscrewed and pulled to pieces. For help, in case of trouble, consult the demonstrator.

(5) Do not scratch or disfigure scales, benches, etc., with ink or pencil marks.

(6) Elaborate cleaning of apparatus with sand-paper, etc., should be left to laboratory attendants. Portions of binding screws which actually touch connecting wires should, however, be rubbed clean and bright.

(7) Do not throw water or acid on the floor. Batteries, voltameters, etc., should not be roughly

carried about. If possible, place them in enamelled trays, so that in case of leakage, the table top, and possibly apparatus underneath, is not damaged.

(8) Avoid meddling with switches, taps, etc.

(9) The habit of leaning on laboratory tables is bad. With sensitive instruments on the table, it is positively annoying.

(10) Do not mix up plugs from keys and resistance boxes, etc.

The C. G. S. System of Units.—The centimetre, gramme and second system of units should be employed for all measurements. Moderate lengths are expressed always in centimetres. Diameters of wires, small thicknesses, etc., are, however, more conveniently measured in millimetres and fractions of millimetres, and finally converted to centimetres prior to calculation of the result. Masses, large and small, (2000 to ·001), are almost always expressed in grammes. Time intervals may be measured in minutes and seconds as a first step, and reduced to seconds immediately after, unless minute intervals play an important part in the experiment, as in cooling curves, etc.

How to Perform an Experiment

Getting ready for work.

(1) The quickest and surest way of starting an experiment properly, and completing it successfully, is to **read up the instruc- tions** for the purpose **beforehand.** When in the laboratory, proceed as follows :—

(2) Place your hat, cap or turban as also all extra books out of the way (not on the working table).

(3) Select a left hand page of the laboratory note book, and put down the date. Next give the name of the experiment, and make a list, complete in every respect, of the apparatus required.

(4) Check the apparatus, provided with the list, to see that all items down to matches, pins, thread,

blocks, clamps, stands, etc., are present. **Nothing that is really wanted is unimportant.**

(5) Next **make a simple sketch of the apparatus,** or a *plain* diagram of connections or light rays, as a help in the correct performance of the experiment.

(6) Draw up a table in which to enter the observations. You will be able to do this, *if you are prepared.* If a table does not suit the experiment, the observations may be recorded with the equality signs in a neat vertical line down the middle of the page, descriptions of the various measurements in the left half, and the figure work in the right half.

(7) *Performance of the Experiment.*

Then, if you are sure you know what to do. (and not before' proceed to arrange and connect up the apparatus, *cleaning carefully with brush and duster every nook and corner of every item.* No matter how clean the apparatus as it comes from the bearer, every student who loves his work, finds room for improvement. **Perfect cleanliness, at all costs, should be your motto.** Everything should be stable, firm and easily reached. If readings cannot be taken in convenient postures, the experiment is likely to be hurried over. Avoid awkward angles. Arrange things horizontal or vertical, if these positions will answer as well as any others.

(8) Proceed to perform the experiment systematically, observing all precautions that are really necessary. **The weakest part of the experiment should receive extra care (see page 8).** If a number of trials with varying amounts or magnitudes are necessary, follow a plan, proceeding step by step, from highest to lowest or *vice versa.* For instance, if you have to determine the period for pendulums of different lengths, adopt the order 60, 90, 120 cm., or, 120, 90, 60 cm., but not 90, 60, 120 cm.

(9) *Recording observations.* **Observations should be entered, directly they are made, into the proper**

table or plan in the note-book. Loose slips of paper for preliminary use, or at any stage, should not be used. A narrow column on one side of the page for small necessary rough notes and calculations can serve the purpose of slips. The ability to make, at short notice, permanently useful memoranda in proper books or diaries, is of service throughout life. Students advance various reasons to justify the use of slips. One is that in case the experiment is unsuccessful, the book is "spoiled." The answer is, that even failures are to be recorded, as the fact of an experiment having been performed, is at least as important as the success attending it. Calculations can be performed in the book, if logarithms are used as recommended (see Model laboratory record, page 16). If a wrong entry is accidentally made, it should be neatly scored through, and the correct entry made above or below it (*e. g.* 15·80cm. ~~15·08cm.~~). *Erasing or concealing the wrong entry is not necessary.* The necessity for corrections is reduced, if the student accustoms himself to the conditions of the experiment, by taking one or two readings without recording them. After confidence is gained, the readings may be recorded. Loose sheets for trial calculations, and rubber erasers for removing observations that appear objectionble are a great temptation to those, who love, above everything else, a result coinciding up to the second place of decimals with the value in Kaye and Laby's Tables. In fact, students often like to try beforehand if their observations lead to good results If things are not rosy, they start on a process of biassed revision and *re*-consideration which very soon lands them into the habits of self-deceit and "cooking" **Make all calculations in the note-book itself. Hate erasers and loose slips.** They encourage prejudice and lead to mixing up of observations, as previous preparation at home and the drawing up of tables are generally omitted, if loose slips are employed in the first stage.

(10) *Care and accuracy necessary.* All measurements should be made with accuracy and care. Tenths of millimetres or of the smallest division (or fifths in the case of very fine divisions), whatever that may happen to be should, *as a rule*, be always estimated. If, on careful measurement, tenths of millimetres are found to be absent, the length should be put down as 25·30 cm. and not as 25·3 cm. The first means that tenths were looked for and found missing, the second that they were either not measured, through carelessness, or that for some reason, it was not considered necessary to measure them. 25·30 is correct to 4 figures, and 25·3 correct to 3. Similarly, if the length of a rod is found to be exactly 25 cm. millimetres and their tenths being found absent after careful measurement, the length should be entered as 25·00 cm., the two zeros signifying that the measurement is considered correct to four figures. *These rules are, however, subject to modification in certain cases discussed under heads* 12 *and* 13.

(11) *Constant errors and errors of observation. Taking means.* All experimenting is, in spite of all care, subject to errors, due to unavoidable imperfections in the apparatus or the observer. These errors are of two kinds. (1) **Constant errors.** Suppose that the graduations on a cm. scale are wrong (as 21, 22, 24, 25......), a whole centimetre being omitted. All lengths affected by this error will be 1 cm. wrong, and as long as the error is not detected, the result is bound to suffer, no matter how careful the work. Such errors are very dangerous, but, fortunately, are not very common. (2) **Errors of observation.** These are due to faults of judgment. One observer may read the temperature of the air to be 25°·7 C while another may think that the reading is 25°·6 C. Even the same observer may record slightly different values of the same quantity (supposed unchanged in

reality), at different times. Such errors are unavoidable. But their effect can be minimised by taking the mean of a large number of readings. As the errors of observation are not of a fixed nature, they are likely to be *as often negative as positive*, some readings being *above* and others *below* the correct reading. The positive errors cancel the negative ones, and the mean may be more or less free from the effect of these. *The student should not yield to the temptation of rejecting those readings which seem to differ considerably from other similar readings.* If there is reason to believe, that such difference is due to the apparatus having moved, or to a scale having been absurdly read (for instance, a length 25·30 may be entered as 25·03), the demonstrator may be consulted, and if he so advises *the faulty reading may be cancelled altogether*, **but no effort should be made to correct it or any other reading from memory only.** If individual readings appear to coincide, they should nevertheless be separately and individually entered, the coincidence indicating, *in case the experimenter is really careful*, that the apparatus and other circumstances are favourable for correct measurement. Often however, *coincident readings merely show the unwillingness of a student to measure tenths, etc.*

(12) The **weakest spot of the experiment** is the part where, for various reasons, either accurate measurement is not possible, or where unusual accuracy is necessary in order to secure a moderate degree of accuracy in the final result. In determining the specific heat of a solid by the method of mixtures, the rise of temperature (about 5°—10°) can be measured, with some difficulty, correct to 0°·1 with an ordinary thermometer. This means an accuracy of less than 2%. This therefore is the weak spot of the experiment, as weighings of the calorimeter, etc., can be easily carried out to within ·2%. Special precautions should, therefore, be observed in measuring the rise of

temperature, and a thermometer graduated to 0°·1C. or 0°·2C. is a great help. Sometimes the square, cube or a higher power of a quantity determines the final result. If the quantity can be measured with a possible error of 2%, this will cause an error of 4% when squared, and 6% when cubed. A higher degree of accuracy is, therefore, indispensable, when measuring such quantities. Diameters of small wires, and distances of magnets from magnetometer needles, are examples of such quantities. Sometimes the *small* difference between two *large* quantities affects the result. For instance, the weight of a lead ball in air and water may be 52·13 and 47·39 grammes, the loss of weight being 4·47 grammes. The individual weights are correct to 1 part in 5000, but the difference is correct only to 1 part in 500. Slight errors in the measurement of the quantities will greatly affect the accuracy of the difference. This is the case when you determine the weight of condensed steam from the initial and final weights of a calorimeter. Special care is necessary in such cases, and the quantities whose difference affects the result may very will be measured with an accuracy greater than usual, and, if possible, mean values of the difference used.

(13) *Reducing observations previous to calculation.* **The student should, from the very beginning, cultivate the habit of estimating accuracy in terms of PERCENTAGE and significant figures.** Suppose the diameter of a wire measured by means of a screw gauge to be 4·51 mm., or correct to ·01 mm. The length of the same wire may be measured as 156·3 cm., or correct to ·1 cm. The beginner generally believes that the diameter has been more accurately measured, because tenths and hundredths of a millimetre have been taken into account, while they have been ignored in the case of the length. The diameter is, however, correct to 1 part in 451 (3 figures), while the length is correct to 1 part in 1563 (4 figures). The length is,

therefore more accurately measured, and the error in the result due to the diameter will be greater than that due to the length. The beginner is similarly very much disappointed, if he gets for the latent heat of steam 550 instead of 537, the difference 13 seeming large. He is not disappointed so much, if he gets for the density of an oil ·82 instead of ·87, the difference ·05 appearing small. The first result, however is only 2·5% out, while the second one is wrong by 5 to 6%.

Accuracy is proportional to the number of correct significant digits obtainable. In physics practical work carry all individual measurements to *four significant figures*, if possible. Extractions of roots, and divisions, etc., should not be carried beyond four figures. If the sum of three lengths measured correct to 4 figures as stated (60·49+50·34+1·64 cm.), reaches 112·47 cm. or 5 figures, it should enter the final expression for the result as 112·5 cm., or correct to 4 figures only. Note that the second decimal figure could not have been dropped earlier in the individual lengths. It should be dropped only in the end. The result is rarely correct to more than 3 figures, on account of the large number of factors entering the expression for the result. **Retain only 3 figures in the result.** For instance, if the velocity of sound turns out to be 33412 cm., enter it as 33400 or $3{\cdot}34 \times 10^4$, 33·4 indicating that accuracy up to 3 figures only is claimed.

The accuracy of individual observations may, in *necessarily* rough experiments, be sometimes safely reduced. To secure the best results for the time and trouble taken, a safe rule is to try to secure nearly the *same* order of accuracy for all quantities entering the final result (but *considerably more for those whose cubes, squares or differences are involved*), and this accuracy should be somewhat higher, though not very much higher, than that possible in the weakest part of the experiment. For instance, if the weak spot observations are correct only to 2 significant figures (1 to 10%), the other observations need only be correct to 3 figures, and lengths though measured as 33·61 and 45·27 should enter the expression for the result as 33·6 and 45·3 cm., (the reduction to 3 figures being carried out only in the last stage). The result will be correct to 2 figures only. **This should not lead the student to believe that he can afford to be rough, as a rule,** because, in the majority of experiments, the weak spot observations can be obtained correct to 3 figures. All other observations should in such cases be obtained correct to 4 figures, as already stated. The result will be correct to 3 figures only.

(14) *Calculating the result.* The student should use tables of four figure logarithms, squares, reciprocals, etc., for all necessary calculations. The use of these labour saving devices is easily learnt, and the *habit* is valuable. Extracting square roots and multiplying in school style should be given up, except for small numbers like 4, 5, 8, etc. Besides using logarithms, the intelligent experimenter will know how easy it is to multiply and divide by 5 ($\frac{10}{2}$), 25 ($\frac{100}{4}$), 99 (100 − 1) and other special numbers Logarithms can be added up in the book, and the result obtained from antilogaritms. The ability to calculate in this neat and pleasant manner will lead to the records being completed in the laboratory, instead of their being postponed to the next week, or, alas ! to the summer or Christmas vacation, or even the preparatory leave days. Another adavantage attending the use of logarithms is that there is no chance of divisions, or other arithmetical processes, being carried to a large number of figures not justified by the nature of the work. More than four figures are not possible with 4 figure logarithms and they are never necessary. For tables other than those given in this book, "Logarithmic and other Tables for Schools" by Frank Castle is recommended.

(15) Very large and very small numbers are best expressed as the product of a number between 0 and 10, and a suitable power of 10. The velocity of light should be expressed as $3{\cdot}0 \times 10^{10}$ cm. per second, instead of 30000000000 cm. per second. The co-efficient of expansion of brass should be $1{\cdot}8 \times 10^{-5}$ and not 000018. If recorded as suggested, the numbers are easier to remember and multiply, etc.

Ink records are neat. Fountain pens are no longer dear, and should be preferred to pencils.

(16) *When to leave the laboratory.* Precautions observed should be noted down, and the book presented to the demonstrator, who will, if all is well, pass the work and assign an experiment for the next time. Remember that the **success of the next experiment depends on your having it assigned now, as otherwise preparation at home is impossible.** Without previous preparation, waste of time and frequent repetitions of the same experiment are unavoidable.

(17) When at home, write out on the right hand side a brief account of the process, as also the result obtained and the % accuracy secured. Unavoidable errors should be mentioned, and a neat diagram (or graph where necessary) drawn. In the opinion of the writer, the rewriting of all the individual observations, in fair, serves no useful purpose. If. for some reason, the original laboratory observations have not been tabulated, or are otherwise mixed up (which should not be), a neat rewriting or tabulation may be advisable, but the course most profitable to the student, and most economical of time (as also more useful in later life), is to learn to enter observations, neatly and intelligently, as soon as they are made. A form of combined home and laboratory record is also recommended (see pages 20, 21).

(18) **The experiment should be throughout looked upon as a valuable mental discipline, and an education in artistic building up.** A good architect will produce a decent

looking, substantial and comfortable house with the minimum of time, trouble and material necessary for such a house. If he builds on sand, costly floors are money thrown away. The foundations must be strong, the walls solid, and the exterior presentable. After this is secured, tiles and marble according to means may come in. An experiment should similarly start with previous preparation, for a faulty experimenter will only abuse even costly apparatus. All parts of the experiment being properly planned, and the apparatus clean and neatly arranged, work becomes like producing a well-proportioned and symmetrical building. Bestow care and labour where they are due. **Microscopic accuracy in one part is of no avail,** if on account of the nature of the apparatus or your own carelessness, accuracy in other parts is absent. A course of practical physics is a valuable discipline, and the familiarity with things mechanical may come in handy to anyone any day in this age of motor cars and electricity.

For specimen pages of a practical note book, see pages 15—21. A plain ruled note book, 10″×8″ is convenient. On a smaller page there may not be enough room for calculations, even if logarithms are employed. A dozen sheets of squared paper perforated for easy detachment should be provided at the end of the book. These may be detached for graphs, and neatly gummed close to the observations to which they refer.

Tables of Constants, etc.—The demonstrator should have these in his private drawer. Tables actually required for calculation, as logarithms, reciprocals, densities of water at different temperatures, are given in the book For other tables, paste the sheets on boards, and keep these on the walls for ready reference, or they should be incorporated in the note-book Logarithms of π, 2π, π^2, $4\pi^2$ and other commonly occurring combinations should also be read up once for all, and added to the usual tables This has been done in case of the logarithm tables given at the end of this book.

Laboratory Assistant's Outfits, etc.--For efficient work in a laboratory, the assistants have to be well-trained, so that the necessary apparatus, without serious omissions, is ready for each experimenter before he arrives. Unexpected contingencies may, however, arise, and the equipment should be so arranged, that the students can supplement, on a small scale, the apparatus provided without bothering the assistants unduly. Blocks of all sizes, matches, match strikers (wood blocks with wire gauze screwed on top), dusters, brushes, pins and thread should be readily accessible. A dozen brushes, and a dozen soft clean dusters (or selvyt cushions like those used for polishing shoes) kept ready at hand, will do much to create in students the love of cleanliness in apparatus. These should be carefully replaced by the students, or at least recovered by the assistant, so that they are always available. A box of sand should be kept in one corner for extinguishing fires.

The laboratory assistant should have always ready for his own use, or for the use of the demonstrator, the following outfits :

(1) *Tool box*—a wooden box about 14″ × 5″ × 5″, containing 1 hammer, 1 or 2 pliers, 2 or 3 screw drivers (one for watchmaker's screws), 1 wrench, 2 three cornered files, sand paper, metal polish, brush, duster, oiling can and vaseline. This box should not be given to students.

(2) *Soldering outfit.*—Very convenient outfits, which make small soldering (gas or electricity used) a pleasure, can be purchased. These contain a very handy soldering iron and autofluxing soldering paste

(3) *Cross wire and galvanometer suspension outfit.*—This is a cardboard box, 6″ × 6″ × 3″, containing 1 reel cocoon silk, 1 or 2 reels phosphor bronze strip, screw driver (watchmaker's), 1 pair scissors, springy brass wire, seccotine.

If these outfits are kept complete, much worry and running about can be saved and undue pressure on the workshop and the mistri avoided.

SPECIMEN PAGE

LABORATORY NOTE BOOK

LEFT HAND SIDE

(Laboratory record)

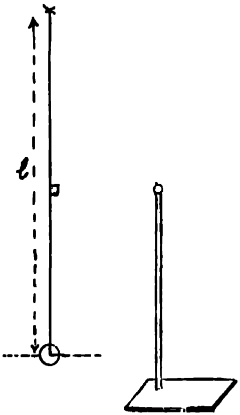
(The student should specially note how corrections are made, and wrong entries neatly scored through without being concealed).

15-10-25.

EXPERIMENT 23.—**(1) To examine the relation between l and t in a simple pendulum. (2) To calculate ‘*g*.’ (3) To calculate the length of a seconds pendulum graphically.**

Apparatus : 3 cm. bob, strong thread, split cork, stop watch, callipers, metre stick, gum paper, clamp on stand.

A retort stand rod was used to indicate passage through mean position.



Observations:

Diameter of bob. : 3·22 cm.
Radius : 1·61 cm.

Length of Pendulum (l).	Time of 30 vibrations.	Period (t)	l/t	l/t^2
Cm.	Sec.			
(1) 63·16 53·45 1 61 ——— 118 22	65·4 65 1 } 65·25	2·175	log 118·2 = 2·0726 log 2·175 = ·3375 ——— antilog: 1·7351 = 54·34	log l/t = 1·7351 log t = ·3375 ——— antilog: 1·3976 = 24·98
(2) 49·05 47·64 1·61 ——— 98 30	59·5 59·4 } 59·45	1·982	log 98·30 = 1 9926 log 1·982 = ·2971 ——— antilog : 1 6955 = 49·61	log l/t = 1·6955 log t = ·2971 ——— antilog : 1 3984 = 25·02
(3) 47 64 35·05 1·61 ——— 84·30	55·8 55 7 } 55·75	1·858	log 84 30 = 1·9258 log 1 858 = ·2690 ——— antilog : 1·6568 = 45·37	log l/t = 1·6568 log t = ·2690 ——— antilog: 1·3878 = 24·42
(4)	...	...	...	25·50
(5)	...	...	...	25·23

log $4\pi^2$ = 1·5964 : special entry
log 25·03 = 1·3982 in table
———
antilog : 2·9956
987·3
= 964 9

$$\text{Mean } \frac{l}{t^2} = \frac{125\cdot15}{5}$$

25·03
= **25·03**

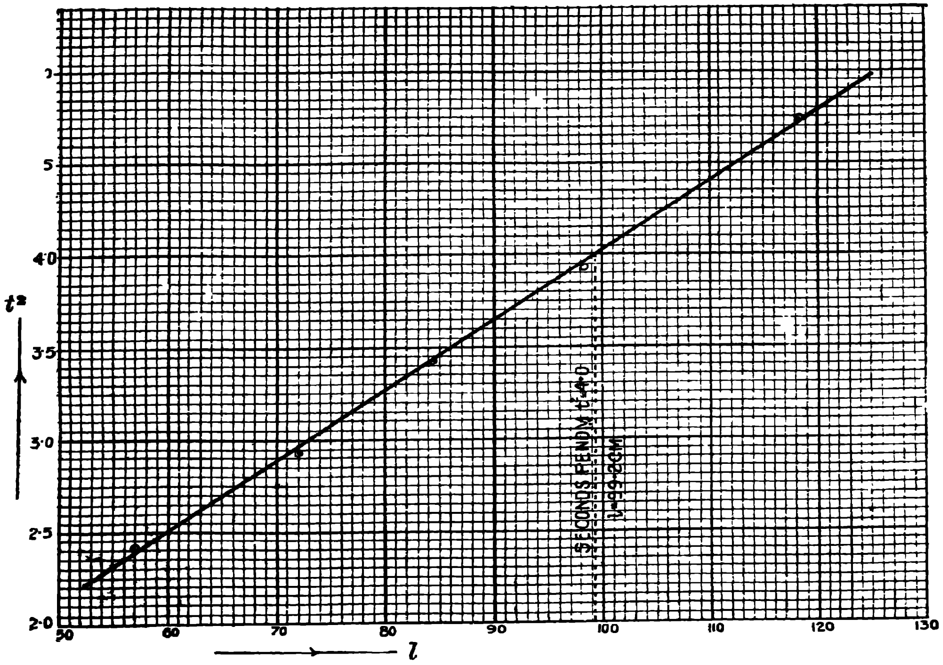
$$\therefore \; `g' = 4\pi^2\left(\frac{l}{t^2}\right) = 4\pi^2 \times 25\cdot03$$

=

987·3 or ... **987**

Values $\frac{l}{t^2}$ constant, l/t changing. Graph drawn for l and t^2.
Length of seconds pendulum ($t^2 = 4$) from graph ... ...**99·2cm.**

Precautions :—Amplitude small. Cork pieces level at bottom.
Counting begins 0, not 1.



Graph for length and period of a simple pendulum.

SPECIMEN PAGE.

LABORATORY NOTE BOOK

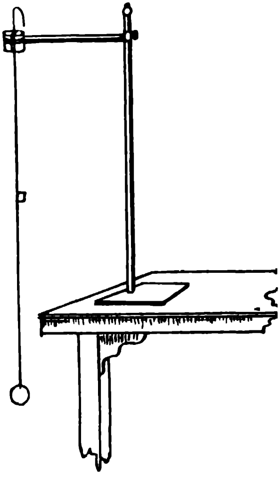
RIGHT HAND SIDE.

(Home record)

EXPERIMENT 23.—(1) **To examine the relation between *l* and *t* in a simple pendulum. (2) To calculate '*g*'. (3) To calculate the length of a seconds pendulum graphically.**

Apparatus : As on the left hand side.

Method: The formula for a simple pendulum is



$t=2\pi\sqrt{\frac{l}{g}}$ where t is the period l the length and g the acceleration of gravity. A brass ball attached to strong thread, clamped in a split cork, was the pendulum. As a graph was required, at least 5 lengths (118·30...50·35) had to be tried, the interval for 30 vibrations being determined twice (thrice would have been better, but time was a consideration). The radius was measured with callipers, and lengths determined correct to 4 figures. A retort stand rod was the reference mark. The amplitude was small, and the counting began with 0.

l/t evidently varied so l and t were unsuitable for a graph. l/t^2 was sensibly constant.

The values of l and t^2 were therefore plotted on graph paper. The points were found to lie evenly round a thread stretched straight between them. A straight line graph was. therefore, drawn.

RESULTS

Mean value of l/t^2 = **25·03**

Length of seconds pendulum (t=2 seconds) read off opposite t^2=4 in the graph = **99·2** cm.

'g' = $4\pi^2\, l/t^2$ = **987**

% accuracy = **0·7**

Sources of error: As the thread used was quite inextensible without being unduly thick, the rate of the stop-watch was, perhaps, the chief source of error.

Rough notes and calculations by logarithms.

Log...... =

Log...... =

30·94 × 40·72

etc.

etc.

Date : 4-1-26. Home record in ink.

EXPERIMENT : **To determine the focal length of a convex lens, using the formula, $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$.**

Apparatus : Optical bench with supports for lens and two parallax needles. Also a knitting needle.

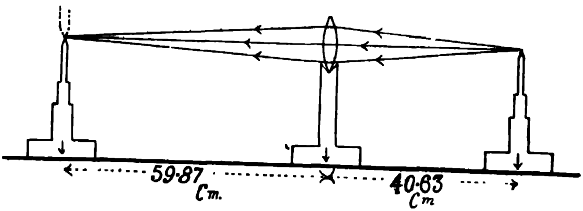
Method : A rough determination of the focal length was made. Images of distant trees were clearly visible on the note book held at about 25 cm Two needles were adjusted on opposite sides of the lens, so that the image of needle O was found to coincide with the needle I. This was done by placing the eye about 25 cm behind I, and moving I till there was no parallax between I and the image of O Two readings for each position were taken. Index correction was applied. Reciprocals were taken from tables, and four values of f found These were........., the mean value being 25·67 or 25·7 cm,

$$\therefore f = -25{\cdot}7 \text{ cm.}$$

The optician's lens meter indicated (2·0 + 1·0) dioptres (f = − 25 cm).

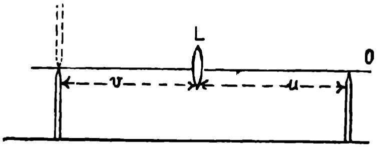
Sources of error : The images were slightly curved and parallax was absent only near the centre The uprights used were not very rigid.

Diagram :



Date : 3-1-26. *Laboratory record in pencil or ink.*

Focal length of a Convex lens



Rough determination : Image of distant trees at about 10 inches or 25 cm. Minimum bench length required 4 to 5 times this distance 2 metre bench ample

Observations : Length of index correction rod =20·00 cm.

Index correction from I to L = 20·00 − (135·05 − 114·10)
= 20·00 − 20·95 = − ·95 cm

" " " O to L = 20·00 − (114·5 − 94·55)
= 20·00 − 19·95 = + ·05 cm.

This last correction ignored as uprights a bit shaky.

u observed taken correct	*v* observed	*v* corrected	$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ (arithmetically)	*f*
1. 100·25 64·62 35·63	187·32 } 187·52 187·72 } 100·25 87·27	87·27 − ·95 86·32 = 83·32	$\frac{1}{35·63} =$ ·02807 $\frac{1}{86·32} =$ ·01159 ·03966 $= \frac{1}{25·21}$ From tables of reciprocals.	− 25·21 cm.
2.				
3.				
4.				

Mean *f* = − 25·67 cm. or **− 25·7 cm.**

Precautions : (1) The no-parallax position to be determined twice in each case.

(2) The reciprocals of *u* and *v* to be arithmetically *added*, as *f* must be less than *u* or *v*.

SUMMARY

1. A true experimenter studies **facts.** *To conceal or misrepresent facts is* **unscientific.**

2. The most important thing about an experiment is **not the final result, but the style of performing it.** *Apparently good results obtained by a careless worker are less reliable than any results based on careful work.*

3. For experimenting with success, previous **preparation** *is necessary.*

4. **Take readings** *carefully and* **without prejudice. Record your data directly in the note-book** *and wherever possible* **in tabular form.** *Loose slips of paper should never be used.*

5. Do not reject without reason observations that appear to be faulty, **nor correct any observations once recorded merely from memory.**

6, **Estimate tenths carefully.** *There are, however, certain exceptions to this rule, though they are rare.* See page 10.

7. Find out the **weak spot** *of every experiment, and give it your special attention. Other parts should also be performed with care, because accuracy in one part is, by itself, useless.*

8. Do not record data to more than **four significant figures.**

9. **Multiply** *and* **divide** *by means of four figure* **logarithms. Record result to three figures only.**

10. **Neat diagrams and graphs** *are a necessary part of all good practical work.*

CHAPTER II

PHYSICAL ARITHMETIC

THE USE OF TABLES OF LOGARITHMS AND ANTILOGARITHMS. ESTIMATING PERCENTAGES

EXERCISE 1.—**Reading four figure logarithm and antilogarithm tables.**

Requirements: Tables of logarithms and antilogarithms, note book.

Tables of logarithms and antilogarithms are series of numbers, with the help of which, multiplications and divisions can be effected by the simple and much less laborious processes of addition and subtraction. The first and second exercises are meant to train the student in the use of these tables, and the time spent on them will be repaid, many times over, later on.

Logarithms. A logarithm may, like any other number, consist of an integer followed by a decimal. There is, however, one difference. The integral part of a logarithm may be negative or positive. The decimal part is *always* positive. For instance 2·1342 and $\bar{1}$·2793 are logarithms of certain numbers, and respectively equal(+2+·1342) and (−1+·2794). $\bar{1}$ represents −1 and is called bar 1.

The integral part, whether positive or negative, is called the **characteristic** of the logarithm. The decimal part, which is *always* positive, is called the **mantissa** of the logarithm.

Every positive number has its logarithm. The characteristic of this logarithm can be obtained from a mere inspection of the number. The mantissa is read from the tables. *Negative numbers have no real logarithms.*

The characteristic depends on the magnitude of the number (number of digits), while the mantissa depends upon the actual digits composing the number. 3796, ·03796 and 37·96 have *different* characteristics, but the *same* mantissa.

To obtain the characteristic: Use the following RULE: **For numbers above 1, the characteristic is + and one less than the number of digits, and for numbers less than 1, the charecteristic is — and one more than the number of zeros after the decimal.**

Characteristic of log 3796 is 3
” ” 37·96 is 1
” ” ·03796 is $\bar{2}$
” ” ·3796 is $\bar{1}$
” ” ·3796000 is 6
” ·00003796 is 5
” ” 3·796 is 0

To obtain the mantissa: **When reading the mantissa from the tables, the position of the decimal point and the zeros at both ends of the number are ignored.** These do not affect the mantissa. For instance, the mantissa for all the numbers whose characteristics are given above is the same, and depends only on the significant figures 3796 common to all these numbers.

On examining 4-figure logarithm tables, you will find in serial order in the first column, numbers 10 to 99. Against these you find rows of four figured mantissæ (supposed to begin with a decimal, which is omitted) under the heads 0 to 9. The rows are continued with "differences" under heads from 1 to 9. To read the mantissa for 3796, we find against 37 under the head 9, 5786. For the fourth figure 6, we read under the difference head 6, the number 7, which when added to 5786 gives 5793. The mantissa, is therefore, ·5793.

The characteristic followed by the mantissa gives the complete logarithm.

Logarithm 37·96 = 1·5793

Log ·3796 = $\bar{1}$·5793

Log ·0003796 = $\bar{4}$·5793

Log 379600 = 5·5793

Antilogarithms. The number corresponding to a given logarithm is called its antilogarithm. To find such a number from the logarithm, antilogarithm tables are used. Let us read the antilogarithms of 2·5793 and $\bar{4}$·5793.

Reading Antilogarithm Tables: **The tables give only the significant figures of the required number.** The mantissa determines these. **The actual magnitude of the number is fixed in the end with the help of the characteristic.** The first column in antilogarithm tables contains mantissæ from ·00 to ·99. Against these are rows of four significant digits, under the heads 0 to 9, supplemented by " differences " under 1 to 9. For digits corresponding to our mantissa 5793, we read against ·57 under the head 9, 3793. For the fourth figure 3, we read under 3 in the difference column the number 3, which when added to 3793 gives 3796, the significant digits of the required number.

To obtain the actual number (that is to obtain the number of zeros or the position of the decimal), we use the

RULE : If **the characteristic is+, the number exceeds one and the integral part contains one digit more than the characteristic; for negative characteristics the number is a fraction, and has one less zero after the decimal than the characteristic.**

The numbers corresponding to the complete logarithms can now be found:

Antilog 2·5793 is 379·6 [Integer digits one more than+characteristic 2].

Antilog 4·5793 is ·0003796 [number of zeros after decimal one less than — characteristic 4].

Write down in your note book the following logarithms and antilogarithms:

Log 79343 (same as log 79340).
Log 23·49.
Log ·004932.
Log ·46457 (read log ·4646).
Log ·000003338.
Log 355·3
Antilog ·5346.
Antilog 2·7945
Antilog $\bar{1}$·73462 (same as antilog $\bar{1}$·7346).
Antilog $\bar{6}$·2338.
Antilog 5·67897 (same as antilog 5·679).
Antilog $\bar{4}$·3496.

Four figure tables are correct to about 1 part in 2500 which is sufficiently accurate for all ordinary experiments.

Under the usual tables, enter for permanent use the following values :

$\text{Log}\ \frac{\pi}{6} = \bar{1}\cdot7190$ $\qquad$ $\log\sqrt{2} = \cdot1505.$

$\text{Log}\ \pi = \cdot4972$ $\qquad$ $\log\sqrt{3} = \cdot2386.$

$\text{Log}\ \frac{4}{3}\pi = \cdot6221$

$\text{Log}\ 2\pi = \cdot7982.$

$\text{Log}\ 4\pi = 1\cdot0993$ ('g' Lahore) $-$ log $979\cdot4 = 2\cdot9910.$

$\text{Log}\ \pi^2 = \cdot9943$

$\text{Log}\ 4\pi^2 = 1\cdot5964$ ('g' Jammu) $-$ log $979\cdot5 = 2\cdot9910.$

EXERCISE 2.—**To multiply and divide by means of logarithm and antilogarithm tables.**

Requirements : Logarithm and antilogarithm tables, note book.

The use of logarithms for mutiplying and dividing is possible on account of the simple rules :

$$\log (a\times b)=\log a+\log b.$$

$$\log \left(\frac{a}{b}\right) =\log a-\log b.$$

$$\log a^{n} =n \log a.$$

To multiply: *Add up* the logarithms of the factors. The antilogarithm corresponding to the sum is the product.

To divide : *Subtract* from the logarithm of the numerator that of the denominator. The antilogarithm corresponding to the difference is the quotient.

To multiply with a square, add the logarithm of the number twice. The logarithm of a root $x^{\frac{1}{2}}$ or $x^{\frac{1}{3}}$ is $\frac{1}{2}$ or $\frac{1}{3}$ of log x. Care is required in doubling and halving logarithms with a negative characteristic. The following examples will illustrate the working in such cases. The operations are conducted so that *the final mantissa always remains +, integers being subtracted from or added to the characteristic as required.*

$$\text{Log } (\cdot 01234)^{\frac{1}{2}} = \frac{1}{2}(\bar{2}\cdot 0913)$$

$$= \bar{1}\cdot 0456$$

$$\text{Log } (\cdot 1234)^{\frac{1}{2}} = \frac{1}{2}(\bar{1}\cdot 0913)$$

$$= \frac{1}{2}(\bar{2}+1\cdot 0913) \quad \left\{ \text{Add}+1 \text{ to mantissa and subtract 1 from characteristic to make it } \bar{2}. \right.$$

$$= \bar{1}\cdot 5456$$

$$\text{Log}\ (\cdot 1234)^{\frac{1}{3}} = \frac{1}{3}(\bar{1}\cdot 0913)$$

$$= \frac{1}{3}(3+2\cdot 0913)$$

$$= \bar{1}\cdot 6771$$

Add+2 to mantissa and subtract 2 from characteristic to make it $\bar{3}$.

$$\text{Log}\ (\cdot 1234)^{12} = \overline{12}(1\cdot 0913)$$

$$= \overline{11}\cdot 0956$$

$\overline{12} + 12(\cdot 0913) = \overline{12}$
$+ 1\cdot 0956 = \overline{11}\cdot 0956$

$$\text{Log}\ (\cdot 1234 \times 99) = \bar{1}\cdot 0913 + 1\cdot 9956$$

$$= 1\cdot 0869$$

$$\begin{array}{r} \bar{1}\cdot 0913 \\ 1\cdot 9956 \\ \hline 1\cdot 0869 \end{array}$$

$$\text{Log}\ \left(\frac{\cdot 0001234}{99}\right) = \bar{4}\cdot 0913 - 1\cdot 9956$$

$$= \bar{6}\cdot 0957$$

$$\begin{array}{r} \bar{6}+2\cdot 0913 \\ 1\cdot 9656 \\ \hline \bar{6}\cdot 0957 \end{array}$$

Calculate :

$$1000 \times \frac{2\pi \times 71\cdot 43}{(3\cdot 9)^2}.$$

$$\begin{array}{lr} \text{Log}\ 1000 = & 3\cdot 0000 \\ \text{Log}\ 2\pi\ = & \cdot 7982 \\ \text{Log}\ 71\cdot 43 = & 1\cdot 8539 \\ \hline & 5\cdot 6521 \\ \text{Log}\ 3\cdot 9\ = & \cdot 5911 \\ \hline & 5\cdot 0610 \\ \text{Log}\ 3\cdot 9\ = & \cdot 5911 \\ \hline & 4\cdot 4699 \end{array}$$

(read directly from the special entry against 2π)

Antilog $4\cdot 4699 = 29500$ (correct to 4 figures only).

$53 \times 34.$

log 53 =
log 34 =

53×34=antilog......................
= ....

$4294 \times 6666.$

log 4294 = ...
log 6666 = ...

4294×6666=antilog......................
= ....

$$(72)^4 \times ·00123.$$

$$4\pi^2 \frac{99}{(2·01)^2}.$$

$$\frac{4}{3}\pi\,(52·9)^{\frac{1}{3}}.$$

$$\frac{(·0029)^{\frac{1}{4}} \times 31·2 \times ·134}{(·000004)^2 \times 73·5}.$$

EXERCISE 3.—**Estimating accuracy in percentage, and locating defects and weak spots in apparatus and method.**

Write down answers to the following in your note book:

(*To judge accuracy, divide total error by quantity measured. To obtain percentage accuracy, multiply the quotient by 100.*)

1. Which is more accurately known
   (1) The population of a town stated as between 33500 and 33600 (uncertainty 100), or
   (2) The number of children in a family stated as either 8 or 9, (uncertainty 1)?
2. Which is more accurately known :
   (1) The capital of a bank=Rs. 37·6 lakhs (Rs. 5,000 or less ignored) or

(2) The income of a firm, between Rs. 600 and 650 per month ?

3. Which is more accurately measured in the same experiment:

(1) The length of a wire stated as 256·3 cm. (correct to the nearest millimetre), or

(2) The diameter of the same wire 3·52 mm. or ·352 cm. ?

For greater accuracy in determining the volume of the wire, do you advise measuring the length correct to ·1 mm. ?

4 The cost of carpeting a room, at the rate of Rs. 6-4 per square metre, has to be estimated correct to half a rupee. The length and breadth are measured as 579·34 and 352·16 cm., and the area carefully determined. How would *you* proceed under the circumstances ?

5. When asked to determine the sp. gr. of glass, a student weighs in air and water a very small bit of glass rod (weights 1·12 and ·81 gram) with average care. The result is much too high. Why ?

SUMMARY

The **accuracy** *of a result cannot be estimated by simply determining the* **difference** *between it and the correct result. It is determined by* **the ratio of this difference to the correct result.** *This ratio should be expressed as a* **percentage.**

Example: *The number of students in a class of 50 is miscounted as 48. The accuracy is* **2 in 50,** *that is,* $\frac{2}{50} \times 100$ or **four per cent.**

PART II

SIMPLE MEASUREMENTS
MECHANICS
HYDROSTATICS

CHAPTER III

MEASUREMENT OF LENGTH AND ANGLES

METRE STICK. BEAM COMPASS. CALLIPERS
DEPTH GAUGE. HOLE GAUGE

The Metre Stick. Lengths and distances are measured by comparison with standard lengths, marked on strips of wood or steel, or linen or steel tape. The commonest of these appliances is the **metre stick,** consisting of a boxwood rule, 1 metre long, divided and sub-divided into centimetres and millimetres. To measure the distance between two points A and B on a flat surface, the metre stick is **placed edge on** along AB so that, while A lies exactly on a cm. mark, say 1 cm. (not 0, because the edge of the stick is always worn), B falls either exactly under some division or between two divisions. In the latter case, the fraction of a millimetre necessary for exact measurement is estimated in tenths, and the distance AB (which may be supposed 25 cm. and 3·7 mm.), represented as 25·37 cm. Had the length been 25 cm. and 3 mm. exactly, it would have been put down as 25·30 cm. (not 25·3 cm., because this would mean that fractions of a millimetre were neglected). **If AB happens to be exactly 25 cm., the length should be put down as 25·00 cm.,** the two zeros signifying that the measurement is supposed correct within a tenth of a millimetre.

The metre stick should be placed edge on with the divisions vertical and not flat, as in the latter case, on account of the thickness of the stick, the points A and B are not close to the division lines, and the reading will depend on how the eye is placed with regard to the stick. If the eye is not exactly above the points A and B, an error, called **the error of parallax,** will be caused. This error is absent if the stick is very thin, or if it is placed *edge on.*

The Dividers and the Beam Compass. The shortest distance between two points A and B, situated on a curved or uneven surface, cannot be measured by a metre stick, on account of the difficulty of applying the straight edge of the stick to the points. The sharp points of a **pair of dividers** can, however, be placed on A and B (Fig. 6), and the distance AB determined by transferring the dividers to a metre stick. For larger distances, a long rod with 2 pointed legs, one or both of which can slide along the rod, and clamped anywhere by set screws, is used. This is called a **beam compass,** and is used like a pair of dividers (see Fig. 6).

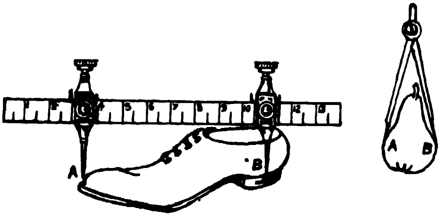
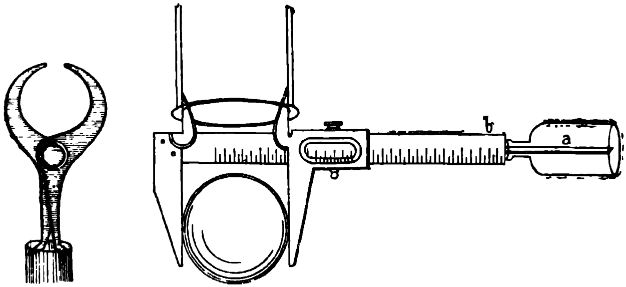


Fig. 6.—Dividers and Beam Compass.

Inside and Outside Callipers. Two hinged arms suitably curved and pointed (Fig. 7) can be used for determining diameters of balls and cylinders, or for measuring the inside and outside diameters of tubes, gas jars, etc. The distances are either read off on curved scales provided, or by transferring to a metre stick. The sliding or vernier callipers (Fig. 7) is also suited for measuring inside and outside diameters.

Depth Gauge and Hole Gauge. A **depth gauge** is used for determining the depth of vessels with curved necks, etc. The vernier callipers is often provided with a depth gauge attachment (Fig. 7). A thin metal strip *a* moves in a groove in the strip *b* so that when the end of *a* is just inside the groove the arrow or vernier 0 on the top reads zero. The edge of the strip *b* rests on the neck of the vessel, while the central strip *a* is pushed till it touches the bottom. The reading of the arrow in this position gives the depth.



(*a*) (*b*)

Fig. 7.—Callipers inside and outside. (*a*) Plain, (*b*) with vernier and depth gauge attachment.

The diameter of small sized tubing can be measured with a **hole gauge.** This is a steel wedge, whose width at any point is one-tenth of the distance of that point from the apex. In figure 8, the distance of A from the apex is 3·90 cm. If, on gently pushing the hole gauge into a piece of glass tubing, the wedge goes in up to the line AB, the diameter of the tube is equal to the width of the wedge at AB, which is one-tenth of 3·90 cm., or 3·90 millimetres.

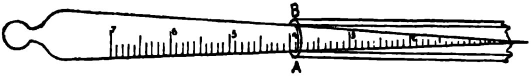


Fig. 8.—Hole Gauge.

EXPERIMENT 4.—**To learn the use of** (*i*) **Metre stick, dividers and beam compass.** (*ii*) **Inside and outside callipers (plain).** (*iii*) **Depth and hole gauges.**

Apparatus : The instruments named above, spheres, cylinders, glass jars, glass tubing, etc. For description and complete directions for the use of the instruments named, read pages 31—33.

(1) **Metre stick:** Measure distance AB between 2 points A and B on a sheet of paper on a drawing board. Lay stick edge on, and estimate tenths of mm. Using different portions of the stick each time take two or three readings. Enter as shown:

Length AB : (1) ... (2) ... (3) ... Mean : ...

(2) *By means of* **dividers** *and* **beam compass,** measure the shortest distance between two points A and B on a large wooden ball.

(3) **Outside and inside callipers:** Adjust the gap between the jaws, so that the gas jar or other cylinder or ball just passes between the jaws. Transfer to a metre stick and measure. Measure three times. Enter:

Diameter : (1) ... (2) ... (3) ... Mean :

(4) Introduce the " **inside** " points, into the gas jar. Adjust gap to fit inside of jar. Transfer to a metre stick, and measure Repeat twice in different directions. Enter :

Inside diameter: (1) ... (2) ... (3) ... Mean :

(5) **Depth gauge** : Rest edge of *b* on neck of flask, and push strip *a* till its point just touches the bottom of the flask. Read depth on scale.

(6) **Hole gauge** : Push the gauge *gently* into glass tubing (Fig. 8.) Read the gauge where it *easily* fits the tubing. *Do not press hard.* Measure in different directions. Enter :

Diameter of glass tubing : (1) ... (2) ... (3) ... Mean :

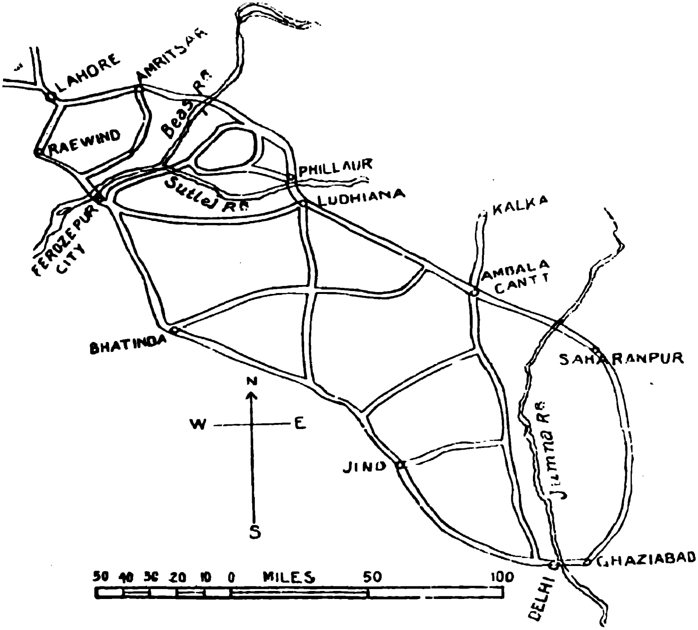


Fig. 11. Duplicate. Detach for Expt. 5.

EXPERIMENT 5.—**To measure the length of curved lines, and to determine distances on a map drawn to scale.**

Apparatus : Thread, pair of dividers, metre rod.

Thread method.

The length of a curved line, say the circumference of a circle, can be measured by applying successive short lengths of a thread (Fig. 9) to small portions of the circumference, and determining the total length of thread required to go once round. The starting point may be marked by a knot on the thread, and the final pont by a pencil mark.

Measure the length of the circumference of a circle of 4 cm. radius, drawn on your note book.

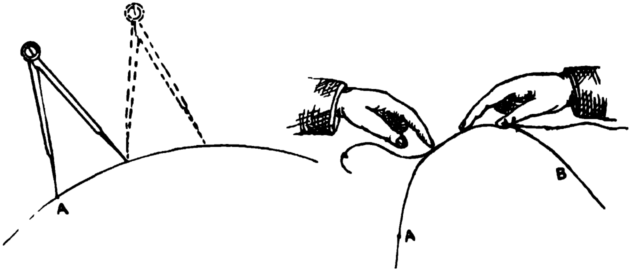


Fig. 9 —Measuring length of curved lines

Dividers method.

The O^{ce} can also be measured with a pair of dividers. Adjust the divider points to a distance of 3 to 4 millimetres, and starting at any point, go round the circumference, counting the number of times the interval between the points is contained in it. Estimate the fraction at the end, and calculate the circumference.

The use of a map : Determine from the map supplied (Fig. 11):

(1) The mileage and time of flight (at 100 miles an hour), for an aeroplane from Delhi to

Lahore (straight line distance).

(2) The same distance by rail, *via* Ambala.

(3) The same distance by rail, *via* Bhatinda.

(4) The same distance by rail, *via* Ghaziabad.

(5) The distance, by boat along the Sutlej, from Phillaur to Ferozepore City.

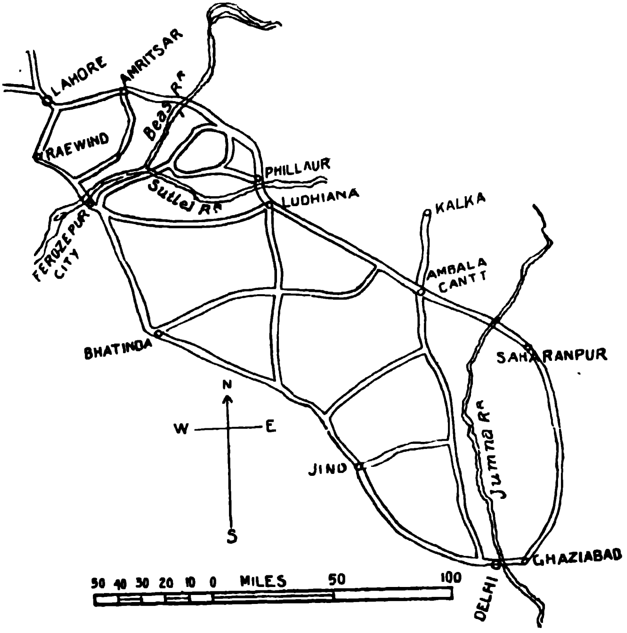


Fig. 10.

Detach from the book the duplicate map supplied.

(1) Draw, and bisect approximately, the straight line from Delhi to Lahore. Transfer, by dividers, each half to the scale below the map, and read the mileage. The sum is the total mileage. [270 miles, 2·7 hours].

(2), (3), (4), (5). Adjust divider points to 10 or 20 miles as represented on the map scale, and determine the number of times this is contained in the journey along the prescribed route. (Estimate the fraction at the end) [310, 297, 348, and 79 miles.]

Gum the map, with distances indicated, into your note book.

EXPERIMENT 6.—**To determine the diameter and circumference of a disc or cylinder, and to deduce the value of π.**

Apparatus: Two or three wooden discs, 1 to 2 cm. thick and varying in diameter from about 5 cm. to 15 cm. Coins Cylinders, wood or metal (pencils). Straight slips of *thin* paper. Thread. Metre rod.

Wooden discs: Wrap a paper slip tightly round the edge of the disc, allowing the ends to overlap. Prick a pin through the overlapping portion; unwrap, and lay the slip flat on the table. Mark the 2 pin pricks by small circles round them, and laying the metre rod edgewise on the holes, determine the distance between them up to ·1 mm. This gives the circumference. Repeat with the same or a different slip, so as to obtain 3 readings. Similarly, measure the other discs.

Q. Is it difficult to measure the circumference with accuracy? The pinpricks are small and the paper quite thin. What is the *percentage error* caused by an error of ·5 mm. in measuring the circumference (which may be 20 to 50 cm.)?

To measure the diameter, find the centre of the disc. This may be done by trial with a pair of dividers. Place the metre rod edgewise across the circular face, so that the 10 cm. line lies exactly on the circumference. Move the rod about so that it passes through the centre of the disc. *With the stick passing through the centre, and one point of the circumference exactly below the 10 cm. line*, read the position of the opposite point of the circumference. The difference between the readings gives the diameter. Similarly, take a reading along the perpendicular diameter.

Q. Which is easier to measure with *accuracy*, the diameter or the circumference? In which measurement is the error likely to be greater ? The diameter is also smaller than the circumference. In which measurement will the *percentage error* be greater? Supposing the disc is not quite circular, how will you proceed? Which is the *weak spot* of this experiment?

Do you think it necessary to obtain a larger number ofreadings for the diameter than for the circumference ?

Take two more readings about 45° away from the first diameter. Similarly measure four diameters on the other face of the disc.

Coin : With a penknife, make a radial scratch near the edge. Place the coin vertical with the scratch on the 10 cm. mark, the metre stick being placed flat. Roll without slipping. Read when the scratch is again on the rod. Obtain the circumference by subtraction. Repeat twice.

Pencils : Wrap round the pencil 10 to 15 turns of fine inextensible thread, taking care to see that the turns are quite close to one another. Draw a pencil line on the turns in a direction parallel to the axis. Unwrap the thread and measure the distance between the 0th and the 10th or 15 mark. An accurate value of π cannot be obtained with pencils. Why ?

Enter results as shown in the table on page 39.

DETERMINATION OF THE RATIO OF CIRCUMFERENCE TO DIAMETER.

CIRCUMFERENCE OF DISC	MEAN CIRCUMFERENCE (CM.)	DIAMETER OF DISC	MEAN DIAMETER (CM.)	$\frac{\text{Circumference}}{\text{Diameter}} = \pi$
Disc No. 1. 1 ... 2 ... $\frac{\text{sum}}{3}$ = 3 ...	46·20	1 ... 5 ... 2 ... 6 ... 3 ... 7 ... 4 ... 8 ... ... + ... $\frac{\text{sum}}{8}$ =	14·75	$\frac{46\cdot20}{14\cdot75}$ Log 46·20 = 1·6646 Log 14·75 = 1·1688 Antilog ·4958 = 3·132
Disc No. 2. 1 ... 2 ... $\frac{\text{sum}}{3}$ = 3 ...		1 ... 5 ... 2 ... 6 ... 3 ... 7 ... 4 ... 8 ... ... + ... $\frac{\text{sum}}{8}$ =	...	... — ... Log ... Log ... Antilog ... = 3·124
Pencils, coins, etc.				

Record mean π correct to *three figures only*

CHAPTER III—(*contd.*)

THE PRINCIPLE AND USE OF THE VERNIER

For reading tenths of millimetres with certainty, as also for reading small fractions of any small divisions, the **vernier** is employed. As even degree students often find difficulty with verniers, if they have not properly studied the principle in the first instance, it is important that the essentials of a vernier be carefully grasped at this stage.

The vernier is a small auxiliary scale sliding along the principal scale. The smallest divisions on the vernier scale are slightly **smaller** or **larger** (usually smaller) than the smallest divisions of the principal scale. The small difference between a vernier division and a scale division is called the **vernier constant.**

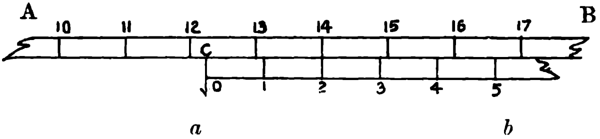
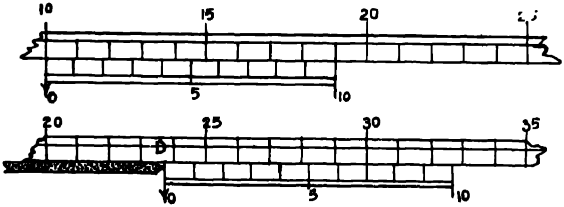


Fig. 12—The theory of the vernier.

Imagine A B (Fig. 12), to be a principal scale and *ab* a vernier scale. Let each vernier division be less than one scale division by an amount x. x, therefore, is the vernier constant. Now suppose the reading of any point c, between two lines on the principal scale is required. The reading is 12 + the fraction between 12 and c. Slide the vernier till its 0 coincides with c. Now look for the vernier line that coincides with a scale division. In the diagram the vernier line marked 2 coincides with a scale line (*which* scale line has nothing to do with the reading). Compare the two vernier divisions between 0 and 2 with the 2 scale division 12—14. Their difference is twice the difference between one vernier division and one scale division, or twice the vernier constant, that is $2x$. But as 2 and 14 coincide, the fraction between 12 and c must be this $2x$. Hence the reading of c is $12+2x$. Had the 4th division of the vernier coincided with a scale division, the reading would have been $12+4x$.

In the common vernier (as in vernier callipers), the vernier constant is ·1 mm., or ·1 of the smallest scale division. There are 10 divisions on the vernier and they equal 9 mm. as shown (Fig. 13). Each vernier division being ·9 mm. is less than a mm. by ·1 mm. ·1 mm. is, therefore, the vernier constant.



OTHER END OF NEEDLE AT ZERO. LENGTH : 23·7 DIV.

Fig. 13.—The common or ten to nine vernier.

Imagine a body with one end on the zero of the principal scale, and the other end at D. The length of the body lies between 23 and 24 mm. Slide the vernier till its 0 coincides with D as shown. As vernier division 7 coincides with a scale line, the difference between 7 vernier divisions from 0 to 7 and 7 scale divisions 23—30 is 7 times ·1 or ·7, and equals the fraction between 23 and D. Hence the length of the body is 23·7 mm. The rules for reading any vernier, may therefore be stated as follows:

Determine the value of 1 *v. d.* in terms of 1 *s.d.* The difference between a *v. d.* and a *s. d.* gives the vernier constant. Read the lower of the two scale divisions between which the point in question lies. Bring the zero of the vernier against this point, and notice which vernier line (*n*th) coincides with a scale line. The reading is given by adding to the scale division already noted, the product of the vernier constant and the number of the vernier division (n) coinciding with a scale line. Or

Total length=scale reading + n × vernier constant where n is the number of the vernier line coinciding with a scale line.

In a barometer vernier (Fig. 14) each inch is divided into 20 parts. On placing the 0 of the vernier opposite a prominent division, say the 29 inches mark, we notice that the 25 divisions of the vernier just equal 24 of the scale.

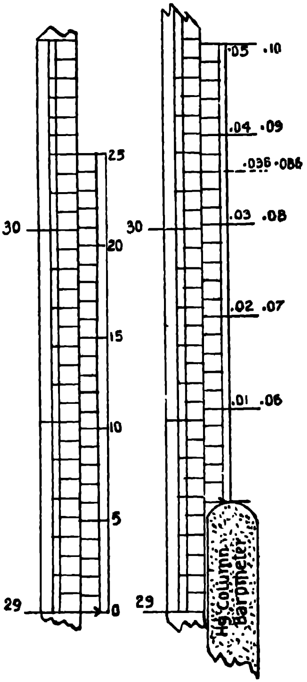


Fig. 14.—The barometer vernier.

Therefore, $1\ v.\ d. = \frac{24}{25}\ s.\ d.$

$$\therefore \text{ Vernier constant} = 1 - \frac{24}{25}\ s.d. = \frac{1}{25}\ s.\ d.$$

$$= \frac{1}{25} \times \frac{1}{20} \text{ inch} = \frac{1}{500} \text{ inch}$$

$$= {\cdot}002 \text{ inch.}$$

The mercury column in Fig. 14 lies between 29·25 and 29·3. The scale reading is, therefore, 29·25 in. To calculate the fraction between 29·25 and the mercury surface, we notice that the 18th line of the vernier coincides with a scale line. The fraction is, therefore, 18 times the vernier constant, or 18 × ·002 inch = ·036 inch. This, added to 29·25, gives 29·25 + ·036 or 29·286 as the reading of the mercury surface.

When, as in this case, the number of divisions on the vernier scale is large, the lines are very often not numbered as 0, 1, 2, 3, but have the fractions corresponding to the prominent lines directly marked on them, so that the fraction can be at once read off and added to the scale reading, without having to multiply the number of the lines with the vernier constant The above barometer vernier would, on this plan, be marked as shown in Fig 15. 5 lines being equal to 5 × ·002 or ·01, the graduations increase by ·01 for each 5 divisions. The fraction opposite the 18th line is ·036.

A second row of figures sometimes provides an extra convenience by making the addition of scale reading and vernier reading practically automatic. This row is to be read in case there is a 5 in the second decimal place in the scale reading (as in case of E in the above illustration) Against the 18th line we will now read 086, which is ·05 + ·036. The scale reading, therefore, need go up to the 1st place only. The second and third places are read off directly from the vernier, the upper or lower row being used according as the point in question is between 29·2 and 29·25 or between 29·25 and 29·30 (that is, the 5 in the 2nd place need not be read The scale reading for the above case would be now 29 2, and the fraction read off, ·086, the total reading being 29·286 as before. Verniers on kathetometers and travelling microscopes are often so graduated

Another common vernier is one in which 10 divisions equal 19 mm. Students, instinctively, regard the 10 divisions as double and made up of 20 imaginary divisions equalling 19 of the scale. The vernier constant for this imaginary vernier of 20 lines is $\frac{1}{20}$ mm., but as only 10 double lines are marked, the actual vernier constant is 2 × $\frac{1}{20}$ mm., or ·1 mm., as in the common vernier. But it is better to apply the original vernier principle in calculating the constant. Each *v.d.* equals 1·9 mm. and is short of the nearest exact number of millimetres (2) by ·1 mm. which, therefore, is the vernier constant. If 10 divisions had equalled 29 of the scale, the vernier constant would still be ·1 mm., because all that is necessary is, that a *v. d.* should be *just a little less* (or more in an unusual type of vernier which reads backwards), *than* 1, 2 *or any integral number of scale divisions.* The only advantage of the 10 to 19 vernier is the larger size of the vernier divisions, which can be more easily read than in the 9 to 10 vernier.

The vernier principle is also applied to the measurement of angles. A spectrometer may be provided with a principal scale of half degrees, and a vernier containing 30 divisions, equalling 29 of the scale. The constant is $\frac{1}{30}$ of $\frac{1^\circ}{2}$ or $\frac{1^\circ}{60}$, which is $1'$. The angle can therefore be read up to $1'$.

EXPERIMENT 7.—**To construct a vernier, and to use it for measuring the length of a sewing needle.**

Apparatus :—White stiff card or thick paper. Pair of scissors. Drawing materials.

Cut off for the principal scale, out of white stiff card, a slip 12 cm.×5 cm. (Fig. 15). Draw lines *a b* and *c d* as shown. Lay off along *a b* with dividers, a simple scale of equal divisions, each equal to about 5 mm. Number as shown. For the vernier cut off a slip 3 cm.×5 cm. and lay off along one edge a length equal to 7 scale divisions. Divide this length into 8 equal parts.

Fold the principal scale card along *cd* to form a ledge, in which the vernier can slide with its divisions close to the divisions of the scale.

The finished vernier and scale are shown in Fig. 15.

Put the 0 line of the vernier opposite 4 of the scale, 8 of the vernier will now lie opposite 11 of the principal scale. No vernier line between 0 and 8 will be found to coincide with any scale line. Slide the vernier a little, so that division 1 on it coincides with division 5 of the principal scale. There will be now, between the 0 of the vernier and 4 of the scale, a small length=$\frac{1}{8}$ of a scale division, because a vernier division is $\frac{7}{8}$th of a *s. d.* The vernier constant of this vernier is $\frac{1}{8}$th of a scale division. Slide the vernier in the same direction a little more, so that division 2 now coincides with 6 of the scale. There will be now, between 4 of the scale and 0 of the vernier, a bigger length just $\frac{2}{8}$ths of a *s. d.* You can, in this

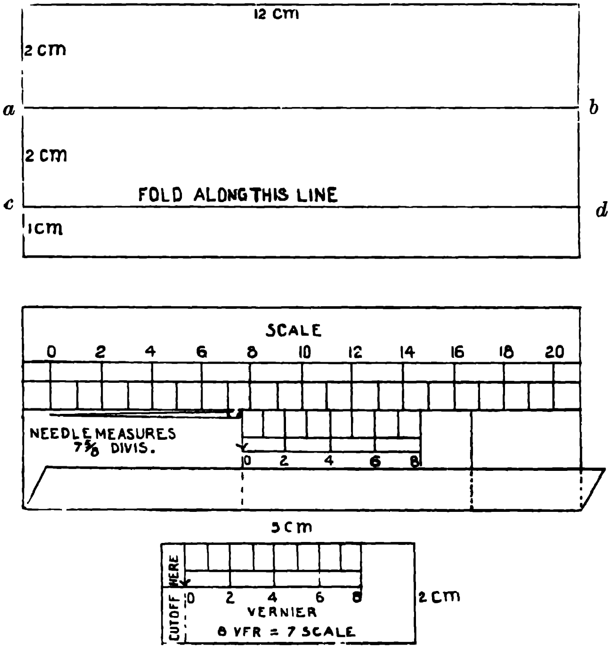


Fig. 15.—A seven to eight cardbord vernier by student.

way, allow between 4 of the scale and 0 of the vernier, fractions equal to 3/8, 4/8, 5/8, 6/8 or 7/8 of *s. d*, by sliding the vernier along, bit by bit, and allowing divisions 3, 5, 6, 7 of the vernier to coincide with the nearest scale divisions. The fraction equals $\frac{1}{8}$ (vernier constant) × the number of the vernier line coinciding with a scale line (*which* scale line is *not* to be read).

To measure the length of a sewing needle, place the needle along the principal scale, with its point opposite the 0. Then slide the vernier, till its 0 just touches the eye end of the needle. The length of the needle is, say, 7 divisions + the fraction between 7 of the scale and 0 of the vernier. Examine the vernier to see which of its divisions coincides with a scale division. Suppose this division is the 5th.

The length of the needle is then 7 $\frac{5}{8}$ divisions.

Write the vernier constant, and the final reading, for the following cases :

Principal scale	Vernier scale	Vernier constant	Divisions principal scale between which point lies	No. of v. line coinciding	Final reading.
$\frac{1}{16}$ inch ...	8 v. = 7 scale		$2\frac{3}{4}$ and $2\frac{13}{16}$ inch.	5	
$\frac{1}{2}$ millimetre ...	25 v. = 24 „		1·95 and 2 cm.	13	
Millimetres ...	10 v. = 19. „		7 and 8 mm.	7	
$\frac{1^\circ}{3}$ ...	40 v. = 39 „		15°.20′ and 15°.40′.	23	
$\frac{1^\circ}{6}$ ...	60 v. = 119 „		31°.40′ and 31°.50′	47	

CHAPTER III—(*contd.*)

THE VERNIER CALLIPERS

EXPERIMENT 8.—**To measure diameters and lengths of cylinders, spheres, etc., with vernier callipers**

Apparatus :—Metal balls, cylinders, and vernier callipers.

To determine the diameter :—The vernier callipers (See Fig. 7) consists of a mm. scale (make sure you are not using the inch side) with a 9 to 10 vernier sliding along it. The vernier and the scale carry plane faced perpendicular jaws, which just close up when the zero of the vernier is opposite the zero of the scale. The reading of the vernier equals, therefore, the width of the gap between the jaws. If the vernier does not read zero with the jaws in contact, take the zero reading. If the vernier reading is +, there is no difficulty. Sometimes, however, the zero of the vernier is below the zero of the scale, say between zero and an *imaginary* graduation −1 mm. below the zero. Suppose in this case that the 8th vernier line coincides with a scale line. The fraction between the *imaginary* graduation —1 mm. of scale and 0 of vernier is ·8 mm. Therefore the distance between the 0 of scale and the 0 of vernier is ·2 mm. This zero reading is evidently negative and is put down as —·2 mm.

Now open the jaws, introduce a ball in the gap and slide the vernier till the ball is just held *without undue pressure*. See that the ball is held along a diameter. Read scale and vernier. Repeat with different directions.

[The radius or diameter has very often to be cubed for calculating the volume of the sphere. How many readings of the diameter do you consequently think necessary? See page 8].

Repeat with the other balls. The smaller the ball, the greater the care necessary. Measure lengths of cylinders similarly. Rotate the cylinder between readings so as to measure the length at different points. Similarly, when measuring the diameter, try different directions to allow for ellipticity of section. 6 to 8 readings of diameters and 3 to 4 readings of lengths will be necessary.

To correct the observed readings for zero :

1. **Change the sign of the zero reading.** This gives the **zero correction.**

2. **Write the observed reading with its proper sign, and after it merely write down the zero correction with its proper sign. No further discussion of signs is necessary. Simplify to obtain the corrected reading.**

In other words, **to get the corrected reading always ADD ALGEBRAICALLY the zero correction WITH ITS PROPER SIGN to the uncorrected reading WITH ITS PROPER SIGN.**

Correction for zero reading :

To correct a reading for the zero should be a simple matter, Difficulty is, however, always experienced by beginners in deciding whether to *add* or *subtract* the zero reading. The indiscriminate use of the three terms, reading, error and correction, is responsible for the doubt and difficulty which give rise to endless questions and answers and often lead to complete failure, in spite of all trouble taken. The following simple rules, if strictly adhered to from the very beginning, will save much anxiety, and ensure confidence at all times :

1. Never use the term, nor calculate, the zero error, as the word error has a doubtful significance as far as the sign is concerned. Use only the terms **zero reading** and **zero correction,** and crorect as directed.

2. **Zero reading** is the actual reading with the proper sign, of an instrument (callipers, screw gauge, spherometer, thermometer), under conditions for which it is designed to read zero. The zero reading for a centigrade thermometer indicating 1° below zero in pure melting ice, is -1°C.

3. **Zero correction** is the name given to the quantity, with its proper sign, which, if *algebraically* **added** to an **observed reading,** will give the **corrected reading.** As thus defined, it equals the zero reading with the sign reversed.

Study carefully the following illustrations:

1. Zero reading of callipers: - ·2 mm. Zero correction = + ·2mm. = + ·02 cm. Observed diameter + 2·57 cm. Corrected diameter = +2·57 + ·02 cm., = + 2·59 cm.

2. Zero reading of thermometer - 1°C. Zero correction is + 1°C. Observed melting point of wax = + 53°C Corrected melting point = + 53°C. + 1°C = + 54°C. Observed temperature of a freezing mixture = - 20°C. Corrected reading = - 20°C + 1°C = - 19°C.

3. The zero end of a metre stick is worn ·4 mm. or ·04 cm This is the zero reading; zero correction = - ·04 cm.

If this stick is held vertical with the zero end on the table, the height above the table of a point object reading 24·79 cm. is 24·79 - ·04 or 24·75 cm.

Record observations as shown on the next page.

***Observations* :**

Value of smallest scale division = 1 mm.
10 vernier divisions = 9 mm.
I. V. D. = ·9 mm.
∴ Vernier constant = 1 − ·9 mm.
= ·1 mm.
= ·01 cm.
Zero reading = (*i*) ... (*ii*) ...
= (*iii*) ... Mean = + ·02 cm.
Zero correction = − ·02 cm.

	Zero of vernier lies between	Vernier division coinciding with a scale division	Total reading	Mean reading	Corrected mean reading
Diameter of sphere	1·9 cm. and 2·0 cm.	6th	1·9 cm. + ·01 × 6 cm = 1·9 + ·06 or 1·96 cm.	Mean of 8 readings = $\frac{\ldots}{8}$ = 1·964 cm.	1·964 cm. − ·02 cm. = 1·944 cm. say 1·94 cm. retaining 3 figures.
	,,	7th	1·97 cm.		
	,,	5th	1·95 cm.		
Length of cylinder	3·0 cm. and 3·1 cm.	4th	3·04 cm.	Mean of 6 readings = 3·05 cm.	3·05 cm. − ·02 cm. = 3·03 cm.
		5th	3·05 cm.		
	,,				

CHAPTER III—(*contd.*)

THE SCREW GAUGE

EXPERIMENT 9.—**To measure the diameter of wires, and thickness of glass slips by means of the screw gauge.**

Apparatus: Copper wire No. 18 used for electric light wiring or other size, pieces of window glass or microscope slides, thin cover glasses, screw gauge in holder.

The screw gauge (Fig. 16) consists of a screw working in a long cylindrical nut, attached to a semicircular or rectangular metal frame. The screw advances along the nut so as to close the gap between itself and a stud fixed at the opposite end of the frame. The gap is closed when the flat end of the screw just touches the flat end of this stud. On the nut is an axial line, along which a scale of divisions (the pitch scale), each equal to the pitch of the screw, is marked. There is a circular scale on the sleeve or cap that rotates with the screw. The screw is turned by the milled head which, in the better instruments, is not rigidly fixed to the end of the screw, but slips on it with a little friction. Ordinarily, on turning the head, the screw will advance; but if the gap is closed, further rotation of the head will cause it to slip on the screw stem, and save the screw threads from undue pressure.

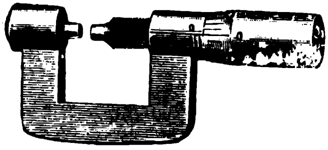


Fig. 16.—Screw gauge.

The pitch of the screw is the distance between the successive threads. This is the distance the screw moves forward in one complete revolution. The pitch is usually 1 or ·5 mm. The sleeve is divided into 100 or 50 parts.

Determine whether *one* or *two* complete revolutions of the screw are necessary to cover or uncover 1 mm. of the pitch scale. The circular scale serves to measure fractions of a turn. Determine the **least count** of the instrument, or the *distance advanced by the screw when the head is turned through one division of the circular scale.* If the graduations on cap give merely the number of divisions as (0,5......90,95,100), it is best to take all screw gauge readings in mm. and *divisions*, and to convert these to centimetres *only when proceeding to calculate*, or *when writing down the final result. If, however, the fractions of a millimetre corresponding to the divisions are directly given (as, ·1, ·2, ·3, etc.) on the cap it is useless to read in divisions.* In this case *the fractions of a millimetre should be read directly,* after the student has once satisfied himself by actual counting that the fractions are correctly marked.

In the ideal condition, the zero of the circular scale, when the gap is just closed, lies on the axial line along which the pitch scale is marked. If this is not so, the zero reading in divisions, with the proper sign, ought to be taken and all subsequent readings corrected for zero, according to rules given in connection with vernier callipers.

To measure the diameter of a wire, introduce the wire into the gap. Screw forward till the wire is *lightly* held. Avoid *undue* pressure. Read the pitch scale in mms. and the circular scale in divisions. Measure the diameter in a direction perpendicular to the first one at the same point. This is to allow for slight ellipticity of cross-section. Similarly, take pairs of readings at points 5 to 10 cm apart, according to the length of the wire, so as to obtain in all about 12 readings. The mean of these, converted to mm., and corrected for zero, gives the corrected diameter.

Enter results thus:

Pitch of screw = ·5 mm. } 2 revolutions cover a mm.
No. of divisions on circular scale = 100

Least count of instrument = $\frac{5}{100}$ = ·005 mm.

Zero reading : +2·1, +2·2, +2·2, +2·1 div ; mean : +2·15 div.

No of complete revolutions 0—0	Observed readings circular scale *divisions*	Zero correction *divisions*	Mean corrected reading of circular scale	Diameter of wire
2 revolutions =2 × ·5 mm. =1 mm.	43·1 43·3 43·9 43·8 43·0 43·6 44·1 44·0 43·8 44·0 44·2 44·0 Mean $= \frac{\quad}{12}$ = 43·65	−2·15	43·65 −2·15 =41·50 div. =41·50 × ·005 mm. = 2075 mm.	1·2075 or 1·208 mm. = 1208 cm.

The use of four significant digits in the result is justified, as the instrument is capable of such accuracy.

Notice the following:

(1) As the circular scale did not give fractions of a mm. directly, **all fractions (including the zero reading) have been taken in divisions.**

(2) *Every observed reading has not been corrected*

for zero or converted to millimetres. This would have required twelve additions and twelve multiplications. **The zero correction should be applied only to the mean observed reading, and this corrected mean alone should be converted to millimetres.**

The thickness of a glass slide or cover glass can be obtained by reading the screw gauge with the slide or cover glass just held between the screw and the stud at different points in turn, and adopting the mean value as the thickness required.

CHAPTER III—(*contd.*)

THE SPHEROMETER.

EXPERIMENT 10.—**To measure with a spherometer the thickness of a glass slip, tin plate, or microscope cover glass.**

Apparatus: Spherometer. piece of plate glass, slips, cover glasses.

The spherometer (Fig. 17) is used for measuring small heights and thicknesses. It consists of a screw of 1 or ·5 mm. pitch, working in a nut at the centre of a small metal tripod of which the screw forms the fourth central leg. The screw, as well as the three legs, which are equidistant, are pointed at the bottom. On the top of the screw is a circular disc often divided into 100 parts. Above one of the legs, is fixed a vertical scale divided into millimetres and half millimetres. This is the pitch scale. The sharp edge of the disc, or a circular line on the edge if it is broad, passes close to this vertical scale. The disc scale is read by noting which of its divisions is opposite the vertical or pitch scale. It will be noticed that the instrument is in all essentials, a screw gauge, with this difference that the screw is vertically arranged, and a tripod replaces the semicircular frame.

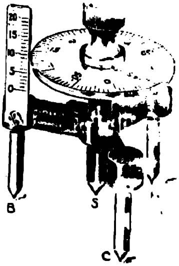


Fig. 17—Spherometer.

The instrument is so designed that when the three legs and the screw-point simultaneously touch a plane surface (that is, are in the same plane), the zero of the circular scale is exactly opposite the zero of the vertical scale. In practice this is rarely the case. A zero correction has, therefore, usually, to be applied. In the method of using the spherometer described below (page 57) the zero correction is applied automatically and *need not be determined separately.*

The theory of the instrument. The theory underlying the use of the instrument is very simple. Let us suppose that the screw has a pitch of .5 mm., and that the disc is divided into 100 parts. Further imagine that no zero correction is necessary. In this case if the instrument is adjusted on a piece of perfectly plane plate glass so that the screw and the three legs all touch it at the same time, the zero of the circular scale will be exactly opposite the zero of the vertical scale. Now suppose we are required to measure the thickness of a small coin. Place the coin under the screw point, the legs remaining on plate glass. Only *two* of the legs will touch the plate glass. In order that the third leg may also touch, the screw will have to be *raised* by turning the disc. If, for this purpose, the disc has to be turned through exactly two whole revolutions, the zero of the disc scale will be exactly opposite the 1 mm. mark on the vertical scale. The vertical scale reading 1 mm. evidently gives the thickness of the coin. If the disc has to be turned a little more, say, in all through two whole revolutions *plus half a revolution*, the division 50 will be *opposite* but somewhat *above* the 1 mm. mark on the vertical scale. The thickness is, in this case, 1 mm. *plus* the amount corresponding to 50 divisions of the circular scale which is one quarter of a millimetre. The thickness is, therefore, 1.25 mm.

Theoretically, the vertical scale reading *plus* the circular scale reading, should, after the zero correction is applied, give the required thickness in every case. A serious difficulty, however, arises in actual practice, when the vertical scale is being read. This reading is uncertain for two reasons: (1) The pitch of the screw is often .5 mm., while the vertical scale is one of whole millimetres, the mm. lines corresponding to 2, 4, 6 .... complete revolutions. For 1, 3, 5, ... revolutions the edge of the disc is opposite the *middle* of a mm. division, and the vertical scale readings are .5, 1.5, 2.5 mm., etc. To know whether the fraction corresponding to the circular scale reading is to be added to an odd number of half millimetres (.5, 1.5, 2.5 mm., etc.) or to an even number (1.0, 2.0, 3.0 mm., etc.) it is necessary to decide whether the edge of the disc is *above* or *below* the *middle* of a mm. division. *This is very difficult.* (2) The disc being rather large, the reading of the edge on the vertical scale changes considerably if the screw moves sideways in the nut, *even a little.* This lateral movement is almost always present, and is, in fact, necessary, as without it, the screw will not work easily in the nut. *The above difficulties frequently result in the reading being wrong by a whole revolution* In practice, it is much more satisfactory to start with the *higher* surface and then screw down to the *lower* surface *actually counting* the whole revolutions as also the number of divisions in the necessary fraction of a revolution. If this commonsense method is adopted, the zero correction can also be applied practically without any extra labour.

Determining the thickness of a glass slip. Prepare columns for recording the data as shown below. Clean the plane glass plate as also the given glass slip. Place the glass slip on the plane plate, pressing gently to secure good contact. If dust or dirt is present between the two surfaces, the thickness cannot be measured with accuracy.

Oil the spherometer screw so that it turns easily in the nut Turn the disc so as to raise the screw point well above the plane of the three legs. Arrange the instrument so that all *the three legs* touch the plane plate and the screw is *above* the slip of glass but *not touching it.* The spherometer will now rest on the plane plate without showing any tendency to turn as a whole. Now lower the screw quickly. As soon as the screw point touches the glass slip, one of the legs will just leave the plane plate and the spherometer will turn as a whole. Screw backwards through a few divisions, so that the three legs touch the plane plate again with the screw point a little *above* the slip of glass. Now screw down *slowly* and *carefully* turning the disc bit by bit without rocking or pushing and stop as soon as the instrument turns as a whole. Read the disc division opposite the vertical scale, *estimating tenths with care.* If the disc is graduated *in fractions of a millimetre* (·1, ·2, ·3,......etc.) *read in millimetres not divisions*. Screw backwards a few divisions, shift the screw point to a different position on the glass slip, and screw down slowly and carefully as before till the spherometer just begins to turn. *Do not look at the divisions when deciding where to* *stop, or you may be prejudiced.* Read the disc again. Repeat so as to obtain five or six readings for different points of the glass slip. Suppose all these readings lie between 78 and 80.

Next remove the glass slip and screw *down* slowly till the zero of the disc scale is opposite the vertical scale. To do so the disc will have to be turned through 78—80 divisions. Continue screwing down and counting (*i*) the whole revolutions from 0 to 0 and (*ii*) the number of divisions in the last fraction of a revolution if any, till the screw touches the plane plate Screw backwards a little, and screw down again carefully and slowly, bit by bit, till the spherometer turns as a whole. Read up to a tenth the disc division opposite the scale. Repeat five or six times on different points of the plane plate. **Do not record or calculate a zero correction.** This will be applied automatically.

Record observations and calculate thickness as follows:

Read in fractions of a mm. directly (not divisions) if the disc is so graduated. The graduations should however be checked once, by actual counting of the divisions.

Pitch of screw = ·5 mm. ————2 revolutions raise the screw 1 mm.

No. of divisions on disc = 100

∴ Least count of the spherometer $= \frac{\cdot 5}{100} = \cdot 005$ mm.

Continued on next page.

Disc reading with screw on glass slip and legs on plane plate (*divisions*)	Details of screwing down to plane plate	Disc reading with screw and legs on plane plate (*divisions*)	Thickness of glass slip
79·1 78·8 79·0 78·2 78·5 78·7	*First incomplete revolution :* From mean value 78·7 downward through 70,60 to 0 or 78·7 divisions *Complete revolutions from 0 to 0 :* TWO	7·5 7·2 7·2 7·5 7·3 7·1	Total divisions in two incomplete revolutions: 78·7 + 92·7 = 171·4 = 1 rev + 71·4 div.
Mean : $70 + \frac{52\cdot3}{6}$ = 70 + 8·7 = 78·7	*Last incomplete revolution:* From 0 or 100 downward through 90,80 etc to mean value 7·3 (*see next column*) or 92·7 divisions	Mean : $7\cdot\frac{18}{6}$ = 7·3	This with two complete revolutions (0—0) amounts to 2 + 1 or 3 whole revolutions and 71·4 divisions = 3 × ·5 mm. + 71·4 × ·005 mm. = 1·5 + ·3570 mm. = 1·857 mm.

The Use of the Spherometer for determining Radius of Curvature of Spherical Surfaces.

If a spherometer, with the screw in the zero position, is placed on a convex spherical surface, the instrument will rest on two legs and the screw, the third leg remaining above the surface. In order that all four may touch the spherical surface simultaneously, the screw has to be pulled up by an amount depending on the curvature of the surface.

To obtain an expression for the radius of curvature in terms of the height of the screw above the plane of the legs, and the distance between the legs, refer to Fig. 18.

Let ABC be the equilateral triangle between the feet of the instrument and S the point of the screw. The distance of S from A, B or C (when in the same plane with them) is

$$l = \frac{a/2}{\cos 30^{c}} = a/2 \times \frac{2}{\sqrt{3}} = \frac{a}{\sqrt{3}} \quad \text{or } a^2 = 3l^2,$$

where $a = AB = BC = CA$,

BES′ is a section of the spherical surface by a plane passing through one of the legs B, the screw point S′, and the centre of the sphere O. Draw S′OE, the diameter through S′. BS drawn perpendicular to S′E represents the plane of the legs SS′ is the height of the screw above the legs.

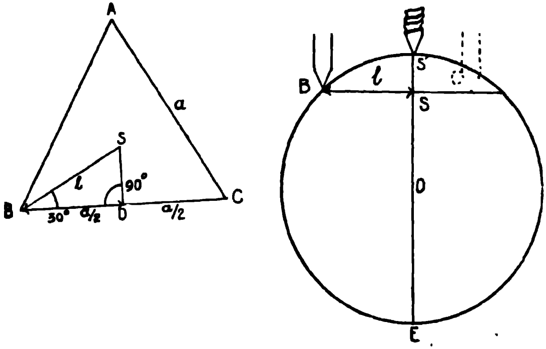


Fig. 18.—Geometry of spherometer.

From the geometry of the circle it follows that

$$BS^2 = SS \,.\, SE = SS'\,(ES' - SS').$$

$$= h\,(2\,r - h),$$ where r is the radius of the circle and h the height SS′.

But $BS = l = \dfrac{a}{\sqrt{3}}$

$$\therefore \quad BS^2 = \frac{a^2}{3} = h(2r - h) = 2rh - h^2$$

or $a^2 = 6rh - 3h^2$
or $6rh = a^2 + 3h^2$

$$\text{or } r = \frac{a^2 + 3h^2}{6h} = \frac{a^2}{6h} + \frac{h}{2} \ldots\ldots\ldots\ldots\ldots$$

As $a^2 = 3l^2$, (*i*) can also be put as

$$r = \frac{3l^2}{6h} + \frac{h}{2} = \frac{l^2}{2h} + \frac{h}{2} \text{ ....}(ii)$$

Formula (*i*) should always be used, because a is greater than l and can thus be measured with greater accuracy.

EXPERIMENT 11.—**To determine the radius of curvature of a convex or concave spherical surface.**

Apparatus : Spherometer, convex and concave mirrors or lenses, plane glass plate, metre stick.

It is best to use formula (*i*), *radius of curvature* $= \frac{a^2}{6h} + \frac{h}{2}$. We have, therefore, to measure h and a. It is always more convenient to measure h by screwing *down* than by screwing *up*. Prepare columns for recording data as shown below, and then proceed as follows :

Convex surface. Adjust the spherometer *first* on the convex surface (not the plane plate). Take five or six readings of the disc when the screw and the legs touch the convex surface at the same time. Different portions of the surface should be used for the various readings. The disc should be turned slowly and carefully, bit by bit, as already explained. *Do not look at the divisions when deciding where to stop, or you may feel unduly inclined to stop at some particular point.* Next screw down to the plane plate counting the whole revolutions, and the divisions in the necessary incomplete revolutions in the usual manner. Record five or six readings for different portions of the plane plate as already explained.

Concave surface. In this case, start with the *plane* surface, and screw *down* to the concave surface. The plane plate readings taken when measuring the h for the convex surface can be used.

To measure a, press the three legs gently on your note-book, and call the three marks produced A, B, C. Measure AB, BC, CA. The mean of these gives a.

Record and calculate as follows :

[In the following record it is assumed that the divisions of the disc scale are numbered in the ordinary way. If however, the disc is graduated in fractions of a mm., it is best to read these fractions *without counting the divisions.* The graduation should, of course, be checked once for all in the beginning.]

Pitch of spherometer screw = ·5 mm. } 2 turns raise the
No of divisions on disc = 250 } screw 1 mm.

$$\therefore \text{ Least count of spherometer} = \frac{\cdot 5}{250} \text{ mm.}$$

$$= \cdot 002 \text{ mm.}$$

Surface to be measured : Convex Mirror.

(Continued on next page.)

Disc reading on convex surface (*divisions*)	Details of screwing down to plane surface	Disc reading on plane surface *divisions*)	h	a
18·0	*First incomplete revolution :*	24·9	*Total divisions in two incomplete revolutions :*	cm.
18·1		24·3		3·99
18·2	From mean value 18·15 downwards through 10... etc. to 0 or 18·15 div.	25·0	18·15 + 225·24	3·97
17·5		25·2	= 243·39 or	3·98
18·1		24·7	243·4 = 243·4 × ·002 mm	Mean :
19·0		24·5	= ·4868 mm.	$3·9\frac{24}{3}$
Mean : $10+\frac{48·9}{6}$	*Complete revolutions (0—0):* *TWO*	Mean : $20+\frac{28·6}{6}$	TWO *complete* revolutions.	= 3·98 cm.
= 10 + 8·15 = 18·15	*Last incomplete revolution :* From 0 or 250 downwards through 240, 230 etc., to mean value 24·76 (see next column) or 250 − 24·76 = 225·24 div.	= 20 + 4·76 = 24·76	= 2 × ·5 mm. = 1·0 mm. h = 1·4868 mm. = ·14868 cm. or ·1487 cm.	

$$\text{Radius of curvature} = \frac{a^2}{6h} + \frac{h}{2} = \frac{(3·98)^2}{6 \times ·1487} + \frac{·1487}{2} \text{ cm.}$$

Log 3·98 = ·5999
Log 3·98 = ·5999

1·1998
Log ·8922 = $\bar{1}$·9505

Antilog 1·2493
= 17·75

$$= \frac{(3·98)^2}{·8922} + ·07435$$

= 17·75 + ·07435 cm.
= 17·824 or retaining 3 figures
17·82 cm.

CHAPTER III—(*contd.*)

CIRCULAR MEASURE AND TRIGONOMETRICAL RATIOS OF ANGLES

It is assumed that the student knows how to measure angles in terms of a right angle and degrees by means of a protractor. He will now be introduced to two other important methods of denoting and measuring angles.

Radian. Imagine an arm OP pivoted at O. If the arm moves from OP to OP_1 so that the length of the circular arc PP_1 is exactly equal to the radius OP_1, the angle POP_1 is called **1 radian.** It is evident that the magnitude of this angle will be the same as long as $PP_1 = OP$, no matter how large or small PP_1 and OP might be. If the arm OP completes one revolution, it will describe the complete circumference, which is 2π times PP_1. The four right angles thus described, therefore, equal 2π radians. A right angle POP_2 is, therefore, $\frac{2\pi}{4}$ or $\frac{\pi}{2}$ radians. The number of degrees in a radian $= \frac{360^\circ}{2\pi}$ or $\frac{180^\circ}{\pi} = 57{\cdot}3$.

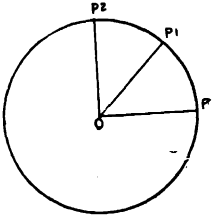


Fig. 19.—The radian.

Trigonometrical ratios :

You perhaps know that if one of the acute angles of a right-angled triangle is 30°, the side opposite this angle of 30° is exactly one half of the hypotenuse. Conversely, if you were told that in a right-angled triangle

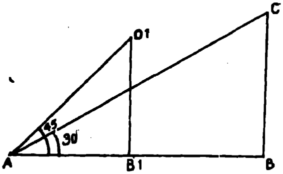


Fig. 20.

ABC (Fig. 20), the height BC was exactly one-half of the hypotenuse AC, you could conclude that the angle opposite BC, that is the angle A, must be 30°. If $\frac{\text{height}}{\text{hypotenuse}}$ equals $\frac{1}{\sqrt{2}}$, you could similarly conclude the angle opposite the height to be 45°. Instead of the ratio $\frac{\text{height}}{\text{hypotenuse}}$, other ratios, for instance, $\frac{\text{base}}{\text{hypotenuse}}$, or $\frac{\text{height}}{\text{base}}$, or the reciprocals of any of these could be used for determining the value of the angle. Such ratios are called **trigonometrical ratios,** and are given the following names:

$$\textbf{Sin BAC} = \frac{\textbf{height}}{\textbf{hypotenuse}} = \frac{\textbf{BC}}{\textbf{AC}}$$

$$\textbf{Cosine BAC} = \frac{\textbf{base}}{\textbf{hypotenuse}} = \frac{\textbf{AB}}{\textbf{AC}}$$

$$\textbf{Tangent BAC} = \frac{\textbf{height}}{\textbf{base}} = \frac{\textbf{BC}}{\textbf{AB}}$$

The reciprocals of the above 3 ratios are called cosecant, secant and cotangent. The ratios most frequently met with in practical physics, are, however, the sine (*sin*), cosine (*cos*) and tangent (*tan*) given above.

The trigonometrical ratios of any angle can be obtained from tables similar to logarithm tables. (See appendix).

EXPERIMENT 12.—**To measure the trigonometrical ratios of angles.**

Apparatus : Protractor, set square, etc., metre stick.

Draw two acute angles θ_1 and θ_2, on a sheet of paper, or on a page of your note book. Let the arms be about 10 cm. long. Draw perpendiculars, as in Fig. 21. Measure all the three sides of both the triangles, up to ·1 mm. Measure the angles also with a protractor.

Calculate the circular measure, the sine, cosine and tangent. For confirmation, consult tables.

Enter observations thus :

Angle	Degrees	$\frac{\text{degrees}}{57.3} =$ *radians*	$\frac{\text{height}}{\text{hypotenuse}} =$ *sine*	$\frac{\text{base}}{\text{hypotenuse}} =$ *cosine*	$\frac{\text{height}}{\text{base}} =$ *tangent*
θ_1	.	...	$\frac{...}{.} =$	$\frac{..}{...} =$	$\frac{..}{} =$
θ_2	.	..	$\frac{...}{...} =$	$\frac{.}{...} =$	$\frac{...}{...} =$

An angle whose sine is ·5 is denoted as $\sin^{-1}(\cdot 5)$. $\text{Sin}^{-1}(\cdot 5) = 30°$, because $\sin 30° = \cdot 5$.

Write down in your book the values of the following ratios and angles:

sin 45°, cos 45°, tan 45°.
sin 0°, cos 0°, tan 0°.
sin 90°, cos 90°, tan 90°.
sin 60°, cos 60°, tan 60°.

$\sin^{-1}\frac{1}{2}$; $\cos^{-1}\frac{1}{2}$; $\tan^{-1}\infty$
$\sin^{-1}\frac{\sqrt{3}}{2}$; $\cos^{-1}\frac{\sqrt{3}}{2}$; $\tan^{-1}0$
$\sin^{-1}\frac{1}{\sqrt{2}}$; $\cos^{-1}\frac{1}{\sqrt{2}}$; $\tan^{-1}1$
$\sin^{-1}1$; $\cos^{-1}1$; $\tan^{-1}\sqrt{3}$
$\tan^{-1}\frac{1}{\sqrt{3}}$

Let us pay a little more attention to the measurement of one of these ratios—the sine.

Draw a circle as shown. Let OP be a horizontal radius. Imagine a rotating radial arm making different angles with OP in different positions OP_1, OP_2, OP_3. Draw perpendiculars P_1N_1, P_2N_2, P_3N_3 The hypotenuses of the various triangles are all equal to the radius

In the first quadrant the perpendicular increases with the angle.

At 90° the perpendicular equals the radius, and therefore the sine, that is sin 90° = 1

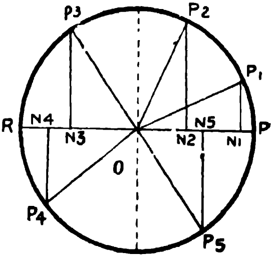


Fig 21.

If the angle exceeds 90° as in the third position of the arm OP_3, $\frac{P_3N_3}{OP_3}$ is still called the sine of the angle P_3OP

This, of course, also equals the sine of the angle $P_3 ON_3$, but the chief thing to note is that the idea of trigonometrical ratios is not limited to acute angles. The sine of 60° and sin 120° are the same Similarly sin 30° and sine 150° (180° - 30°) are the same. Therefore, an angle has a fixed sine, but a particular value of the sine may correspond to different angles

The sine rises from 0 to 1, as the angle increases from 0° to 90°. from 90° to 180° the sine falls from 1 to 0 When the angle exceeds 180°, the perpendicular is below the diameter ROP, but its magnitude increases. The sine is now called negative and changes from 0 at 180° to - 1 at 270°. From 270° to 360° the sine changes from - 1 to 0 so that we have

$$\text{Sin } 0^\circ = \sin 180^\circ = \sin 360^\circ = 0$$
$$\text{Sin } 90^\circ = 1, \sin 270^\circ = -1.$$

The cosine and the tangent go through similar periodic changes, as the angle steadily increases.

Estimation of angles, distances and heights. It is quite easy, with very simple apparatus, to estimate approximately from a distance, heights of trees, buildings, etc, by making use of the properties of angles, or what is the same thing, properties of similar triangles. A number of interesting measurements can be made with a single pencil and a bit of string as the only apparatus The pencil and string can be converted into a powerful measuring appliance as follows:

With a penknife, make, as near one end of the pencil as you can, a round notch, and then equidistant notches, half an inch apart. Fasten a piece of string round the end notch, and at exactly 20 inches from this notch, knot the thread, so that the piece of string between knot and pencil provides us with a fixed distance of 20 inches. Next engrave, with a needle, on the notch next to the thread, the number 40, because 20 inches of string is exactly 40 times ½ inch, the distance between this notch and the notch round which the thread is tied. On the next notch engrave

$$02\ \left(\frac{20}{1}\right), \text{ and then } 13\left(\frac{20}{1{\cdot}5}=13{\cdot}3\right). \quad \text{Follow with}$$

$$10\ \left(\frac{20}{2}\right), 8\left(\frac{20}{2{\cdot}5}\right), 7\left(\frac{20}{3}=6{\cdot}7\right), 6\left(\frac{20}{3{\cdot}5}=5{\cdot}7\right), 5,\ \frac{9}{2}\ 4, \text{ and so on.}$$

A simple paper scale with the above factors marked successively on lines half an inch apart can be pasted round the pencil, and is equally useful. Now we are ready for experimenting.

EXPERIMENT 13. **To estimate (1) the distance of a building, (2) the height of a tree, (3) the width of the laboratory.**

Apparatus. Pencil and string prepared as above.

The experiment consists in comparing the distance or height to be estimated, with a known length, say the height of a man near the building. If the distance is known, the height can be found and *vice versa.*

(1). **The distance of a building.** We will compare this distance with the height of the building (if known), or with the height of a friend near the building. Close one eye; place the knot on the cheek exactly under the other. Hold the pencil vertical with the string taut and horizontal, and slide the thumb nail on the pencil so that when the bottom of the building and the end notch (or end) are in a line with the eye, the thumb nail and the top of the building (or the head of the friend) are also in a line with it. Read the factor on the pencil nearest the thumb nail; suppose this is 8; then the distance of the building is 8 times the height of the building or the friend.

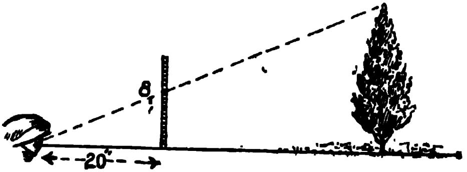


Fig. 23.—Estimating heights with simple apparatus.

Conversely, if the distance is known, the height will be 1/8th of this distance. The distance could also have been compared with the width.

(2). **Height of a tree.** Measure your distance from the tree, by pacing or by a tape. Suppose the factor is 6. The height is one-sixth of the measured distance.

(3). **Width of the laboratory.** If your distance from the wall whose width is being measured is known the width =this distance/factor.

Another way is to determine the factor for a student or an almirah of known size, standing against the wall. If the factor for width is 3, and for almirah 10, the width is 10/3 times the almirah dimension measured.

A flat strip of wood or stiff cardboard, with the factors marked at intervals of 1/8th of an inch, is much more accurate and useful than the essentially crude appliance described above.*

---

*The above descriptions are based on the inexpensive "scalometer," a beautifully finished celluloid rule, with factors from 500 to 5 clearly marked. The instrument is made by the "Standard Scien[illegible] Company," New York.

CHAPTER IV

THE MEASUREMENT OF MASS

Standard Masses or "Weights." The mass of a body is obtained by comparing it with standard masses called "**weights.**" These are contained in a box with holes, into which the brass "weights" from 1 gram upward are fitted. The fractions from ·5 to ·001, generally of aluminium, are placed in shallow compartments under a slip of glass. The box contains a pair of forceps. Fractions below ·01 gm. are not required for Intermediate work. As to the fractions required (·5—·01 gms.), every student (or pair of students) should purchase a set, as there is practically no other way of ensuring efficient work. Fractions, if supplied with laboratory sets, are invariably lost, and in consequence, boxes are often far from complete.

The use of a rider does away altogether with the difficulty of missing fractional weights.

The Balance. The comparison of the mass of a body with the "weights," is carried out by means of a balance. This consists of a beam with three agate knife-edges. The central knife-edge rests on agate planes above the pillar, while from the end knife-edges are suspended agate topped stirrups carrying the pans.

A vertical pointer attached to the middle of the beam swings in front of a small scale. The balance beam with the pans can be set swinging by means of the milled head or lever provided. When not is use, the beam rests on special supports, which relieve

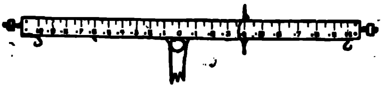


Fig. 25.—A captive one gramme rider. The beam can be graduated to read centigrams.

the pressure on the agate knife-edges.

For measuring fractions of a gramme, the beam is graduated, and a mass called the rider slides along it and thus serves the purpose of an adjustable weight. A one gm. rider (preferably captive, to prevent loss), may slide from the centre to the right hand knife-edge, and if this half of the beam is divided into 100 parts, any fraction up to ·01 gm. can be read off.

EXPERIMENT 14.—**To test the accuracy of the weights and the balance, and to weigh a body.**

Apparatus: Balance, weight box (200—·01 gm.), a glass or metal ball, or a cylinder weighing about 100 gm.

Clean the balance, if necessary. Next, level it by means of a spirit level or by means of the plumb line attached to the pillar. See that the stirrups are not displaced. Try the lever to see that the beam is in order. Next, gently raise the beam, shut the case and watch the pointer. If the pointer is at rest, set it swinging by raising the glass door a little, and moving a piece of paper near it. If the pointer does not make equal swings on the two sides of the zero, adjust carefully the small nuts at the ends of the beam till the swings are very nearly equal.

Next, let us prepare for temporary use in this experiment, a set of milligram weights. Place in the left hand pan ·01 gm., and in the right about two inches or more of ordinary cotton thread or very fine wire. Adjust the length of this by trial till the pointer swings equally on both sides (shut case when watching this). One half this length is 5 milligrammes. Subdivide the other half into 2, 2, and 1 mgm.

To test the accuracy of the balance and weights: Place in the left hand pan the 200 gramme weight, and in the right hand pan another 200 gm. weight, or, if that is not available, 100 gm.+50 gm.+20 gm. + 20 gm. + 10 gm. Carefully lift the beam. If the pointer does not make equal swings, adjust by adding fractions to one side or the other. If ·01 gm. is too much for adjusting the difference, use the prepared mgm. fractions. Next interchange the weights (carry those in the left hand pan to the right and *vice versa*) and adjust for equality of swings. Repeat interchanging as before, with 100 gms. on one side and 50+20+20+10 gms. on the other side. Interchange. Try in this way different combinations. Record thus:—

No.	LEFT PAN	RIGHT PAN	DIFFERENCE	% ACCURACY
1	200 gm. wt. ...	100 + 50 + 20 + 20 + 10		
2	100 + 50 + 20 + 20 + 10	200 gm. wt.		
3	100 gm. wt.	50 + 20 + 20 + 10		
4	50 + 20 + 20 + 10 ..	100 gm. wt.		
5				

Record anwers to the following questions in your note-book:—

(i) Can the balance and "weights" weigh correctly up to (i) 1 mgm. (·001 gm.) (ii) upto 1 cgm. (·01 gm.)?

(ii) Why do the weights seem to differ on changing places? Is the difference much? Do you consider the balance fairly well constructed?

If the balance beam is not graduated, it can be fitted with an attachment consisting of a light celluloid scale directly graduated in centigrammes. With a rider, weighings take much less time than with small fractions that have to be carried from box to pan and *vice versa.*

With a moderately good balance and set of weights, it is very easy to determine a mass of about 100 grammes correct to within ·1 gramme or even ·01 gm. This means an accuracy of ·01 % or 1 part in 10,000. We cannot expect to be more accurate than this in any other observations we are likely to make. Even for small masses, the accuracy is 1 in 500 or so. To weigh correctly up to ·001 gm., a good balance, an expensive weight box, and the application of a number of corrections, depending on the temperature, humidity and pressure of the air, are necessary. A sensitive instrument, like the balance, has to be treated with care. The following rules should be observed :—

(*i*) Before weighing, dust the base and the pans with a soft camel hair brush. Next, level the instrument with the help of the levelling screws till the plumb line indicates adjustment.

(*ii*) Arrest the beam, by means of the lever, before attempting to place articles or weights in the pans, or when removing them.

(*iii*) Raise and lower the beam *gently*. Keep the case shut, except when changing the weights.

(*iv*) Do not allow liquids, common salt or any other chemicals to touch the pans. Weigh all such bodies in watch glasses, and other receptacles. If soiled, the pans should be carefully washed and dried.

(*v*) The "weights" should, as far as possible, be lifted with the forceps only.

To weigh a body : Place the body in the left hand pan, and a weight considered heavier than this, say 100 gms. in the other. Raise the lever a little, to see whether 100 gms. is too much or too little. If too little, substitute for it 200 gms. wt. If this is too much, replace 100 gms. weight along with 50 grammes weight. If insufficient, add 20, or if too much, remove 50 and add 20. Proceed in this way, trying the weights systematically, in descending order of magnitude, down to ·01 gm. It will very likely happen, that in the end, ·02 gm. is found too much, and ·01 too little. Adopt ·015 as the required weight in this case.

To add up, look at the empty holes in the box, and note the weights in the book.

Now remove the biggest weight, check off the note made, and replace in the box. Check the whole list in this way, and finally total up.

Enter weights thus :

Holes empty: 50′+20′+10′+2′+2′+·5′+·2′+2′+·05′ +·02′. Weights removed (list checked) = 84·7 grammes.

CHAPTER V

MEASUREMENT OF TIME

The Standard of Time. The standard of time is the **mean solar day,** or the mean time interval elapsing between two consecutive passages of the sun across the same meridian. Convenient fractions of this interval are, **hours, minutes** and **seconds.**

The Unit of Time. The C. G. S. unit of time is the **second,** and equals $\frac{1}{86400}$ of the mean solar day.

Time-keepers. Time can be measured with very great accuracy. A **standard clock** or **chronometer** will lose or gain one or two seconds a day, and this rate is generally fixed and can be allowed for. A good pocket watch may be correct to a minute a month, and is thus nearly as accurate as a chronometer. Ordinary watches gain or lose a minute a day, which means an accuracy of 1 in about 1500.

Stop-watches and stop-clocks become necessary when *small* intervals of time have to be measured. They are, generally, *inferior* timekeepers, and may lose or gain up to a second a minute, or one part in sixty. For intervals lasting more than three minutes or so, an ordinary watch with a seconds-hand is, generally, more accurate. The physics student who intends purchasing a watch, should select a centre-seconds one, in which a central hand measures the seconds on the large dial.

Where *absolute* time is required, as in determining ,*g*', the *rate* (loss or gain per hour) of a stop-watch should be measured and allowed for. When time intervals have only to be *compared*, as when drawing cooling curves, etc., the correction for rate is unnecessary.

EXPERIMENT 15.—**To compare with a standard**

clock or chronometer: (1) a pocket watch with a seconds-hand, (2) a stop-watch.

Apparatus : Pocket watch, stop-watch, chronometer or standard clock.

The chronometer or clock is on no account to be touched by any student. The ticks can be heard and the dials read, without handling. To rate an ordinary watch, two observers are required. One looks at the clock or chronometer, and says "ready" to the other observer, 5 seconds before the seconds-hand is due at 60 or 0. When the hand is at 0 or 60, say "now" or "go." At the latter signal, the other observer reads the seconds on the pocket watch and notes down the time. In the case of a stop-watch, the zero reading should be taken, and the watch started at "now" or "go." Wait for 10 minutes. When 10 or 15 seconds remain to the completion of 10 minutes, get ready for the final observation. Say "ready" and "go" as before, and read the seconds (or stop the watch), when the chronometer hand is at 60 or 0, after completing 10 minutes. *Enter thus* :

CHRONOMETER OR CLOCK READING	POCKET WATCH	STOP-WATCH	1 SECOND POCKET WATCH EQUALS	1 SECOND STOP-WATCH EQUALS
h *m* *s.*	*h* *m* *s.*	*m.* *s.*		
2 35 0	2 29 15	0 +1·2		
2 45 0	2 39 16·5	9 54·1		
Interval 10 m =600 sec.	10 m. 1·5 sec =601 5 sec.	9m. 52 9 sec. or 592·9 sec	$\frac{600}{601\cdot5}=$	$\frac{600}{592\ 9}=$
Loss or gain per day.	144 × 1 5 sec. =216 sec.= 3·6 mts.	144 × 7·1 = 1022 sec = 17m. 2 sec.		

$$\%\ \text{error pocket watch} = \frac{1{\cdot}5}{6} \text{ or } \underline{{\cdot}25}$$

$$\text{,,} \qquad \text{stop-watch} = \frac{7{\cdot}1}{6} \text{ or } \underline{1{\cdot}2}.$$

Record answers to the following:

(1) Which is the better time-keeper, the stop-watch or the pocket watch?

(2) Which would you use for determining the amount of hydrogen evolved by a current in half an hour?

(3) Which would you use for determining the time of fall of a body from the college tower?

CHAPTER VI

MEASUREMENT OF DERIVED UNITS

MEASUREMENT OF AREA

EXPERIMENT 16.—**To measure the area of a circle by (*i*) squared paper, (*ii*) weighing, and to verify the formula : area of a circle=πr^2.**

Apparatus : (*i*) Squared paper, compasses. (*ii*) Thick paper, compasses, set squares, etc., pair of scissors.

(*i*) *By squared paper* :

With the intersection of two heavy lines as centre, draw a circle of exactly 5 cm. radius. Examine the horizontal and vertical radii to see if they are both exactly equal to 5 cm. lines If not, the paper is inaccurately ruled. Better paper, if available, should be employed.

Proceed to measure the area of one quadrant by counting the number of sq. cms. and sq. mms occupied by it Count the complete cm. squares, and then the complete 5 mm. squares. Follow up by counting the remaining 1 mm. squares one by one. For portions of mm. squares. count those bigger than half as if they were complete sq. mms., and ignore those less than half. Calculate the area of one quadrant as shown, and multiply by 4. If the squared paper is inaccurately ruled, count out the squares over the whole cricle.

Enter thus:

Area of one quadrant : sq. cm.
Complete cm. squares = ...
,, 5 mm. squares = ... = $\frac{}{4}$ sq. cm. = ...
Millimetre squares = ... = $\frac{...}{100}$,, ,, = ...

Area = ...

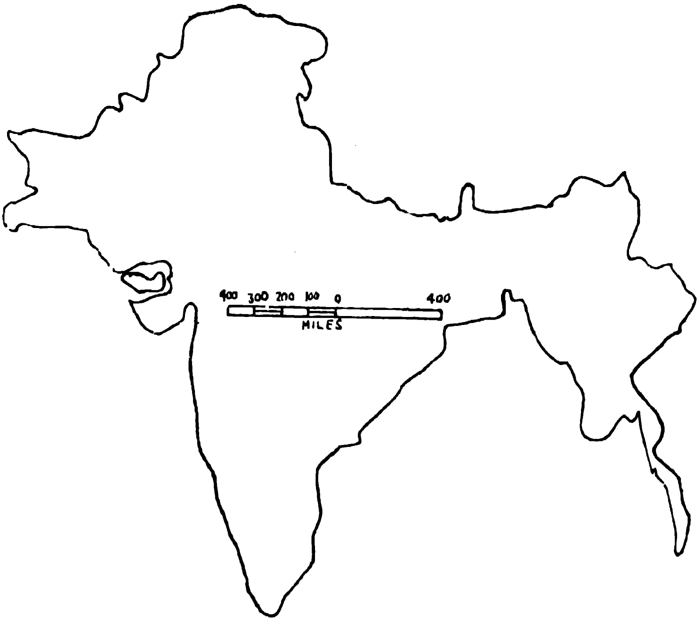


Fig. 26. Duplicate. Detach for Expt. 17.

The area of the circle (*Measured*) $= 4 \times$ area of quadrant

$= \ldots$ sq. cm

Calculated area $= \pi \times 5^2$

$$= \pi \frac{100}{4}$$

$$= \frac{314 \cdot 16}{4}$$

$= 78 \cdot 54$ sq cm.

(ii) *By weighing*:

Draw accurately, on thick paper, with a pointed pencil, a square of exactly 10 cm. side. If the square is correctly drawn the two diagonals will be found equal. Draw also, on a piece of the same paper, a circle of 5 cm radius. Cut out accurately, the square and the circle, and weigh separately on a balance.

Enter and calculate thus:

Weight Square	Area Square	Weight per cm.^{2}	Weight circle.	Calculated area circle.	Area from formula $a=\pi r^2$

Question. A student asked to find out the weight of 1 sq. cm. of a given piece of cardboard, cut out a square of 1 cm. side which he weighed with great care. Give your opinion about the method adopted.

EXPERIMENT 17.—**To determine from a map drawn to scale the area of India.**

Detach from the book the duplicate map supplied (Fig. 26). Determine its actual area in sq. cm. either by means of (1) squared paper, or by (2) weighing. To do so place the map on (1) squared paper, or (2) thick plain paper, and trace out the boundary in each case by pin pricks through the map.

(1) Count the squares or (2) cut out and weigh the piece of plain paper. Weigh also a large square piece from the same paper. The area is thus determined. Next determine how many miles each centimetre on the map represents by transferring with dividers a space of 2 or 3 cm. to the mile scale on the map.

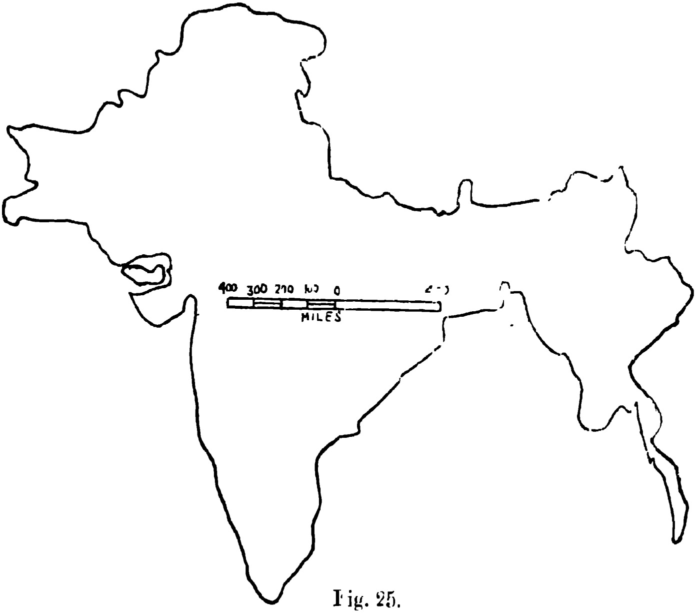


Fig. 25.

Enter thus:

Area of map by square paper = (A) sq. cm.
Scale of map: 3 cm. = 770 miles
or 1 cm. = 256·7 miles
∴ 1 sq. cm. = $(256{\cdot}7)^2$ sq. miles
∴ Area of India = $A \times (256{\cdot}7)^2$ square miles
=..................sq. miles.

CHAPTER VI—(*contd.*)

MEASUREMENT OF VOLUME

Bodies of Regular Shape. The volumes of spheres, cylinders, and bodies of other regular geometrical shape can be found by using appropriate formulæ, after the lengths, diameters and other dimensions, have been measured accurately by callipers, screw gauge, etc.

Bodies of Irregular Shape. It is easily seen that a body immersed in a liquid will displace a volume of it equal to its own. The volume of a body of any shape can, therefore, be determined by measuring or weighing the liquid displaced. Water is very convenient for this determination, because, 1 c.c. of it weighs exactly 1 gramme at 4°C. The volume of a body in c.c. is therefore, numerically equal to the weight in grammes of the water displaced by it.

Burettes and **graduated jars** are used when the volume of the water displaced has to be directly **measured**.

The **water displaced** can, however, be actually **weighed**. This weight can also be calculated by applying

The Principle of Archimedes : On immersion in a liquid, a body suffers a loss of weight equal to the weight of the liquid displaced. If the liquid is water, *the difference in grammes between the weight in air and the weight in water is numerically equal to the volume of the body in c.c.* (strictly true at 4°C only).

EXPERIMENT 18.—**To determine the volume of an irregular solid by measuring the water displaced.**

This and the following experiment with simple apparatus provide the student an opportunity of exercising his *critical* faculty with regard to the relative merits of various methods of determining volume and density, etc

Apparatus: 100 c.c. burette, narrow graduated jar, overflow can, small beaker, graduated test tube. the irregular solid (glass stopper) or metal ball.

Wash a 100 c.c. burette (50 c.c. size may have to be repeatedly filled) and the graduated jar and test tube with caustic potash solution or soap. Rinse thoroughly. The water surface is deformed in greasy vessels, and readings cannot, consequently, be taken accurately

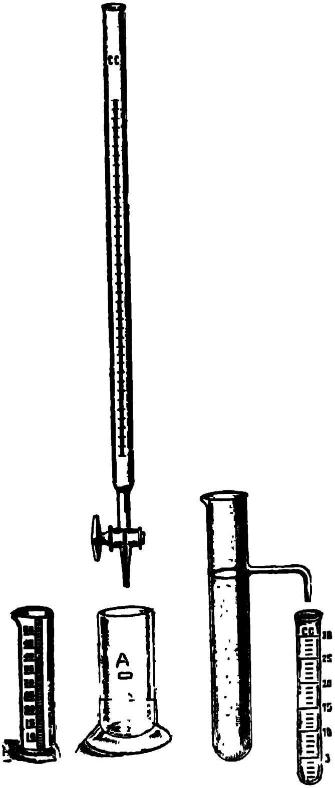


Fig. 27.—Measurement of water displaced by irregular solid.

1st method: Put in the graduated jar an amount of water which you think will cover the stopper, *when placed in it later on.* Hold white paper behind the jar, place the eye at the same level as the water surface, and read the position of the lowest point of the curved surface (tangent to the meniscus). *Tenths of the smallest division should be estimated with the greatest care.*

Incline the jar, and gently *slide* the body in. Replace, and read the level as before, tenths being estimated with the greatest care. The difference in the two readings gives the volume.

Repeat both readings to see how far the two values of volume agree.

2nd method. Another way is to use a *narrow* ungraduated vessel, with a gum paper mark *A* on it. Set up the burette so that it is held vertically (use a plumb line) in a clamp above the vessel. Fill burette with water, place an extra beaker underneath, and run water into it till the level reads zero. The part of the burette below the cock should be free from air bubbles. Run water into the marked vessel, quickly at first, drop by drop later on, till the mark is reached. Read burette (eye level, white paper behind), estimating tenths with great care. Empty and dry the marked vessel. Refill burette. Adjust to zero. Place stopper in the marked vessel, and fill up to mark again from burette. Read carefully again. The difference gives the volume of the solid. Repeat both readings to see how far the two values of volume agree. Do not look at the burette when deciding if mark *A* is reached.

3rd method: Fill up the overflow can till water flows freely from the spout. When all dripping is over, place under the spout a graduated test tube, or a burette filled to a known mark near the bottom, so as to catch the overflow. *Gently* lower the solid, by means of a fine silk thread, into the water, and read the volume of the water that overflows. Obtain a second value.

Enter observations thus:

	First determination of volume	Second determination of volume	Difference	Mean volume
1st method.	c c	c c	c c	c c
Reading of jar without solid	50·24	43·90		
Reading of jar with solid	60·42	54·00		
Volume	10·18	10·10	·08	10·14
2nd method				
Volume required to fill jar up to A without solid	......	......		
Volume required with solid in	......	......		
Volume	......	......	......	......
3rd method.				
Volume of overflow	......	......	......	......

Study the differences between the two determinations in each case and *record* answers to the following:

(1) Which method is, on the whole, in your opinion, the most accurate? Mention defects if any, of this method.

(2) How does the diameter of the measuring vessel affect the amount of probable error? Is it easy to measure correct to ·01 c.c.?

(3) Supposing you could place the solid in the burette, could you expect a more accurate value?

EXPERIMENT 19.—**To determine the volume of an irregular solid by weighing the water displaced.**

Apparatus: Overflow can, catch bucket or beaker, cubical vessel with ground top and cover, balance stool, balance and weights.

1st method:

Use the overflow can as in the last experiment, except that the overflow water is to be caught in a weighed catch bucket or beaker. Weigh beaker or bucket with the overflow. The increase in weight gives the mass of the water displaced. Each gramme of this water occupies 1 c.c. at 4°C. At the temperature of the laboratory it must occupy more. With the help of tables (page 100) calculate the volume.

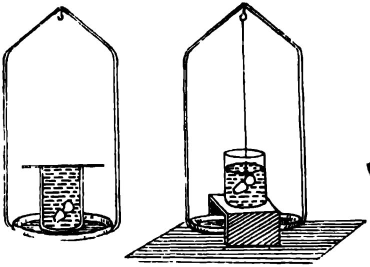


Fig. 28. The balance used for measuring volumes.

Make a second determination to see how far the values agree.

2nd method :

Fill to the brim the cubical vessel, and close up with cover provided, excluding all air bubbles. Wipe and dry outside with blotting paper. Place in left hand pan carefully. Place the irregular solid in the pan alongside the vessel. Weigh carefully up to ·01 gm. Remove vessel and solid. Put the solid in the vessel. Fill up and cover again, excluding bubbles Wipe dry and weigh as before. The decrease of weight gives the mass of the displaced water. Make a second determination. Calculate the volume with the help of tables.

3rd method :

Suspend the body from the hook of the left hand pan by means of a **single** *fine* silk thread. Do not use a *loop*. Weigh. Next place on a small stool (Fig. 28) under the suspended solid a beaker containing water free from bubbles (water freshly obtained from the tap contains bubbles, and should not be used). Remove with a small brush all air bubbles from the suspended body. Now weigh, taking care to see that the body is *completely* immersed *even when the balance beam is raised.* The stool or the beaker should not touch the pan. The body should not touch the sides of the beaker. The difference in weight (Archimedes' principle) gives the mass of the displaced water. Make a second determination. Calculate the volume with the help of tables.

Observations:

Temp. of laboratory = 20°C
Volume of 1 gm. of water at 20 C = 1·002 c.c.

	Weight of water displaced		Difference	Mean Volume
	First determination	Second determination		
1st method.	Grammes	Grammes	Grammes	C C
Weight empty beaker	. ..	....		
Wt with overflow	. ..	.. .		Mean weight
Wt of overflow	..	....	.... .	× 1·002 ......
2nd method				
Wt of vessel (filled) + solid in the pan	.	......		
Wt. of vessel (filled) + solid in vessel	.	......		Mean weight
Wt. of water displaced	. .	. ....	......	× 1·002 ......
3rd method				
Wt of body in air	.. .	......		
Wt. of body in water	.. .	......		Mean loss
Loss of weight	. ....	......	......	× 1·002 ......

[For density and specific volume of water at different temperatures use table on page 100].

Study the difference between the two determinations in each case and *record* answers to the following in your note-book:

(1) What is your opinion as to the relative accuracy of the two methods of determining volume: measuring and weighing the displaced water? Which is easier: to determine weights correct to 01 gm. (and hence volumes to ·01 c.c.), or to measure volumes correct to ·01 c.c. directly with graduated jars, etc.?

(2) Of the three methods of weighing the displaced water, directly or indirectly, which is the most convenient and accurate? Give reasons.

(3) If the body were of small size (lead shot), would it be possible to use a stoppered sp. gr. bottle, instead of the cubical vessel and cover, for determining the volume?

EXPERIMENT 20.—**To verify the formulæ:**

Volume of a cylinder= $\pi \times$ **(radius)^2** $\times$ **length**

Volume of a sphere $= \frac{4}{3} \pi \times$ **(radius)^3.**

Apparatus: Vernier callipers, hydrostatic balance, burette or graduated jar (narrow), metal cylinder, sphere 3 cm. diameter or 5 or 6 equal sized spheres just small enough to go into the burette or narrow graduated jar (use motor car ball bearings).

The experiment consists in measuring the dimensions of the cylinders and spheres provided, with the callipers, calculating the volumes with the help of the formulæ, and comparing these with the volumes determined by any one of the methods described in experiments 18 and 19.

The volume is best found by determining loss of weight with the balance as in experiment 19. A sphere is rather difficult to suspend. If a sufficient number of equal sized small spheres is available, they can all be dropped into the burette or *narrow* graduated jar, and the volume of one determined by division. The diameter and length can be determined with callipers as in experiment 8

Observations :

Volume of 1 gm. of water at 20° = 1·002 c.c

	DIAMETER AND LENGTH	VOLUME FROM FORMULÆ	VOLUME FROM BURETTE OR LOSS OF WT.
SPHERE	Diameter 1 ... 2 ... 3 ... 4 ... 5 ... 6 ... 7 ... 8 ... Mean:		... – ... =
CYLINDER	Diameter 1 ... 2 ... 3 ... 4 ...... Mean: ...... Length 1 ... ... 3 ... Mean:		... – ... =

CHAPTER VII

MEASUREMENT OF DENSITY

Density. The mass, in grammes, of 1 c.c. of a substance is called its density, or

$$\textbf{Density}=\textbf{D}=\frac{\textbf{Mass of substance in grammes}}{\textbf{Volume of substance in c.c}}$$

Specific gravity is the ratio of the density of a substance to the density of some standard substance. For solids and liquids, this standard is water at 4°C. For gases, hydrogen at N. T. P. is adopted as the standard substance.

$$\textbf{Sp. gr. of a solid or liquid}=\frac{\textbf{Density of the solid or liquid}}{\textbf{Density of water at 4°C.}}$$

Density of water at 4°C (mass of 1 c.c. at 4°C) is however, 1. Consequently, the sp. gr. of a solid or liquid is numerically equal to its density, if the C.G.S. system of uits is employed. This is not the case in the F.P.S. system.

According to the F.P.S. system, the unit of mass is a ℔, and the unit of volume a cubic foot. The density of water, on this system, is the mass in ℔s. of one cubic foot of it, that is 62·5. The *density* of lead is 11·3 × 62·5 ℔s per ft^3. On the other hand, *specific gravity* of lead

$$=\frac{\text{Density of lead}}{\text{Density of water}}=\frac{62\cdot5\times11\cdot3}{62\cdot5}$$

$$=11\cdot3.$$

Therefore, *the sp. gr. of a given body is the same, no matter what system we employ The density varies with the system used.* On the C.G.S. system, the two happen to be equal. In practical work, we are generally concerned with the ratio specific gravity. It is better however, to use the term **density**, to emphasize the more important idea of **mass per c.c.**, which is also easier to remember and understand than the idea of a ratio between the densities of substances.

Determination of Density. We have learnt already how to measure masses by means of the balance, and volumes by measuring dimensions, or in the case of an irregular body, by measuring or weighing the water displaced. We are, therefore, in a position to determine densities experimentally by means of apparatus already employed or suggested. Detailed directions for using these will, therefore, not be repeated. The following remarks apply in general. to the methods used in experiments described in this chapter.

$$D = \frac{M}{V} \text{ (always)}$$

To determine M : The balance is employed for this purpose.

To determine V : The following methods already described, are employed:

1. By calculation from dimensions by use of the appropriate formula, if the body is a sphere, cylinder, or other solid of simple geometrical shape.
2. By graduated jar, or if the substance is in fragments, by direct introduction into the burette.
3. By overflow can. and burette or graduated test tube.
4. By overflow can, and weighed beaker or catch bucket.
5. By specific gravity bottle (used similarly to cubical vessel with cover used in experiment 19).
6. By hydrostatic balance (loss of weight method).

Densities of solids Lighter than, or Soluble in water. Solids lighter than water are kept immersed in it with the help of a sinker. Such solids may, sometimes, sink in oil. Volumes of solids soluble in water have also to be found by displacement of oil (kerosene). To get the volume of the solid from the loss of weight, when immersed in oil, consider the following: Let the volume of the solid be v c.c.

By Archimedes' Principle,

Loss of wt. in oil=weight of v c.c. of oil

$$= v \times \text{density of oil}$$

$$\therefore v = \frac{\text{Loss of wt. in oil}}{\text{Density of oil}}$$

Hence, **density of solid=**

$$\frac{\mathbf{M}}{\mathbf{V}} = \frac{\text{Mass of solid}}{\text{Loss of wt. in oil/density of oil}}$$

$$= \frac{\textbf{Mass of solid}}{\textbf{Loss of wt. in oil}} \times \textbf{density of oil.}$$

Density of liquids.

$$\text{Density of a liquid} = \frac{\text{Mass of liquid in grammes}}{\text{Volume of liquid in c.c.}}.$$

The volume, and sometimes both volume and mass, are indirectly obtained. In **the sp. gr. bottle method,** we obtain the mass of a bottleful of oil directly. The volume of this amount in c.c., is numerically equal to the number of grammes of water the bottle contains

Hence, **density of liquid=**

$$\frac{\mathbf{M}}{\mathbf{V}} = \frac{\textbf{Mass of a bottleful (V. c.c.) of liquid}}{\textbf{Mass of a bottleful (V. c.c.) of water}}.$$

In **the hydrostatic balance method**, both mass and volume are indirectly obtained. Loss of weight of a solid, when weighed in oil, numerically equals the mass of the oil displaced (**M**). Loss of weight in grammes of the same solid in water, equals numerically, the volume of the solid and hence, volume of the oil displaced, in c.c. (**V**). Hence.

$$\textbf{Density of liquid} = \frac{\mathbf{M}}{\mathbf{V}} = \frac{\textbf{Loss of wt. in liquid}}{\textbf{Loss of wt. in water}}.$$

To avoid undue strain on the memory in remembering formulæ, the best policy is to start with the relation $d = \frac{M}{V}$ and substitute for M and V, step by step, in terms of the various weighings recorded. This is much easier and more instructive, than remembering formulæ like: Density of light soild $= \dfrac{\text{weight of solid}}{\text{weight of solid in air and sinker in water - wt. of solid and sinker in water.}}$

or $d = \dfrac{w_1}{w_2 - w_3}$ where w_1, w_2, w_3 are etc., etc

EXPERIMENT 20 (*a*).—**To determine the density of a solid of simple geometrical shape by measurement of mass and dimensions.**

Apparatus : Metal sphere or cylinder, vernier callipers ; or, screw gauge and a long piece of thick wire. Balance and weights.

Weigh the given body. This gives **M**

To determine **V**, measure the diameter and length etc.. by callipers as in expt. 20 and then calculate the volume in c.c., by means of appropriate formula. In measuring a wire, the diameter should be determined with the screw gauge and the length determined with a metre stick. If the wire is 60 to 70 cm. long, measure the length correct to half a mm. If only 15—20 cm. long, estimate the tenths. Small thicknesses and diameters should be repeatedly measured, and the mean of at least ten readings taken

Enter thus :

	MASS M.	DIAMETER d.	LENGTH .	VOLUME V.	DENSITY ·M/V.
SPHERE		1... 2... 3... 4... 5... 6... 7... 8... 9... 10... Mean·		$\frac{4}{3}\pi r^3 = \frac{\pi}{6}d^3$, For log $\frac{\pi}{6}$ see page 26, or special entry in log table.	
CYLINDER		1... 2... 3... 4... 5... 6... Mean :	1... 2... 3... 4... 5... 6... Mean :	$\pi r^2 l = \frac{\pi}{4}d^2 l$.	

EXPERIMENT 20 (*b*).—**To determine the sp. gr. of lead shot or other finely divided solid.**

Apparatus: Lead shot, beaker, or light shallow dish of cardboard, burette on stand, balance and weights.

Weigh the small beaker or cardboard dish as container for the lead shot. Place in it from 100 to 150 grammes of lead shot and re-weigh. The difference gives the mass of the lead shot. If the lead shot is placed directly in the pan, much handling of the pan, when removing the shot, is unavoidable. Clean, and support vertical, a 50 c.c. burette. Adjust the water level to about the 30 c.c. mark, read carefully, and slowly drop into the burette, *all* the weighed lead shot. See that none roll away. Read the water level as before.

Observations:

Mass of dish :	15·59 gms	30·23 c.c :	Burette reading
Mass of dish with shot :	149·23 gms.	18·41 c.c :	„ „ with shot
Mass of shot:	133·64 gms.	11·82 c.c.:	Volume of shot.

$$\therefore \quad \text{Density of shot} = \frac{133\cdot6}{11\cdot82} = \underline{11\cdot3}.$$

Note that in the final fraction, 133·6 is used instead of 133·64, because knowledge of the mass to 5 significant digits is not necessary. The fifth figure should, however, be dropped only at the last stage (it cannot be dropped in 149·23), because, as the mass of shot has been obtained by *subtraction* of one weight from another, the dropping of the fifth figure in the very beginning may effect the fourth, third or even the second figure of the difference.

For instance, 149·23 − 149·00 = **·23**, while 149·2 − 149·0 = **·20**, the difference between ·23 and ·20 amounting to 15%.

, In other words, when *small* differences between two quantities enter the expression for the final result, the quantities concerned should be measured with special care, and an extra digit determined.

Question, A student performed the above experiment using ten pieces of fine lead shot. The weight and volume were carefully determined. The density turned out to be 16·4 (nearly 50 % wrong). Explain the cause of failure.

Question. You are asked to determine the *average* weight of *one* lead shot. How will you proceed?

CHAPTER VII—(*contd.*)

THE SPECIFIC GRAVITY BOTTLE

This is a small glass bottle fitted with a stopper with a central hole. The bottle is used for weighing a constant volume (equal to the capacity of the bottle) of different liquids. The bottle is filled to the brim, and the stopper ***very gently*** pressed in.

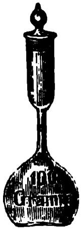


Fig. 30.—Specific gravity bottle.

This form of bottle is very defective. The stopper can be pressed in more or less, especially if the neck expands due to handling, so that the capacity of the bottle is not quite constant. A better form has a long neck with a mark on it (Fig. 30). The bottle is always filled up to the mark, the excess of the liquid being removed with filter paper.

Specific gravity bottles have, often, to be dried. Do this by leading a current of warm air from the bellows, or the air supply system, *down to the bottom of the bottle*, by means of a thin delivery tube (preferably metal, for permanency). A little alcohol (expensive) will hasten the drying. To clean bottles soiled with oil, rinse with petrol (very inflammable) and dry. The old

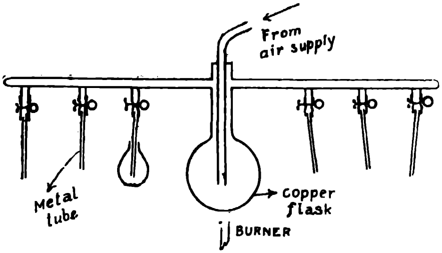


Fig. 29.—Hot air tube for six students.

fashioned hot air tube is very inefficient. The arrangement shown in the sketch (Fig 29) is very effective, and a number of students can obtain their supply of hot air simultaneously

EXPERIMENT 21.—**To determine the density of sand with the specific gravity bottle.**

Apparatus: Sand free from dust (washed and dried), water, sp. gr. bottle, balance and weights.

Clean and dry the bottle by leading a current of warm air *down to the bottom of the bottle* (see Fig. 29). Weigh. Put in enough sand to fill ⅓ to ½ of the bottle, and reweigh. Next, put in a little water into the bottle, shake up to wet the sand and let out all air bubbles, and finally fill up, and weigh. In the end, wash out the sand, fill up with water, and weigh. The outside of the bottle should be dried each time before weighing, and the bottle handled as little as possible. The heat of the hand warms the liquid, and may expel some of it. This is shown by the bore becoming partly empty, as the weighing goes on. Always interpose a *dry* duster or handkerchief between the bottle and the hand.

Take the temperature as soon as weighings are over. Four weighings are necessary with sand and shot, as it is convenient to weigh them in containers. Three weighings suffice, if the sand or shot is weighed *directly in the pan.* This, however, cannot be recommended.

Record and calculate as shown below.

[When applying the temperature correction remember that *this correction is practically the same for a range of one or two degrees.* Do not therefore spend much time in calculating proportional parts, etc. See paragraph below on temperature correction].

Weight of bottle = ... (1)
" " + sand = ... (2)
" " filled with water and sand = .. (3)
" " filled with water = .. (4)

Temp of water = 30°C.
Mass of sand (M) = (2) − (1) ... (5)
Weight of sand + weight of bottle filled with water (5) + (4) ... (6)
Weight of bottle filled with sand and water same as (3). (7)
Hence, weight of displaced water = (6) − (7) ... (8)
Volume of displaced water (V) (8) × volume of 1 gm water at 30°C
(8) × 1·004 ... (9)
Hence, corrected density of sand $\frac{M}{V}$
Log (5) (5)
Log (9) (9) ...

NOTE. **The density, corrected for the temperature of the water used, is always LESS** (even **for temperatures below 0° C), than the uncorrected density.** If this is remembered, blunders will be impossible.

The density corrected as above represents the **true density**, or the number of grammes per c.c. of the substance **at room temperature**. To determine the **mass per c.c. of the substance at 0°C**, another correction involving a knowledge of the coefficient of expansion of the substance is necessary. Such correction is unusual.

Question. A student performed the above experiment using about one gramme of sand. All weighings were carried out with care. The density of sand turned out to be 100% wrong. How should the experiment be repeated?

Temperature Correction. The density of solids is often found by weighing the body to obtain its mass **M**, and determining the *weight* of the displaced water to obtain **V** the *volume* of the body immersed. The weight of the displaced water in grammes is numerically equal to its volume in c.c. only at 4°C.

At higher temperatures, the volume is greater than the mass. At the highest temperature likely to be met with, 35°C, 1 gramme of water occupies 1·006 c.c. The correction, therefore, amounts to 6 in a thousand or ·6%, and must be applied, if three correct significant digits are expected in the result.

In less accurate experiments, or at lower temperatures, the application of the correction may not be justified. The student should carefully think out this matter for himself, in each case. To apply the correction, use the following rule:

Density corrected for temperature equals
Uncorrected density × Density of water at room temp.

Calculation of Small Corrections. When applying *small* corrections, students often waste a lot of time and labour in elaborate multiplications and divisions which do not improve the final result at all If a correction amounts to 1 percent. of the quantity corrected, it is not necessary that the correction also should be calculated with an accuracy of 1 percent A mistake of 5 percent in the calculation of the correction will affect the final result only by one part in two thousands or $\frac{1}{20}$ percent. which is quite negligible This is best illustrated by an example.

Imagine 100·0 as the uncorrected determination of some quantity and 101·0 as the corrected value. The correction 1 amounts to 1 percent If, in the calculation of the correction 1, you allow an error of 10 percent, and obtain 1·1 instead of 1, the corrected result will be 101·1 instead of 101·0, or wrong by one part in about one thousand or 1 percent, which you can afford to ignore. In other words, an error of 10 percent. in calculating the small correction affects the result only by 1 percent which is negligible. *Hence much time need not be spent* in trying to calculate *small* corrections *with accuracy*.

Example: Uncorrected determination of density of lead, 11·39 Temperature, 35°·2C. Suppose that the density tables given do not supply figures for 36° C Use figure for 35° C Density thus corrected will be 11·32.

A more exact calculation of the correction would involve the use of proportional parts, etc, and the final result would be unchanged as far as four digits (more than which we are not justified in keeping even in the most accurate determinations) are concerned Hence in consulting tables for applying the temperature correction fractions of a degree, and if convenient, even 1°C may be ignored.

DENSITY AND SPECIFIC VOLUME OF WATER FROM 20°C TO 41°C.

[To correct densities for temperature, use d and log d. To correct volumes obtained by loss of weight method, use v and log v. Example: Uncorrected density of sand at 35°C=2·64. Corrected density $=2{\cdot}64\times d_{35}$ = antilog (log 2·64+log d_{35}) = antilog (·4216+$\bar{1}$·9974) =antilog ·4190=2·62. Loss of weight of a glass stopper in water at 30°C=6·94 grammes. Volume of stopper=6·94× v_{30} = antilog (log 6·94+log v_{30})=antilog (·8414+·0017) = antilog ·8431=6·968 or 6·97 c.c.

[*No correction need be applied for temperatures below 20°C*].

Temp °C.	Density or grammes per c c. $d=1/v$.	Log d.	Specific volume or c.c. per gramme $v=1/d$	Log v.
22–23	·998	$\bar{1}$·9991	1·002	·0009
24–27	·997	$\bar{1}$·9987	1·003	·0013
28–30	·996	$\bar{1}$·9983	1·004	·0017
31–33	·995	$\bar{1}$·9978	1·005	·0021
34–36	·994	$\bar{1}$·9974	1·006	·0026
37–39	·993	$\bar{1}$·9969	1·007	·0030
40–41	·992	$\bar{1}$·9965	1·008	·0034

EXPERIMENT 21*a*. - **To determine the density of a liquid with a sp gr bottle.**

Apparatus. Sp. gr. bottle, balance, weights, liquid (kerosene), water, etc.

Weigh the bottle empty, (w_1), and then filled with water, (w_2), and oil, (w_3), respectively. Clean and dry each time. Take the temperature.

Mass of kerosene $= w_3 - w_1$..................................**(M)**

Volume of kerosene in c.c. = weight of water filling bottle

$= w_2 - w_1$..................................**(V)**

$\therefore$ Density of kerosene $= \dfrac{\mathbf{M}}{\mathbf{V}} = \dfrac{w_3 - w_1}{w_2 - w_1}$ (uncorrected for temp.)

Temp : 30°C.

Density of water at 30°C ·996.

Density (of kerosene at 30°C) in grammes per c.c. (as compared with water at 4°C)

$$= \frac{w_3 - w_1}{w_2 - w_1} \times \cdot 996$$

EXPERIMENT 22.—**To determine the density of sugar crystals.**

Apparatus.—Sp. gr. bottle, balance, weights, sugar, kerosene, water, etc.

Carry out four weighings as in expt. 19, substituting sugar for sand and kerosene for water. Then, *if the density of kerosene is not supplied*, carry out a *fifth* weighing, *viz.* bottle filled with water

$$\text{Density of sugar} = \frac{\text{Mass of sugar}}{\text{Volume of sugar}} = \frac{\mathbf{M}}{\mathbf{V}}$$

M is obtained from the 1st 2 weighings.

$$\text{Volume of sugar } \mathbf{V} = \text{Volume of kerosene displaced}$$

$$= \frac{\text{mass of kerosene displaced}}{\text{Density of kerosene}}$$

(See page 93.)

Hence, density of sugar

$$= \frac{\text{Mass of sugar}}{\text{Mass of kerosene displaced}} \text{ density of kerosene}$$

$$= \frac{\text{Mass of sugar}}{\text{Mass of kerosene displaced}} \times \frac{\text{Mass of kerosene filling bottle}}{\text{Mass of water filling bottle}}.$$

Observations:

Mass of empty bottle = ... (1)
Mass of bottle + sugar ... (2)
Mass of bottle + sugar + kerosene = ... (3)
Mass of bottle + kerosene = ... (4)
Mass of bottle + water = ... (5)
Mass of sugar (**M**) = (2) - (1) = ... (6)
Mass of bottle filled with kerosene + sugar (outside) = (4) + (6) = ... (7)

Mass of kerosene displaced $= (7) - (3) = \ldots$ (8)

Mass of kerosene filling bottle $= (4) - (1) = \ldots$ (9)

Mass of water filling bottle $= (5) - (1) = \ldots$ (10)

Vol. of sugar (V) = Vol. of kerosene displaced

$$= \frac{(8)}{\text{density of kerosene}}$$

Hence, density of sugar =

$$\frac{M}{V} = \frac{(6)}{(8)/\text{density of kerosene}}$$

$$= \frac{(6)}{(8)} \times \text{density of kerosene. This}$$

$$= \frac{(6)}{(8)} \times \frac{(9)}{(10)} \qquad \ldots\ldots\ldots\ldots (11)$$

Corrected density = (11) × density of water at laboratory temperature.

Use of the Sp. Gr. Bottle for determining Volumes. It is evident that, in the above experiments, we have been employing the sp. gr. bottle for determining the *volume* of finely divided substances. As *density* was required, certain additional weighings, for determining the *mass*, were necessary. The *volume* of the solid in case water is used), equals the mass of the displaced water. Two weighings alone suffice for this: bottle filled with water+solid in pan, and bottle filled with water and solid inside. The difference gives the mass of the water displaced. In case of soluble solids, the density of the intermediate liquid (kerosene) or extra weighings for determining the latter will be required.

THE HYDROSTATIC BALANCE

The use of the hydrostatic balance for determining volumes by the loss of weight method has already been explained. To determine densities, we require, in addition, the mass which the balance will also give. The following rules should be observed in all experiments with the hydrostatic balance.

RULES FOR USE OF THE HYDROSTATIC BALANCE

(1) For suspensions, use a *single* fine silk thread. Silk is stronger than cotton; hence, a finer thread can be employed. It also does not absorb water so easily.

Be sure, however, to see that the thread is strong enough.

(2) Do not use water freshly obtained from the tap, as it is charged with air bubbles, which stick to bodies suspended in it.

Water that has been kept standing some time in the room, should be used.

(3) Remove air bubbles from bodies suspended in water, with a small brush. Pencils and wires will not do. The beaker and sp. gr. stool should not touch the suspended solid or the pan. The solid should be *completely* submerged, ***even when the balance beam is raised.***

EXPERIMENT 23*a*.—**To determine the density of a moderately large glass or metal object.**

Apparatus: Balance, weights, stool, beaker, silk thread, small brush.

Weigh the body (*i*) in air in the ordinary way, and (*ii*) in water. Take the temperature of the water.

$$\text{Density} = \frac{\mathbf{M}}{\mathbf{V}} = \frac{\text{Mass of solid}}{\text{Volume of solid}} = \frac{\text{Mass of solid}}{\text{Vol. of water displaced}}$$

$$= \frac{\text{Mass of solid}}{\text{Loss of wt. of solid}}.$$

Density corrected for temperature $\dfrac{\text{Mass of solid}}{\text{Loss of weight}}$ × density of water at...°C.

EXPERIMENT 23*b*. To determine the density of a liquid.

Apparatus: Same as in expt. 22; in addition, a liquid (kerosene).

Weigh the solid body (vol. v c.c.) in air, water and kerosene. Take the temperature

$$\text{Density of liquid} = \frac{\mathbf{M}}{\mathbf{V}} = \frac{\text{Mass of } v \text{ c.c. of liquid}}{\text{Vol. of } v \text{ c.c. of liquid}} = \frac{\text{Loss of wt. of solid in liquid}}{\text{Loss of wt. of solid in water}}.$$

Correct for temperature as usual.

EXPERIMENT 23.—To determine the density of a solid lighter than water.

Apparatus: Hydrostatic balance arrangement, weights, piece of varnished wood, a sinker (lead ball or metal cylinder)

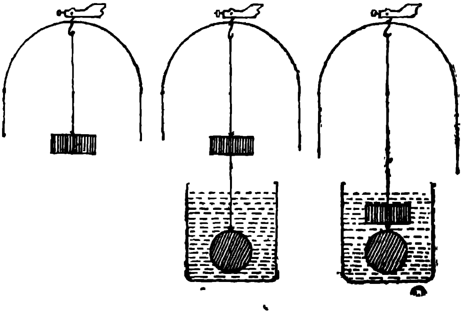


Fig. 31. Determining the density of a solid lighter than water.

Carry out three weighings, as represented in the diagram. The body weighed in water should be completely immersed (without touching the sides) when the beam is raised, and all air bubbles should be removed with a small brush.

Record observations thus:

Weight of wood in air ....(1)
Weight of wood in air + sinker in water ....(2)
Weight of wood and sinker in water ....(3)
Temp...°C

$$\text{Density of wood} = \frac{M}{V} = \frac{\text{Mass of wood}}{\text{Vol. of wood}}$$

$$= \frac{(1)}{\text{Vol. of displaced water}}$$

$$= \frac{(1)}{(2)-(3)} \times \text{density of water at laboratory temperature.}$$

If the solid can sink in oil, the sinker need not be used. Use oil, instead of water, in the above experiment. Density of solid equals

$$\frac{\text{Mass of solid}}{\text{Loss of wt. in oil}} \times \text{density of oil.} \quad \text{(See page 93).}$$

EXPERIMENT 23*c*.—To determine the density of a solid soluble in water.

Employ oil and use formula given above.

If density of oil is not given, determine it as in Expt. 23*b*.

This means an additional 3 weighings, as a different solid has to be weighed in air, water and oil.

EXPERIMENT 23*d*.—To determine the density of a solid soluble in water and lighter than oil.

Determine mass of oil displaced by use of a sinker as in Expt. 23. Obtain or determine density of oil.

Density of solid can then be deduced by formula given above.

CHAPTER VIII

ARCHIMEDES' PRINCIPLE AS APPLIED TO FLOATING BODIES

If a body of volume v c.c is immersed in water, the loss of weight is v grammes. If the body itself weighs less than v grammes, the whole of it will not remain immersed. It will float. That is, it will have no weight. A thread supporting it will not be pulled, and is unnecessary. This means that the loss of weight due to partial immersion, just equals the weight of the body. Hence, for floating bodies

Mass of liquid displaced due to partial immersion of a floating body = mass of floating body.

A floating body will, therefore, come to rest when as much of it is immersed as will displace a mass of the liquid equal to its own. The volume immersed will, therefore. vary inversely as the density of the liquid.

Fig. 32.—The floating tube or rod : a simple hydrometer.

EXPERIMENT 24.—**To determine the density of kerosene oil with a cylindrical floating body.**

Apparatus : Cylindrical floating body (hollow glass tube weighted with shot, or varnished wood cylinder weighted at bottom). Jars containing kerosene and water.

A hollow glass tube can have its lower end closed and flattened on the blowpipe ; or the bottom of a test tube may be flattened. Put in a suitable amount of lead shot to enable the tube to float vertical. Enclose a paper scale of 10 equal divisions with 0 at bottom.

Float in water, and take the reading opposite the surface. (say, 7·1). Float in oil, and similarly read (8·1)

Enter thus:

Temp.........°C.
Volume of tube per division $= a$ c.c
$\therefore$ Mass of $7{\cdot}1 \times a$ c c of water = mass of tube
= Mass of $8{\cdot}1 \times a$ c c. of oil. Or
$8{\cdot}1 \times a \times$ density of oil $= 7{\cdot}1 \times a \times 1$

$$\therefore \quad \text{Density of oil} = \frac{7{\cdot}1}{8{\cdot}1} = {\cdot}876 \text{ or } \underline{{\cdot}88}.$$

Question. Is it worthwhile correcting for temperature? Give reasons.

Hydrometers are floating bodies, used like the tube in experiment 24 for density determinations. For greater sensitiveness, the shape is that of a bulb at the bottom, and a narrow graduated stem at the top. They are graduated by having their zeros fixed in water, or salt solutions of known concentration, or in other empirical ways. The densities of liquids in which they float can be obtained from the readings of the surface, sometimes directly, oftener by using special formulæ. The systems in use are the following:

(1) **Direct reading Hydrometer.** The graduations are numbered as 1000, 1020, 1030. The density is obtained by fixing a decimal after the first 1: 1000, 1020, 1030 respectively stand for 1·000, 1·020, 1·030.

(2) **Beaume's Hydrometer**: **Heavy** liquids: sp. gr. $= \dfrac{146}{146 - \text{reading}}$;

Light liquids „ $= \dfrac{146}{136 + \text{reading}}$.

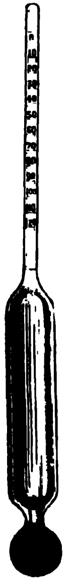


Fig. 33.
Hydrometer.

(3) **Twaddel's Hydrometer,** used for **heavy** liquids only.

$$\text{Sp. gr.} = \frac{1000+5\times\text{reading}}{1000}$$

(5) **Lactometers, salinometers, urinometers,** are used for milk, salt solutions, and urine. They are, sometimes, specially graduated to give directly the concentration of the solution concerned.

EXPERIMENT 25.—**To determine the density of**

(1) **Saturated salt solution, by**
  (*i*) **Direct reading,**
  (*ii*) **Beaume's and**
  (*iii*) **Twaddel's hydrometers.**

(2) **Oil, by Beaume's hydrometer.**

Float hydrometer carefully. Cotton wood or sand at the bottom of jars is advisable. Wipe and dry after each reading. The hydrometer should not touch the jar, and bubbles sticking to it should be removed with a brush Use appropriate formulæ.

Nicholson's Hydrometer. The above are called variable immersion (but constant mass) hydrometers, because different volumes are immersed in different liquids. Nicholson's hydrometer is a constant immersion (but variable mass) instrument. It is immersed up to a fixed mark in all liquids, weights being added to it, as necessary. It is, in fact, nothing but a **floating balance.** and, like the hydrostatic balance, can be used for determining masses, volumes, densities, etc. It cannot, however, approach the balance in accuracy. It has, also, the disadvantage of requiring large quantities of liquids.

Figure 34 shows a Nicholson's hydrometer. The instrument is always immersed up to the mark F. There is a pan on the top, and the conical bulb at the bottom, serves as the second pan for weighings in the liquid. A wire gauze cage round the lower pan is of use when weighing bodies are lighter than the liquid. The mass and volume of the hydrometer are so arranged that *some* weight in the upper pan is necessary before the instrument will sink up to the mark F If the volume of the hydrometer below F is **V** c.c., the mass of the hydrometer + any mass added to it must equal the mass of **V** c.c. of the liquid used.

EXPERIMENT 26. **To determine the density of kerosene with a Nicholson's hydrometer.**

Weigh the hydrometer. Place the hydrometer in kerosene in a jar, and add weights on the top, so that the liquid surface is at F. A U-shaped piece of wire on the top of the jar, will prevent the pan from accidentally touching the liquid surface Remove, clean, dry, and determine weights required for sinking the hydrometer up to the mark F, in water

By Archimedes' Principle.

Mass of **V** c.c. of liquid

M = Mass of hydrometer + mass in pan when floating in liquid = ... (1).

Mass of **V** c.c. of water

V = Mass of hydrometer + mass in pan when floating in water = ... (2).

The ratio of (1) and (2) gives $\frac{\mathbf{M}}{\mathbf{V}}$ or the density of the liquid.

Correct for temperature as usual.

Avoid bubbles on the hydrometer. Water fresh from the tap should not be used. Use water that has been standing some time.

EXPERIMENT 27. **To determine the density of a solid (lighter or heavier than water) with a Nicholson's hydrometer.**

Take a small piece of the solid (2 or 3 gms.).

Float the instrument in water, and record weights as under

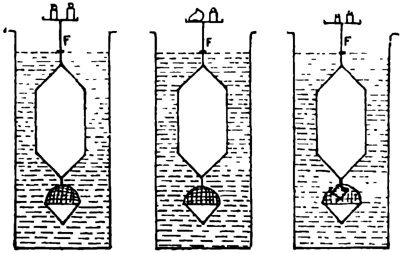


Fig. 34.--Nicholson's hydrometer, a floating balance.

Observations :

Weights on top, hydrometer in water up to F = ... (1)
,, ,, with solid in top pan ,, ,, = ... (2)
,, ,, with solid in water in lower pan ,, ,, = ... (3)
Mass of solid = **M** = (1) -(2) = ... (4)

Volume of solid = **V** = Volume of water displaced by solid
= loss of weight = (3) − (2) = (5)

$$\text{Density} = \frac{\mathbf{M}}{\mathbf{V}} = \frac{(1)-(2)}{(3)-(2)} = \frac{(4)}{(5)}.$$

Correct for temperature if you think it justified

Record in your note-book answers to the following questions

1. What is the smallest weight which definitely affects the hydrometer ?
2. What % is this minimum weight, of the weight of the solid used ?
3. Why is the ordinary balance considered more accurate than this floating balance ?

Jolly's balance is essentially a spring balance with a specially sensitive spring There are two pans at the bottom, the upper one for use in air, and the lower one for determining loss of weight in liquids The spring and pans are mounted in front of a mm scale on looking glass

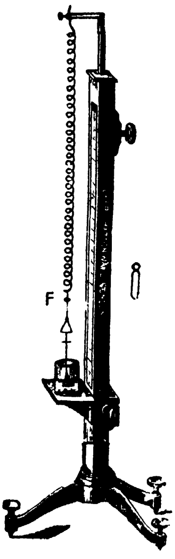


Fig 35 —A spring balance (Jolly's) used at an hydrostatic balance

There are two ways of using the instrument In one, the readings on the scale, of a fixed point F on the spring, indicate the weights or masses in the pans In the other way, weights from a box are added and densities determined as in Nicholson's hydrometer, a small piece of the solid being first placed in the upper pan, and then in the lower, and weights adjusted each time till the reading is the same The pans can be removed, and a small sinker attached for comparing densities of liquids by the loss of weight method The instrument is often used by geologists, for rapidly determining densities of minerals, when great accuracy is not necessary For a physics laboratory, nothing is so accurate as the ordinary balance

EXPERIMENT 28 —**To determine with a Jolly's balance the sp gr. of kerosene**

Remove the pans and attach the small sinker provided Read the position of the bead or other mark When reading, *place your eye in a line with the bead and its image in the mirror, so as to avoid* **the error of parallax.** Repeat reading with sinker in (*a*) water (*b*) kerosene

Density of kerosene =

$$\frac{\text{Loss of wt of sinker in kerosene}}{\text{Loss of wt of sinker in water}}$$

$$= \frac{\text{Reading with sinker in air} - \text{reading with sinker in kerosene}}{\text{Reading with sinker in air} - \text{reading with sinker in water}}.$$

The fine wire above the sinker should be immersed approximately up to the same point in the two liquids.

EXPERIMENT 28*a*. **To determine with a Jolly's balance, the sp. gr. of a quartz crystal.**

Place the crystal in the upper pan, immerse the lower pan in water, and read the bead F. Remove the crystal and substitute weights, till the reading is the same. The weights evidently equal the mass of the crystal. Now place the crystal in the lower pan, and adjust the weights in the upper pan, till the reading is the same. The weights in the upper pan now equal the loss of weight. Hence

$$\text{Density of crystal} = \frac{\text{weights in first case}}{\text{weights in second case}}.$$

Record answers to the following :

(1) What part is the minimum weight to which the spring easily answers, of the weight of the crystal used ?

(2) How many digits in the value of density would you consider correct?

CHAPTER X

HYDROSTATIC PRESSURE

COMPARISON OF DENSITIES OF LIQUIDS BY BALANCING COLUMNS

U-tubes erect or inverted and various combinations of them, can be used for comparing densities of liquids. The principles, underlying the method, are the two well-known laws of hydrostatic pressure :

(1) **The pressure at any point in a liquid is proportional to the density of the liquid, and the depth of the point below the surface.** It equals (in grammes weight) $h\,d$ where h is the depth in cm., and d the density in grammes per c.c. In absolute units, the pressure is hdg, 'g' being the acceleration of gravity.

(2) **The pressure at two points in a horizontal plane is the same, provided they are both situated in a continuous mass of the same liquid.**

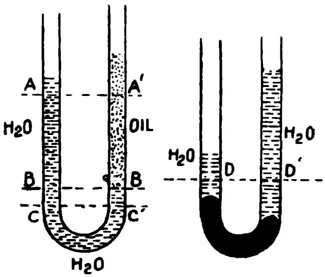


Fig. 36.—Hydrostatic pressure.

Illustration. Pressure at A and A′ (Fig. 36) in the same horizontal plane, is *not* the same, because A and A′ are not in the *same* liquid. B and B′ have the same pressure; so have C and C′.

Pressure at D and D′ in the same horizontal plane and in the same liquid, water, is *not* the same because D and D′ are not in the same *continuous* liquid.

From (1) and (2) it follows that if h, h', are the heights of two liquid columns (densities d, d',) measured from their point of junction B (Fig. 36) in a U tube,

$$h\,d = h'\,d'.$$

Besides the **plain U tube,** the **double U or W tube** (easily made out of 2 U tubes and a bit of rubber tubing), and the inverted U tube or **Hare's apparatus** are employed. The plain U tube cannot be used for comparing liquids that are miscible, or act on one another It is not well suited even for immiscible liquids, for the junction of the two liquids is often distorted, and water will not move freely in tubes soiled by oil, etc. In Hare's apparatus, it is not easy to read the position of the liquids in the beakers. The double U tube is free from these drawbacks. It is easily read, and can be used for all liquids, miscible or immiscible.

If the tubes are mounted on scales, the positions of various points on them can be read with ease. If merely supported in clamps, readings are best taken on a metre-stick held vertically against the tubes, with the lower end resting on the table. A set square if suitably used increases the accuracy of the readings.

EXPERIMENT 29—**To determine the density of oil by means of a plain U tube.**

Apparatus: Plain U tube about 1 cm. bore and 50 cm. high, oil, water, metre rod, set square.

Support the clean and dry tube vertically, by means of clamps and a plumb-line. Half fill the limbs with water. With a small funnel, pour oil *gently* down one side, till the

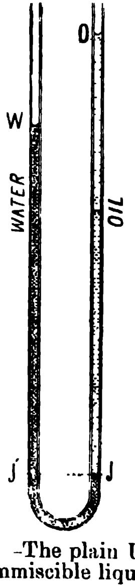


Fig. 37. -The plain U tube for immiscible liquids.

junction is only 4 or 5 cm. above the bend of the U. Place a vertical metre stick, *with one end on the table*, parallel to the tube, and read the position of the junction J. Next read the free surfaces of oil and water, O, W. Calculate OJ and WJ′ (J and J′ are on the same level).

Now as pressure at J′ and J is the same,

$$\mathbf{WJ' \times density\ of\ water = OJ \times density\ of\ oil}$$

$$\text{or } \mathbf{Density\ of\ oil} = \frac{\mathbf{WJ'}}{\mathbf{OJ}} \times 1 \text{ (uncorrected)}$$

$$= \frac{\textbf{Height of water column}}{\textbf{Height of oil column}}.$$

(NOTE:—Densities are *inversely* as heights of balancing columns).

Empty the tube, dry, rinse with petrol, and dry. Refill and determine the density a second time. *Small columns should not be used.*

Observations:

All heights read on a metre stick with zero end on table.

	Height of junction	Height of oil surface	Height of water surface	Length of oil column	Length of water column	Density of oil
1	10·15	42·36	39·01	32·21	28·86	$\frac{28·86}{32·21}$ = ...
2	...	...	...	...	...	...

Mean density = ...

Temp. = ...°C. Density corrected for temperature = ...

Record in your note-book answers to the following:

(1) How will *small* bubbles on the sides affect the result? How will *large* bubbles, that break up

the columns, affect the result (see law 2, p. 112)?

(2) Suppose the two tubes have different diameters. What corrections, if any, will be necessary?

EXPERIMENT 30. - **To determine the density of syrup, sulphuric acid, or methylated spirit with the double U tube.**

Apparatus: W tube, meter stick, clamps, sulphuric acid, water

To construct the W tube, connect two U tubes, like those used in the previous experiment, with a piece of rubber tubing six inches long.

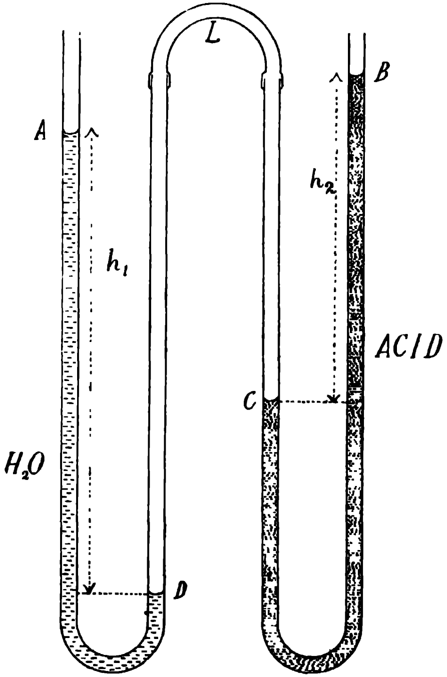


Fig. 38.—The W or double U tube for miscible liquids.

Clean the tubes and clamp vertical. Pour water at A till the two adjacent limbs are half filled. Pour sulphuric acid carefully down B till liquids stand as shown.

Read the heights of A, B, C, D above the table. Calculate h_1 and h_2. Read the temperature. Density of sulphuric acid $= \frac{h_1}{h^2} \times$ density of water at laboratory temperature.

Repeat the experiment so as to obtain a second value. It is evident that the air in the bend of the W is compressed. The extra air pressure (above atmospheric pressure) at C and D is balanced by the equal pressures exerted by the two liquid columns.

Record observations thus:

All heights read on a metre stick with zero end on the table.

Height of water in open tube	Height of water in closed tube	Length of water column	Height of acid in open tube	Height of acid in closed tube	Length of acid column	Density of acid
cm.	cm.	cm	cm.	cm.	cm.	
42·19	24·07	18·12	38·94	28·90	10·04	$\frac{18\cdot12}{10\cdot04} = 1\cdot80$
...	...	...	...	...	...	...

Mean density = ... Temp = ...°C.

Density corrected for temperature = ... × .... = ...

If a side tube is provided at L, C and D can be made to stand *above* A and B by sucking at this side tube. The mode of calculation will remain unchanged.

Record answers to the following in your book :

(1) Can you read the columns correctly to tenths of a millimetre? Give reasons.

(2) Suppose the columns 40 cm. long, and that you could, in any case, depend on correctly measuring within ·5 mm. What is the percentage accuracy?

(3) Place the W tube, the hydrometer, Jolly's balance, and the ordinary balance in descending order of accuracy attainable with each.

EXPERIMENT 30*a*.—**To determine the density of a liquid with Hare's apparatus.**

Apparatus : Hare's apparatus, water and salt solution in two beakers, metre stick, two pointed rods or knitting needles, clamps, stands etc.

Clean the apparatus, if necessary. Strengthen all joints by means of thread tied round the rubber tubes. Clamp the tubes vertical, and place the beakers as shown, with the ends of the tubes well below the surfaces. With the bulb shown, or with a clean well washed piece of glass tubing in the mouth, suck till the liquids are near the top, and close the pinch cock firmly. Watch if the columns are going down. If so the pinch cock or the joints are defective. Do whatever seems necessary *till the liquid surfaces in the tubes are stationary.* Let the higher water surface in the tube be called A and the lower water surface in the beaker A′. Let B and B′ represent the corresponding surfaces for the salt solution.

Measuring the columns AA′ *and* BB′. The usual method of reading the heights of A, A,′ B and B′ above the table is unsuitable, because (*i*) on account of the presence of the beakers the metre stick cannot be brought close to the columns, and (*ii*) the surfaces in the beakers are not well defined, and their position cannot be read with certainty.

To avoid these difficulties, fix the two pointed rods in corks and support these firmly in clamps. Arrange the rods at a distance of about 1 to 2 cm. from, and parallel to, the tubes and adjust their

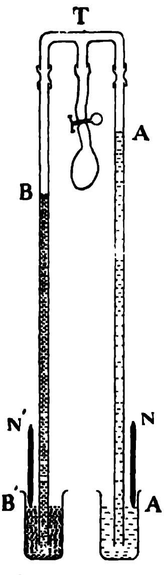


Fig. 39.—Hare's apparatus.

position till the lower points just touch the liquid surfaces in the beakers. *If the rods are kept very close to the glass tubes the liquid may rise between them.*

To obtain the height AA′ measure AN the height of A above the upper end of the rod. This can be done by placing the zero end of a vertical metre stick on the pointed end of the rod and reading the stick opposite A the top of the column. The rod must be firmly clamped, so as not to yield to the weight of the stick. Read similarly B N the height of the upper salt solution surface above the end of the rod. To these heights the lengths of the respective rods will be added *in the end.*

Question. Suppose that the zero end of the stick is worn ·5 mm. Estimate the % error caused in the result if the shortest column used is 20 cm. long. Is this error negligible ?

Next reduce the columns by about five centimetres. Make sure that the columns are stationary, and repeat the readings. Obtain in this way six pairs of readings with columns varying from about 50 cm. to 20 cm. but not less. The lower ends of the rods should be frequently examined to see that they just touch the liquid surfaces, without dipping. Finally, remove the rods, and measure their lengths with a metre stick. Record as below and finally draw a graph.

To draw the graph, place *the zero at the origin as a special case and select equal scales for abscissae and ordinates.* Make l_1 (water column) the ordinate and

l_2 (salt solution column) the abscissa. The graph will be a straight line, passing through the origin. Draw the graph with the help of a stretched thread so that the points lie *on it or evenly round it.* $\frac{l_1}{l_2}$ is the tangent of the angle made by this graph with the horizontal axis. Read l_1 and l_2 on the squared paper for any point *high up on the graph.* The ratio $\frac{l_1}{l_2}$ thus calculated gives the *mean* density of the salt solution (see page 114).

Record as follows:

Temperature = ...°C.

AA	LENGTH OF ROD A	LENGTH OF WATER COLUMN l_1	BA′	LENGTH OF ROD A′	LENGTH OF SALT SOLUTION COLUMN l_2	DENSITY OF SALT SOLUTION l_1/l_2
cm.	cm.	cm.	cm.	cm.	cm.	
30·44	20·00	50·44	25·02	20·12	45·14	$\frac{50·44}{45·14} = 1·11$
...	...	...	...	...	...	...
Such six readings						

Mean density from last column = $\frac{\dots\dots\dots}{6}$ = ...

Mean density from graph = $\frac{\dots\dots\dots}{\dots\dots\dots}$ = ...

Final mean = $\frac{\dots\dots\dots}{2}$ = ...

True density of solution *at laboratory temperature* = ... × density of water at laboratory temperature.

Balancing columns 50 cm. long provide a convenient and rapid method of comparing densities of liquids with moderate accuracy. The sources of error (especially with a W tube) are very few. The shape and size of the bore are, within limits, immaterial. Small quantities of liquids suffice (*cf.* hydrometers), weights are unnecessary, and small bubbles can be ignored.

CHAPTER IX—*contd.*

THE BAROMETER

Atmospheric Pressure is a Force. When we state that the atmosphere exerts a **pressure,** we mean that it exerts a certain **force per sq. centimetre** (or per square inch) of surface. Now the weight of water, mercury, and in fact anything, is a **force.** We can, therefore, compare the force called **atmospheric pressure** with the force **represented by the weight of columns of mercury or water**.

In the siphon barometer (Fig. 40), the barometric height is shown as equal to 76 cm. of mercury. This means that the pressure due to the weight of a mercury column of 76 cm. height, equals the pressure due to the weight of a column of the atmosphere, supposed to be contained in an imaginary continuation of the other limb of the barometer (U tube principle).

If the area of cross section of both limbs be regarded as 1 $cm.^2$, the mercury in the column will have a volume of 76 c.c., and we can say that the

Atmospheric pressure per sq cm. = Weight of 76 c.c. of mercury.

Temperature Correction. The weight of 76 c.c. of mercury, however, depends on temperature. Hot mercury is lighter than cold mercury. The temperature of mercury is. therefore, a cause of uncertainty. For instance if we cool the mercury column in the above siphon barometer with ice, the column of heavier mercury required to balance the atmospheric pressure would be shorter. The atmospheric pressure is however, the same It is the practice, therefore, to measure atmospheric pressure in terms

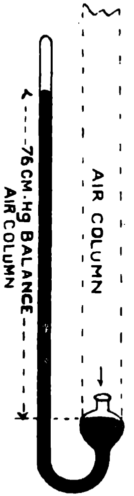


Fig. 40.—The barometer, an illustration of the U tube principle.

of mercury at 0°C A mercury column of 76 cm at 30 C is equal in pressure to a mercury column of somewhat less than 76 cm at 0°C In India (the plains) the temperature is, generally, above 0°C The effect of correcting a barometric height for temperature is therefore, always to *lessen* it (Remember this to avoid blunders)

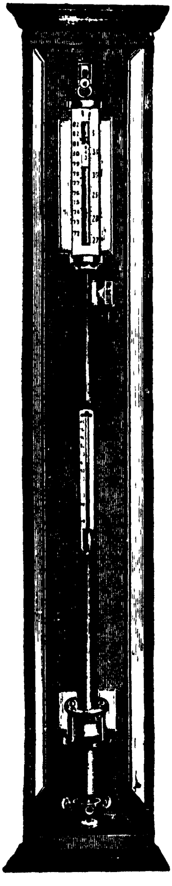


Fig 40a
Fortin's barometer.

Temperature also affects the length of the divisions on the brass scale on which the height is read If the scale be correct at 0°C, then, if the barometer reads 76 cm at 30°C, the real height is more than 76, because the distance between the 0 and 76 cm mark is, at 30°C, greater than 76 cm *The correction due to the expansion of the scale is, therefore, always positive* This correction is generally combined with the correction for change in the density of mercury, and both applied in one operation

Latitude Correction. 76 c c of mercury at 0°C weigh more (and consequently exert more pressure) at the poles than at the equator, because the poles are nearer the centre of the earth than the equator This cause of uncertainty is removed by applying a latitude correction, which reduces the height of the barometer at any latitude to the height of a column at standard lat 45° (mean between pole and equator) exerting the same pressure The latitude correction is nearly −1 mm for the Punjab and Kashmir

The Fortin's Barometer. In a siphon barometer, the level of mercury in the cistern rises and falls as the column changes in height. This makes the measurement of the height by a fixed scale difficult, as the zero reading is always changing. In a Fortin's barometer (Fig. 40a) the tube dips in a cistern with a leather bottom The bottom can be raised or lowered by a screw, so as to bring the surface of the mercury always to the zero of the fixed scale. This zero coincides with the tip of the ivory peg just above the mercury in the cistern.

When the mercury surface just touches the tip of this peg, the reading of this surface is zero, and the reading of the top surface on the scale gives the barometric height at once.

EXPERIMENT 31.—**To read the Fortin's barometer, to correct the observed height for temperature and latitude, and to calculate the pressure of air.**

Apparatus : Fortin's barometer

Near the bottom of the barometer is a horizontal ring with three screws directed towards its centre. These should be screwed back so as to let the barometer hang freely in the vertical position. The screws should then be pushed in, so as to fix the barometer in this vertical position. This adjustment is generally supposed to be made, and the student should consult the demonstrator as to whether it is necessary.

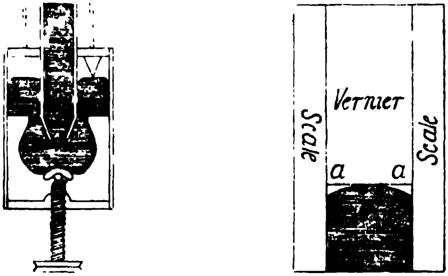


Fig. 40*b*. Top and bottom adjustment in Fortin's barometer.

Now lower the mercury surface so that the peg is clearly seen against the white milk glass background provided. Next, gently turn the adjusting screw up till the peg, as seen against the white background, just touches the mercury surface (Fig. 40*b* left). The reading device for the top consists of a short metal tube, which surrounds the barometer tube. The front lower edge of this tube is the zero of a vernier scale engraved on it. Raise this tube above the mercury surface, and then carefully lower it so that looking in a line with the front and back edges of the tube the white background behind, just disappears at the top of the convex surface of the mercury, but remains visible at the corners if these are not covered. If the vernier is on a *flat* plate, adjust this plate so that, looking *horizontally* the zero of the vernier just touches the *convex top* of the mercury column. The vernier is to be read in this position (Fig. 40*b* right .

To calculate the vernier constant, consult the directions given in connection with verniers (pages 40—42).

Next read the thermometer (centigrade) in the case to the nearest degree (for reason see page...).

The height corrected for temperature and latitude (reduced to 0°C and 45° latitude) is in cm.

$$_{45}H_0 = H_t\ (1 - \cdot 00016t) - \cdot 19\ \cos 2\phi\,.$$

where ϕ is the latitude of the laboratory.

The coefficient of expansion of mercury is $1\cdot 8 \times 10^{-4}$, and that of brass (linear) $1\cdot 8 \times 10^{-5}$. The difference of the two or ·00016 is used, so that the height may be simultaneously corrected for expansion of mercury and the brass scale. To correct for latitude *subtract* from the height corrected for temperature $1\cdot 9 \cos 2\phi$ millimetres or $\cdot 19 \cos 2\phi$ cm., where ϕ is the latitude of the place.

The latitude correction for principal laboratories in the Punjab and Kashmir is

Delhi	(lat. 28°·7)	...	−1·0 mm
Lahore	(„ 31°·6)	...	−0·9 „
Amritsar	(„ 31°·7)	...	−0·9 „
Jammu	(„ 52°·6)	...	−0·8 „
Peshawar	(„ 34°·0)	...	−0·7 „
Srinagar	(„ 34°·2)	...	−0·7 „

Use in practice, the following simplified relation applicable to barometric heights of 72—76 cm. The latitude correction varies from −0·7 mm. in Srinagar to −1·0 mm. in Delhi.

Corrected height (*millimetres*)		Observed height (*millimetres*)		Temperature correction (*millimetres*)		Latitude correction (*millimetres*)
${}_{45}H_0$	$=$	H_t	$-$	$\cdot 12t$	$-$	$(\cdot 7 \text{ to } 1)$

The weight of ${}_{45}H_0$ c.c. of ice cold (0°C) mercury in latitude 45° equals the pressure of the atmosphere in the laboratory at the time the barometer is read. If the density of mercury at 0° be 13·60 grammes per c.c., the mass of ${}_{45}H_0$ c.c. is ${}_{45}H_0 \times 13{\cdot}60$ grammes. The weight of one gramme in latitude 45° is 980·6 dynes.

∴ Atmospheric pressure in dynes $= {}_{45}H_0 \times 13{\cdot}6 \times 980{\cdot}6$.

Observations :

If the barometer vernier is graduated to give fractions of a mm. *directly*, the vernier constant should, nevertheless, be calculated, and the values directly given *checked*. *After this*, the fractions can be read directly *without actually counting the divisions*.

Determination of vernier constant :

Principal scale divisions = 1 mm.
25 *v. d.* = 24 *s.d.*
1 *v. d.* = 24/25 *s.d.* or less than 1 *s.d.* by 1/25 mm.
∴ Vernier constant = 1/25 mm. = ·04 mm.
Observed height = 734·9 mm.
Temperature = 29°·5C (say 30°C)
Latitude of Lahore 31°·6
∴ Barometric height corrected for temperature and latitude (reduced to 0°C and lat 45°) ${}_{45}H_0$ = 734·9 − ·12 × 30 − ·9 mm.
= 730·4 mm.
= 73·04 cm.

Atmospheric pressure in dynes per sq. cm. = 73·04 × 13·60 × 980·6
= ... (retain 3 significant figures only).

CHAPTER X

GRAPHS AND THEIR USES

Suppose you were, for some reason, very much interested in your weight, and kept in your diary a record of monthly weighings, carried out on the 15th day of every month, for a whole year. The record, if put together with some additional memoranda from the diary, may run like this :

January	15	1925.	120·5	lbs.	
Feb.	15	...	120·7	,,	7th, getting warm,
March	15	...	119·6	,,	
April	15	...	118·0	,,	Very hot.
May	15	...	112·0	,,	
June	15	...	104·6	,,	11th, sleepless night.
July	15	...	107·2	,,	5th, left for Simla.
Aug.	15	...	115·3	,,	Putting on weight.
Sep.	15	...	125·3	,,	More and more. Weather fine.
Oct.	15	...	125·9	,,	Return to college.
Nov.	15	...	117·2	,,	Must lose in plains.
Dec.	15	...	119·0	,,	Healthy winter.
January	15,	1926.	122·5	,,	Better than last year.

A study of the record may enable you to find out the way in which your weight varied with the time of the year and the weather, etc. You would, for instance, gather that you lost in summer, and gained at Simla, on account of the cooler weather. On return to the warmer plains, you lost some of the weight put on, though the healthy winter soon set things right, so that on January 15, 1926, you weighed 2 lbs. more than on January 15, 1925.

The way in which the weight varied with the time of the year, and other related causes, could have been more easily seen on a **chart,** similar to the temperature chart prepared by a doctor for watching the career of a fever patient. The above 13 entries are shown plotted in the accompanying chart or "**graph**" (Fig. 41) on squared paper.

How to Plot a Graph. A horizontal line *OX*, and a vertical line *OY* intersecting at *O* towards the left hand bottom corner, are the axes. The factor *whose changes interest you less* (in this case, *time*), is the **independent variable.** This is represented by 13 equal lengths, laid out along *OX* the *horizontal axis* to represent 13 months (supposed equal). The factor *in whose changes you are particularly interested* is the weight. This is the

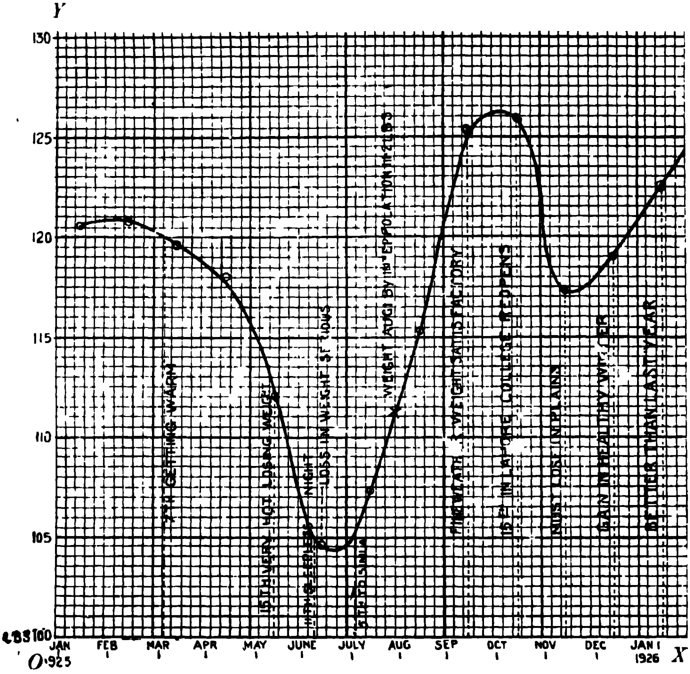


Fig. 41.—Graphical representation of health memoranda from a private diary.

dependent variable. This is marked out along the *vertical axis* OY, by 5 convenient lengths, representing 5 lbs. each. The origin O stands for 100 lbs., the nearest round number less than your lowest entry, 104·6 lbs. The horizontal lengths are called **abscissæ,** and the vertical lengths **ordinates.**

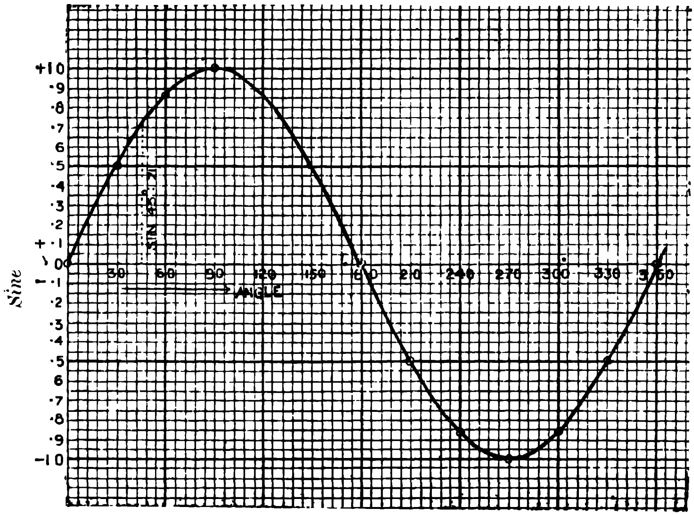
To start plotting the entries in the list, we take the first pair of values, January 15 and 120·5 lbs. The middle of the length representing January stands for January 15. Draw (or imagine drawn) through this point a vertical line. On OY, fix the point representing 120·5 lbs., and draw (or imagine) through this a horizontal line parallel to OX. These 2 lines intersect at a point whose height above OX is proportional to 120·5 lbs., and whose horizontal distance from OY represents January 15. The point, therefore, stands for the entry, January 15—120·5 lbs., *because by its* **abscissa** *and its* **ordinate** *it can represent January 15 and 120·5 lbs., respectively* The other pairs of values are plotted similarly. Thirteen points, marked by small circles, having been thus determined, a smooth line or "**graph**" can be drawn through them. The height of this graph above any date is proportional to your weight on that date.

The maxima and minima can be read at a glance (*and very much better than in a list of data*), and noticing the *worst* minimum of 104·6 lbs. above *June 15*, you conclude at once, that the weight is least when the summer is at its worst. In winter, the curve is high. The highest point corresponds to September 15 in Simla. On return to the warmer plains, the curve descends. As the winter approaches, it begins to rise again. CONCLUSION:

The warmer the weather the less the weight.

Graphs are used in this way, to illustrate more complicated relations. A dozen pairs of numbers will often disclose nothing, and may look unintelligible. If plotted on squared paper, however, the shape of the curve may disclose some law, according to which one set of data vary with the other set.

As another example, we shall study graphically how the sine of an angle varies, as the angle steadily increases from 0° to 360°. We have already seen that, as the angle increases, the sine rises up to 1 at 90°, falls to 0 at 180°, changes from 0 to − 1 from 180° to 270°, and is again 0 at 360°.



. Fig. 42.- -A sine curve.

The following table gives the sine for 0°, 30°, 60°, ... 360°.

Angle	Sine	Angle	Sine	Angle	Sine
0°	0·00	120°	·87	240°	− ·87
30°	·50	150°	·50	270°	− 1·00
60°	·87	180°	0·00	300°	− ·87
90°	1·00	210°	− ·50	330°	− ·50
				360°	− 0·00

As some of the ordinates in this curve will be *negative,* the y axis will be produced *downwards.* The ordinates vary between 1 and -1: so equal lengths below and above the x axis will be necessary. These lengths may conveniently be divided into 10 parts, each representing ·1. Twelve equal lengths, each representing 30°, may be laid out along the x axis to represent the various angles. (See Fig. 42).

Any convenient lengths can be chosen to represent 30° and ·1. The scale actually selected depends on the size of the paper available.

The importance of graphs in physics experiments lies in that the shape of a graph may, *at a glance,* show the nature of the relation between the quantities represented by the abscissæ and the ordinates. This relation is then called **the equation of the graph**. For instance, **a straight line graph shows that for fixed equal increments of the abscissa the ordinate also increases by equal amounts.** In addition, *graphs can be used for obtaining the values of ordinates corresponding to those values of the abscissae, which are not included in the list of data supplied.* For instance, from the sine curve (Fig. 42) it is possible to read sin 45° (not given in the list). The vertical through 45° cuts the curve at ·71, which, therefore, is sin 45°. Similarly, the weight corresponding to August 1, though not in your diary, can be read off from the curve (Fig. 41) as 111·2 lbs. This is called **reading by interpolation,** and corresponds, in reading tables, to **calculation by proportional parts.**

Graphs or curves most frequently employed in physics are **straight lines,** *because they are the easiest to study and recognise.* Mère inspection will show if a graph is a straight line, while it is not so easy to say if a curved graph forms a portion of an ellipse or a circle. When studying data by the graphical method, reciprocals, squares, cubes or logarithms of one or both the sets of data are, therefore, often plotted, instead of the numbers themselves, the functions selected being such as to yield a *straight line* as the 'curve'.

Important Rules about Graphs.

Graphs are plotted on paper ruled in mm. squares.

To obtain a neat, well proportioned curve in any case, the following rules should be kept in view:

(1) Horizontal and vertical lines, intersecting at the bottom left hand corner of the page (if all the data are positive), should be selected as axes. If negative quantities are included, the point of intersection (called the **"origin"**) may be in the middle of the page, or in the middle of the left hand vertical or lower horizontal side.

(2) The quantity whose changes do not interest you much, or the independent variable, should be plotted along the horizontal axis or as abscissæ. *The quantity whose changes are the special object of study should be plotted along the* **vertical axis** *or as* **ordinates**.

(3) **It is not necessary that the origin should read zero.** *The origin should stand for the nearest round numbers less than the smallest abscissa and ordinate.* For instance, in the first curve, the origin represented 100 lbs. If the origin had represented 0 lb., the portion of the paper between 0 and 100 would have been wasted, and the portion from 100 to 125 alone (one-fifth of the whole), utilized. *The curve would have been flat and unduly compressed.*

(4) *The scale selected should be such that the curve is well within the paper, without much of it lying useless.* To do this, lay off along the axes, lengths to represent a little more than the difference between maximum and minimum values of the quantities, respectively represented by the axes, and figure, in increasing order, the points representing equidistant round numbers. For instance, if the abscissæ run from 22 to 43, the origin may represent 20 and the figuring of OX may be 20, 25, 30, 35, 40, 45,

the last figure 45 being towards the extreme right hand end of OX. The ordinates range from 62·1 to 78·3. The figuring of OY should be from 60 (at the origin) to 80. Do not number the axes for the actual data. For instance, do not mark on the axes, 22, 43, 62·1, etc. It is difficult to check the accuracy of the plotting, if this method is adopted.

(5) **The plotted points**, representing the pairs of data, **should be marked by small crosses, or circles** round them, to make them easily distinguishable from accidental spots on the paper.

(6) After the points have been plotted, **the graph** is to be drawn. The exact manner of doing this depends on the object for which the graph is required. In junior work, the usual plan is to **draw a smooth line or curve, so that the plotted points are either actually on the curve, or more or less symmetrically situated on both sides of the curve.** Very often, the plotted points will be found to lie very approximately, but not exactly, on a straight line, circle, parabola or other regular smooth curve. If the experiment has been performed to verify a simple relation or formula connecting the two sets of quantities, the relation is likely to be represented by the straight line, circle, or other smooth curve, on which the points seem approximately to lie, and *the best course is to draw between the points the best line or circle that is possible, the plotted points lying either on the curve, or ranged equally on both sides of it at small distances.* These deviations from the smooth curve are, in this case, looked upon as unavoidable experimental errors.

If "the best curve" appears to be a straight line, **stretch a fine thread so that the plotted points lie evenly on both sides of it.** Mark the ends of the thread. A line connecting these is the graph. A glass or celluloid ruler (so that points on both sides are visible) is also very convenient for this purpose.

Other curves are drawn freehand, as best as may be possible.

Fig. 43 shows a bad curve, drawn by a beginner, to represent the evening temperature of a typhoid patient on seven consecutive days. The data are as follows :

February	21	...	98°·8F
,,	22	...	100°·3
,,	23	...	103°·6
,,	24	...	104°·3
,,	25	...	103°·2
,,	26	...	103°·0
,,	27	...	105°·5

Defects.

(1) The student starts mechanically with zero at the origin. The result is that the paper is more or less thrown away, a tiny area being used to draw a ridiculously small curve.

(2) The variations of temperature, which were the object of study, are not easy to read, on account *of the very small space left for the numbers 98—106.* The same applies to the other axis. February 1 to February 19 need not have been laid out at all.

(3) The ordinate scale is badly numbered. ***The integral values 98°, 99°, 100°, etc. should have been marked, and not the actual data 98°·8, 100°·3, etc.***

In the proper curve for this case (Fig. 44)

(1) The origin stands for 98°F and February 20. Tenths of a degree are represented by the smallest space, and the scales extend to 106°F and February 27.

(2) Integral degrees alone are numbered.

EXPERIMENT 32.—**Draw a graph for the following data:**

Length of water column (U tube)			Length of oil column (U tube)
8·7 cm.	...	...	10·0 cm.
10·5	...	...	12·1 „

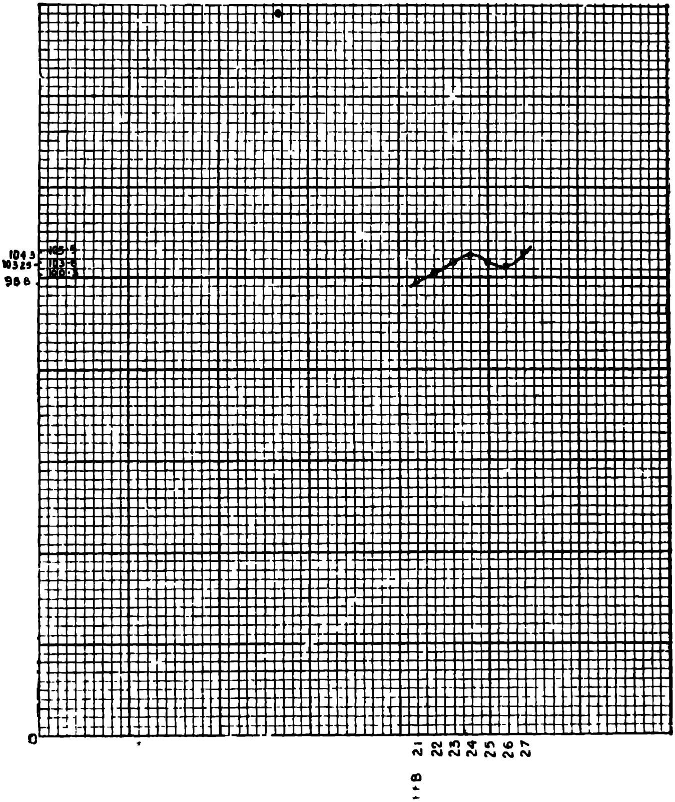


Fig. 43.—An ill-designed graph.

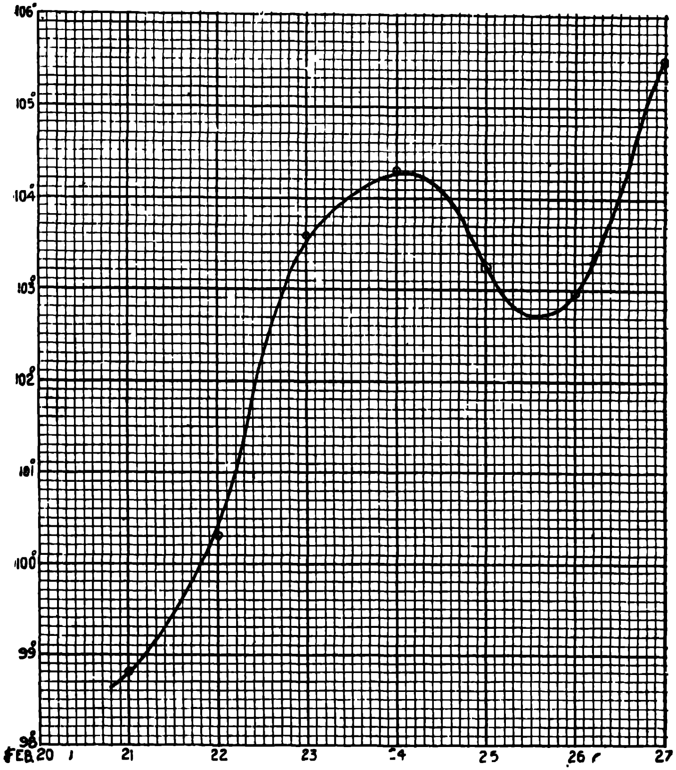


Fig. 44. —A well designed graph (same data as Fig. 43)

13·3 ,,	...	...	15·3 ,,
15·2 ..	...	...	17·5 ..
18·2 ..	...	...	20·1 ,,
21·0 ..	...	...	24·2 ,,

For directions, read pages 131-133.

Read from the graph the length of an oil column corresponding to a water column 16 cm. long Measure the tangent of the angle of slope, and state how this is related to the density of the oil. (See Fig. 45).

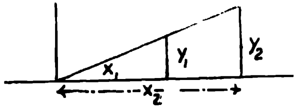


Fig. 45.—A straight line graph $\frac{y_1}{x_1} = \frac{y_2}{x_2} =$ constant (tangent of the angle of slope).

When to expect a Straight Line Graph. We shall consider only one case, the one that occurs most commonly in physics practical work. Fig. 45 shows a straight line graph passing through the origin.

The ratio $\frac{\text{ordinate}}{\text{abscissa}}$ or $\frac{y}{x}$ is constant for every point, and equals $\tan \theta$ **Therefore, whenever we find or expect to find by experiment, the ratio of two quantities $\frac{y}{x}$ to be constant, the graph on plotting them may be expected to be a straight line.**

If y/x is not constant, but xy is constant, the relation should be put in the form $\frac{x}{1/y}$ $(= xy) =$ constant. Plot, therefore, x, and $1/y$ instead of y, and a straight line graph will result. Sometimes, the square of one quantity may be selected instead of the quantity itself, for plotting. This happens in the case of the length and period of a simple pendulum (see page 17).

SUMMARY

To draw a proper graph

(1) Select scales so that the graph covers as much of the paper available as possible. **The origin need not represent zero,** unless this is specially necessary. It should represent round numbers nearest to but less than the smallest values of the ordinate and the abscissa, that have to be plotted.

(2) That quantity should be made the **ordinate**

whose changes you **particularly** wish to study.

(3) Do not actually number the points which represent your data on the axes. Use only round numbers, and indicate the position of your data between these by small dots.

(4) Mark the plotted points by circles or crosses, which are easily visible without being too large.

(5) The list of data plotted should be placed in one corner on the graph for easy reference and checking.

(6) If the graph is a straight line, draw it with a stretched thread or a transparent ruler.

(7) In every case draw a **smooth curve** so that **the plotted points lie either on the curve or evenly round it.**

CHAPTER XI

ELASTICITY

BOYLE'S LAW. HOOKE'S LAW

EXPERIMENT 33.—**To verify that the volume of a given mass of gas at constant temperature varies inversely as the pressure, or that PV=constant, if the temperature is constant.**

Apparatus : Boyle's law apparatus of the usual type, with cylindrical air tube and movable cistern. Thermometer.

Fix the air tube near the middle of the scale, so that pressures below and above the atmosphere can be tried. The tube should contain an amount of *dry* air that will nearly fill it, when the cistern is at its lowest point. Read the position of the inside of the closed end of the air tube on the millimetre scale, as accurately as you can. Suspend the thermometer near the air tube, and read accurately.

Place the cistern at its lowest, and read accurately with a set square, or other help, the positions of the mercury in the air tube, and in the cistern or open tube, up to ·1 mm. Repeat these readings, raising the open tube each time by about 8 to 10 cm., so as to obtain 8 to 10 readings by the time the top of the scale is reached. Read the barometer. Read the thermometer to see if the temperature has changed considerably. The volume of the air is proportional to the length of air tube occupied. The pressure of the air is obtained by adding to, or subtracting from, the barometric height, the difference of the readings of the mercury in the air tube and the open tube.

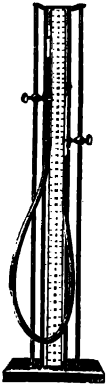


Fig. 46.—Boyle's law apparatus.

Multiply p and v to see if the law (Boyle's) is verified. Also prepare a graph.

Observations:

Temperature of air tube : Initial ...... Final ......
Position of closed end of air tube : ....
Barometric height : ....

Mercury reading air tube	Mercury reading open tube	Head of pressure + or −	Total pressure p	Cm. air tube occupied v	$pv = \frac{p}{1/v} = \frac{v}{1/p}$
1					
2					
3					
4					
5					
6					
7					
8					

Q.—To verify the law graphically, what functions of p and v would you plot, in order to get a straight line. Determine, by interpolation, the volume of the gas at atmospheric pressure.

Hooke's Law.—When a body resists change of shape or size, it is said to be **elastic**. The air in the last experiment was elastic. The *greater* the **resistance** offered, the *more* **elastic** the body is said to be. The change of shape or size called **strain** is generally proportional to the **stress** or force producing the change. This principle is known as **Hooke's Law** and may be stated briefly thus:

$$\frac{\textbf{Stress (applied force)}}{\textbf{Strain (change of shape or size)}} = \textbf{constant.}$$

EXPERIMENT 34.—**To verify Hooke's law with a spiral spring or rubber cord.**

Apparatus : Rubber cord or spiral spring (Jolly's balance is excellent), weight box, scale, stand and clamp.

Suspend the spring (brass or steel) from a clamp in front of a scale. The rubber cord can be securely held between the halves of a split cork, as in the case of a pendulum string. A mark near the lowest point of the spring or the rubber cord is necessary. Attach a scale pan, and read the position of the fixed mark against the scale. Add 5 or 10 gms. to the scale pan, so as to stretch the spring or cord by about one-tenth of its length. Read the position of the mark. Add a weight nearly equal to that present in the pan and read again. Take 5 or 6 such readings, increasing the load by equal amounts each time, and taking care not to stretch the spring or cord, in all, by more than about half of its unstretched length.

Enter observations thus :

Load in pan (*Stress*)	Reading of mark on spring or cord	Extension (*Strain*)	$\frac{Stress}{Strain}$
0	r_0		
10	r_1	$r_1 - r_0$	
20	r_2	$r_2 - r_0$	
30	r_3	$r_3 - r_0$	
40	r_4	$r_4 - r_0$	
50	r_5	$r_5 - r_0$	
60	r_6	$r_6 - r_0$	

Plot stress and strain on graph paper. What do you learn from the graph, or from a study of the values in the last column, in the above table?

CHAPTER XII

ACCELERATION AND FORCE

THE SECOND AND THIRD LAWS OF MOTION

In a junior laboratory, the attention paid to practical work in dynamics is often inversely proportional to the importance of the subject. This is, no doubt, due to the difficulty of performing accurate experiments with simple apparatus. With care, however, good work can be done with simple inclined planes. We will use the inclined plane to verify the second law of motion.

EXPERIMENT 35.—**To determine the acceleration of a body rolling down an inclined plane, and its relation with the sine of the angle of slope.**

Apparatus : Grooved strip of wood 8 ft. × 3″ × 1½″. steel ball, stop-watch, metre stick.

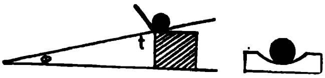


Fig. 47. Rolling ball and inclined plane.

Place the inclined plane flat on the table, and adjust by cardboard under one end, till the ball shows no tendency to roll one way or the other. Raise one end of the plane by means of a wooden block 2 to 3 cm. thick. Hold the ball captive by means of a pencil, just above the point where the edge of the block touches the plane. Record the zero reading of the stop-watch Release ball, start the watch simultaneously, and stop it when the ball reaches the bottom of the plane. Repeat so as to obtain three readings. Measure the distance traversed by the ball along the plane.

Similarly, measure the time taken by the ball to roll down from a lower point to the bottom three

THE FOLLOWING FIGURES ARE FROM A STUDENT'S NOTE-BOOK.

Distance of line of contact of block with plane from bottom of plane: 160 cm. (fixed).

Distance in centimetres s	Time t in seconds	Mean t	Acceleration $\left(=\frac{2s}{t^2}\right)$ a	Mean a	Mean height block h	$h/160$ or $\sin\theta$	$\frac{a \times 160}{h}$ or $a/\sin\theta$
240	6·6 6·4 6·4	6·46	log 480 = 2·6812 log 6·46 = ·8102 1·8710 log 6·46 = ·8102 antilog 1·0608 = 11·5	10·84	2·63	$\frac{2\cdot63}{160}$	660
160	5·3 5·4 5·0	5·23	log 320 = 2·5051 log 5·23 = ·7185 1·7866 log 5·23 = ·7185 antilog 1·0681 = 11·69				
100	4·6 4·7 4·6	4·63	log 200 = 2·3010 log 4·63 = ·6656 1·6354 log 4·63 = ·6656 antilog ·9698 = 9·33				

240		5·53		=15·70	15·46	3·67	$\frac{3·67}{160}$	674
160		4·33		=17·06				
100		3·83		=13·63				
240		4·76		=21·18	20·11	4·72	$\frac{4·72}{160}$	681·7
160		4·0		=19·99				
100		3·23		=19·17				

Record answers to the following :

Q. What conclusion do you draw regarding the relation between a and $\sin\theta$?

Q. Have you enough data for a graph ? What would the graph, if plotted with sufficient data, be ?

[Values of acceleration from the 100 cm. fall are distinctly lower than from bigger falls for the same inclination. Probably friction is larger in the first two or three seconds than later on. It would be desirable, therefore, not to experiment with distances less than 150 cm. The theoretical value of $a/\sin\theta$ for a frictionless ball and plane is 5/7 'g' or 700.]

times. Measure thickness of block, carefully, at 3 points along the touching edge.

Repeat with blocks of about twice and three times the thickness, obtaining the time thrice for 2 or 3 distances in each case.

The formula for accelerated motion. $s = \frac{1}{2} a t^2$, applies here. Calculate $a\left(= \frac{2s}{t^2}\right)$ for the various cases.

The sine of the angle of slope is easily measured, as it equals $\frac{\text{height of block}}{\text{length of plane}}$. Calculate $\frac{a}{\sin\theta}$.

Observations:

(See pages 142-143).

The Force causing Acceleration down an Inclined Plane. To measure this, we determine what pull will prevent a body from rolling down an inclined plane at various inclinations. This force must evidently be equal to the force that causes the acceleration in each case.

EXPERIMENT 36.—**To determine, statically, the force causing the acceleration down an inclined plane, and its relation to the sine of the angle of slope.**

Apparatus : Inclined plane of the usual type with glass top. 200 gms. spring balance reading in steps of 10 gms. or less ; or pulley and weight arrangement. Two rollers (200—400 gms.), or trolley with tray for weights.

Clean and oil the bearings of the roller or trolley, so that it is smooth running. Clean the glass top, and set up the plane at an angle of about 20°. Attach the spring balance as shown, and holding it by the outer case (not ring), determine by pulling steadily at the balance the reading when the roller just moves *up* the plane. On account of friction, the pull necessary will be higher than if friction were absent. Now relax your hold steadily, and read the

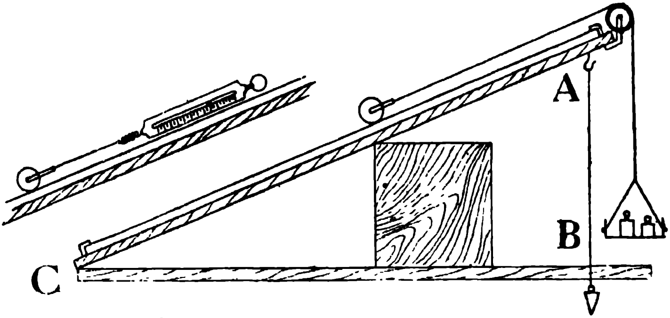


Fig. 48. Measuring the force down an inclined plane

pointer as the roller just begins moving *down*. Friction will cause this reading to be lower than it would be otherwise. The mean of the readings will be approximately free from the effects of friction. Try three times both ways. *Pull steadily without jerks.* The force indicated by the mean reading is evidently keeping the roller in equilibrium, and therefore, equals the force tending to cause acceleration down the plane. Weigh the roller by lifting it off the plane, and reading the balance.

A plumb line meets the lower surface of the plane and the upper surface of the lower board at A and B. C is the hinge. Measure AB and AC. $\frac{AB}{AC}$ is *sin* θ. Multiply mass of roller and *sin* θ in each case.

Perform four such experiments, 2 with one roller at 2 different angles and 2 with the other at 2 different angles.

The force F, in grammes weight, may also be determined by adjusting weights, in a scale pan at the end of a thread attached to the roller and passing round a pulley on the top of the plane, till the roller just moves (1) up, (2) down. The weight of the scale pan should be added to the mean of these.

NOTE :—The spring balance, though not quite so accurate, is more convenient than the pulley arrangement; and as a large number of readings can be rapidly taken, results are, on the whole, satisfactory.

Observations:

Mass of roller or trolley M	Balance reading (F)			AB/AC or $\sin\theta$	$M \sin\theta = M.\frac{AB}{AC}$
	Moving up	Moving down	Mean		
M_2	1	1			
	2	2			
	3	3		...	..
M_3	1	1			
	2	2			F will be found to
	3	3 ...			equal $M.\frac{AB}{AC}$
M_1	1	1			
	2	2			
	3	3 ..			
M_4	1	1			
	2	2			
	3	3			

The Second Law of Motion. From the above experiment we learn that the force along a plane necessary for supporting a smooth roller, is equal in grammes weight, to the mass of the roller multiplied by the sine of the angle of slope, or

$$\mathbf{F = M \sin \theta} \text{ ....} \mathrm{I}$$

If F is absent, there is an acceleration a, such that $\frac{a}{\sin\theta}$ is constant ($=K$) [Expt. 35]

$$\text{or } \mathbf{a = K \sin\theta} \text{ ....} \mathrm{II}$$

From I and II we have, by division

$$\mathbf{\frac{F}{a} = \frac{M}{K}}$$

$$\text{or } \mathbf{F = \frac{Ma}{K}}, \text{ or } \underline{\mathbf{F} \textit{ varies as } \mathbf{Ma}}.$$

The Third Law of Motion. The third law of motion states: "**When two bodies A and B act on one another, the force (F) exerted by A on B is exactly equal and opposite to the force (—F) exerted by B on A.**"

A body suspended in water is pressed *upwards*. In consequence, its apparent weight is less than its weight in air (more correctly vacuo), by the weight of the water displaced. In virtue of the third law of motion the body must be pressing the water *downwards*, and the weight of the beaker containing the water should appear to increase, by an amount equal to the loss of weight suffered by the body.

EXPERIMENT 37. –**To verify the third law of motion by means of the hydrostatic balance.**

Apparatus : Balance and weights, stool, small light beaker, 1 inch metal ball and thread.

Weigh the ball in air and then in water, in a *small* and *light* beaker, placing the latter on the stool placed across the pan, in the usual way (Fig. 49). The beaker or stool should not touch the pan. Bubbles should be removed. Determine the loss of weight. This equals, in grammes weight, the pressure exerted by the water on the ball.

Next, weigh the beaker in the pan. Place no more water in the beaker than is necessary for the complete immersion of the ball, with a little room to spare above and below. A layer about 2 inches deep should suffice. Now support the ball (Fig. 49), from a small suitable clamp, so that it is completely immersed in the water in the beaker. The thread, clamp, or ball should touch neither the beaker nor the pan. There should be no bubbles. Weigh again. The increase in weight is the downward thrust of the ball on the water.

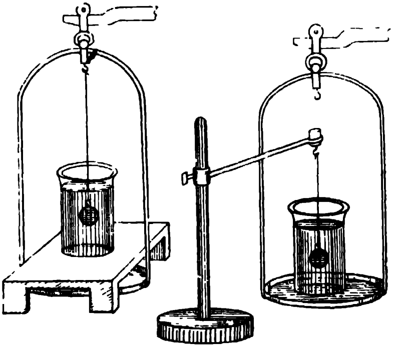


Fig. 49 Action and reaction are equal and opposite

Enter thus :

Beaker and stool not touching pan	Weight of ball in air	=	...	(1)
	" " " " water	=	...	(2)
	Loss of weight or upward thrust of water on ball in gms. wt. (F)	= (1) (2)		
		=	......	I
Ball, thread, and clamp not touching pan or beaker.	Weight of beaker in pan	=	...	(3)
	" " " with ball suspended in water	=	...	(4)
	Gain of weight or downward thrust of ball on water (F)	= (4) (3)		
		=	......	II

I and II will be nearly equal

Answer the following in your book :—

(1) Suppose you weigh a beaker containing some water and a metal ball placed alongside in the same pan. If the ball is now suspended from the hook of the pan into the water in the beaker, and the whole reweighed, what difference in the weight do you expect? Explain.

CHAPTER XII—*contd.*

ACCELERATION AND FORCE

THE SIMPLE PENDULUM

The Simple Pendulum. The acceleration of gravity, or 'g' can be determined accurately by timing the vibrations of a simple pendulum. For ordinary work, a metal ball of 2 to 3 cm. diameter, attached to a piece of *inextensible* twine, constitutes the "simple pendulum." For suspending the pendulum, a simple clamping device is a split cork between the two halves of which the thread is securely held. This cork can be supported in a retort stand and clamp on the table, so as to be at a height of about 4 or 5 ft. from the floor.

If let go, after being displaced from the vertical, the pendulum moves to and fro, or **vibrates**. The maximum angle that the pendulum makes with the vertical is the "**amplitude**." In Fig. 50, the angle between the vertical OP and the extreme position of the pendulum OP′ is the "amplitude." The time of a complete vibration is the **period**. It is defined more precisely as **the time elapsing between two consecutive passages in the same direction through the mean position**. For instance, the time between the instant of passage through P towards P′, and the next instant of passage through P when moving towards P′, is the "period." The **length** of a simple pendulum is the distance of the **point of suspension** from the centre of the ball or **'bob.'**

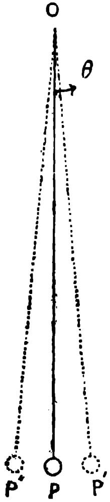


Fig. 50.—Period and amplitude.

The period of a pendulum vibrating *with small amplitude* depends only on its length and the value of the acceleration due to gravity 'g' at the place. The mass of the bob does not affect the period ; the amplitude, as long as it does not exceed a few degrees, is also immaterial. The relation between the period (t). length (l), and g is, $t=2\pi\sqrt{\frac{l}{g}}$. If the amplitude is large, the time is appreciably more than this.

EXPERIMENT 38*a*.—**To see if the period of a simple pendulum is affected by length, mass of bob, and amplitude.**

Apparatus : Two simple pendulums, bobs of various sizes, thread, split cork, metre-stick, stop-watch, or pocket watch with seconds-hand

Length :

Suspend two pendulums with similar bobs, one in front of the other. The thread is best held between the two halves of a split cork, the halves being pressed together by a clamp, *so that their bottoms are on the same level.* Let one be about 50 cm. long, and the other about 100 cm. Set them both vibrating with amplitude. A mere glance should convince you that small the longer pendulum has the longer period. Pull them to one side, and release simultaneously. The longer pendulum will be found to lag behind the shorter one.

Mass of bob :

Attach two bobs, of 1·5 cm. and 3 cm. diameter, to two pieces of string, and support the two pendulums thus formed, in two separate stands, so that the cork suspensions are close to one another, and the pendulums hang side by side. Adjust them to be of equal length (about 90 cm.), by arranging the two points of suspension, as also the centres of the two bobs, to be on the same level. This can be done by pulling the thread of one through the split cork. Now arrange the pendulums, so that one is behind the other, and place yourself in a line with the two strings. Displace both the bobs 5 cm. to one side, by means of a wooden block, and release simultaneously. The two pendulums will continue to vibrate together, passing through their mean positions simultaneously for a long time. This shows that the mass of the bob does not sensibly affect the period.

Amplitude:

The period has now to be measured with accuracy. Arrange a pendulum about 100 cm. long, so that the bob is near the floor, and place a retort stand rod behind the string. Place a metre stick under the bob parallel to the direction of vibration. Take up a position, where the string and rod appear in a line. Pull the bob aside about 4 cm., and release without pushing. Take up the position selected, let the pendulum vibrate once or twice, and then at a passage of the string towards the right in front of the rod, say "zero," and *simultaneously* start the watch. At the next passage to right in front of the rod, say one, then two and so on, up to one hundred. At one hundred, stop the watch and note the time. Repeat with amplitudes, 7, 10, 15, 20. 40 and 60 cm., noting down the time of one hundred vibrations in each case

The counting should begin with *zero* and not one. Another way to avoid possibility of error in counting is to count in the order: 3, 2. 1, $\underline{0}$, 1, 2. 3......$\underline{100}$. Note the time at 0 and 100.

Observations:

Length of pendulum: about 100 cm.

AMPLITUDE IN CM. OF ARC	TIME OF 100 VIBRATIONS	PERIOD	REMARKS
4	...	...	Here state up to what amplitude the period appears unchanged and the amount of change in % when the amplitude is large
7	...	...	
10	...	...	
15	...	...	
20	...	.	
40	...	...	
60	...	...	
100	...	...	

EXPERIMENT 38.—**(1) To examine the relation between the length and period of a simple pendulum, (2) to calculate 'g', (3) to calculate the length of a seconds pendulum graphically.**

Apparatus: Simple pendulum with 3 cm. bob, metre stick, stop-watch, clamp, callipers, split cork.

If the previous experiment (38a) has not been performed, read the complete directions given there for suspending the pendulum, and counting the vibrations.

Arrange a pendulum about 120 cm. long. To measure the length, make a mark, with pencil or gum paper, near the middle of the thread. Place the zero of the metre stick against the cork suspension, lay the thread along the scale and read the position of the mark. Next place the zero on the bob just below the hook, and measure similarly up to the mark. Measure diameter of bob by callipers and add the radius to the sum of the first two lengths. This gives the length of the pendulum. Arrange a retort stand rod behind the string, or draw a chalk line on the floor as a reference mark. Measure the time of 30 vibrations three times, taking care throughout to *keep the amplitude quite small* (2 inches or 5 cm. on each side is enough). Repeat with pendulums of length 100, 80, 65, and 50 cm., measuring the length as above, and the time of 30 vibrations three times (twice only if pressed for time) in each case.

For '*g*' the rate of the stop-watch should, if possible, be determined and the period corrected.

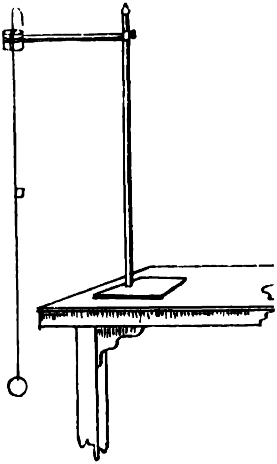


Fig. 51. - The simple pendulum.

Observations :

For record of observations and graph see pp. 16-17.

Answer the following in your book:

Q. Which would you plot for a graph: l and t, or l and t^2? Why?

Prepare such a graph, and from it determine by interpolation, the length of a seconds pendulum (a pendulum for which $t=2$ seconds). (See pp. 16-17).

CHAPTER XIII

THE COMPOSITION OF FORCES

PARALLELOGRAM OF FORCES

The parallelogram of forces states that **if two forces P and Q act at a point A in directions AB and AC, the resultant R of P and Q is proportional to and along AD, where AD is the diagonal of the parallelogram constructed on AB and AC, taken proportional to P and Q respectively.**

In other words (Fig. 52)

$$\frac{\mathbf{P}}{\mathbf{AB}}=\frac{\mathbf{Q}}{\mathbf{AC}}=\frac{\mathbf{R}}{\mathbf{AD}}.$$

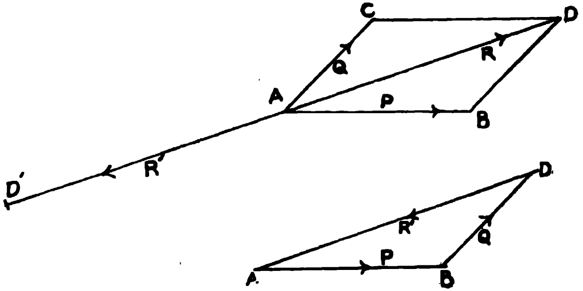


Fig. 52. The parallelogram and triangle of forces.

If this principle is accepted, another important relation follows.

Produce DA to D′ so that AD=AD′.

The force AD′ (R') is equal and opposite to R and will therefore cancel it. But R equals P and Q.

Hence P, Q, R' are together equal to zero.

The triangle ABD is identically redrawn in the lower figure, with arrows in the directions AB, BD, DA. The three sides of this triangle, taken in the direction of the arrows, represent in magnitude and direction *P*, *Q*, *R′*, because BD is equal and parallel to AC (*Q*) and DA is equal and parallel to AD′ or *R′*. But *P*, *Q*, *R′* are together equal to zero. Hence forces AB, BD, DA are equal to zero This modification of the parallelogram of forces is called

The Triangle of Forces and states that

If a particle is acted on by 3 forces represented in magnitude and direction by the three sides of a triangle, taken the same way round (or in order), the particle will be at rest

Lami's Theorem Even a third relation (the same as the first two, in a different form) exists. Each one of the three sides of a triangle is proportional to the sine of the angle between the other two. Hence **If three forces at a point neutralize one another, each is proportional to the sine of the angle between the other two**

EXPERIMENT 39.—**To verify the parallelogram and triangle of forces, as also Lami's theorem.**

Apparatus : Vertical board with 2 pulleys, weight box, scale pans, (or weights with hangers), metre stick, wooden block etc.

Parallelogram of forces : Knot together 3 pieces of fine string and arrange on pulleys as shown (Fig. 53) with weights on hangers or in pans (200, 300, 400 gms. or some such), so that the knot O is near the centre. Pin on to the board, behind the strings, a sheet of white paper. With the help of a rectangular wooden block, placed against each string, draw three fine lines on the board under each piece of string. These, when produced, should meet in a point corresponding to the knot O. Let C′O, B′O and A′O be the 3 lines. On a scale of 1 cm.=20 to 50 gms. cut off OA, OB, OC to represent the 3 forces used. On OA, and OB, construct the parallelogram OADB. OD will be found equal to OC, and in prolongation of it. As OC neutralizes OA and OB it must be equal and opposite to their resultant. OD is therefore the resultant of OA and OB

Triangle of forces : Redraw the triangle OA′D below as OAD, and put in arrows in the directions OA, AD, DO. The three lines OA, AD and DO, taken in order, are parallel and proportional to the three forces *P*, *Q*, *R*, which are together equal to zero. The triangle of forces is therefore also verified.

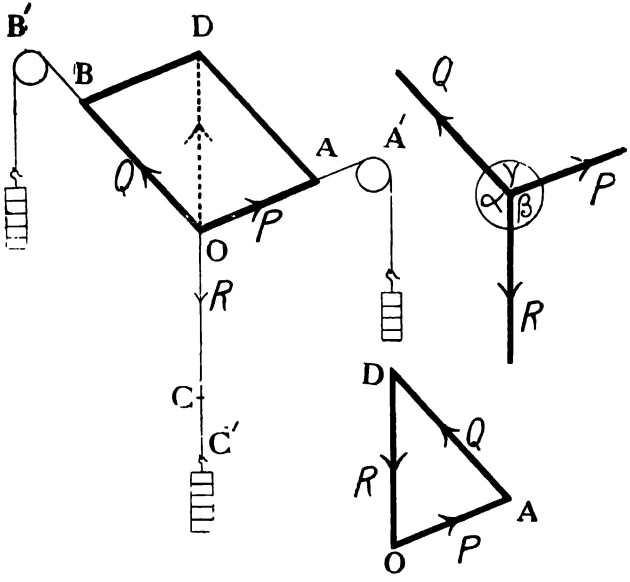


Fig. 53 – Verification of the parallelogram and triangle of forces, etc.

Lami's theorem : Measure the angles α, β, γ between the 3 pairs of forces and obtain their sines from tables, or measure the sines directly. The sines of α, β, γ equal the sines of the acute angles 180° − α, 180° − β, 180° − γ. Calculate the ratio of each force to the sine of the angle between the other two. These ratios will be found equal. Hence, if 3 forces on a particle keep it in equilibrium, each is proportional to the sine of the angle between the other two.

Observations :

Force P = ... gms.
,, Q = ...
,, R = ...

Scale selected for sides of parallelogram
1 cm. = ... gms.

Diagonal = ... cm.
Force represented by diagonal
(Calculated resultant) = ... gms. wt.
Actual observed value of resultant = ... gms. wt.
Angle between CO produced, and OD = (2°. say)
Lami's Theorem :

FORCE	ANGLE BETWEEN THE OTHER TWO	SINE	$\frac{\text{FORCE}}{\text{SINE}}$
P = ...	...	...	...
Q = ...	...	...	...
R = ...	...	...	...

CHAPTER XIII–*contd.*

THE COMPOSITION OF FORCES

MOMENTS. PARALLEL FORCES

Moments. The tendency of a force to turn a body round a fixed axis is called the **moment** of that force round that fixed axis. We measured forces producing certain accelerations by determining what known forces could neutralize them. We will use a similar method for measuring moments. A body supported at its centre of gravity remains at rest. If supported elsewhere, it tends to turn round the point of support. We will measure this tendency.

EXPERIMENT 40.—**To determine the turning tendency of a body round any point (not the C. G.), and to determine the weight of the body.**

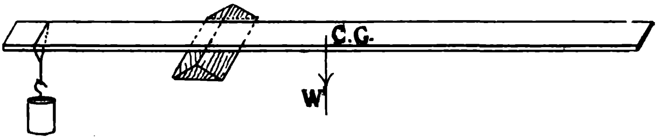


Fig. 54.—A simple lever: a metre stick on a plain wooden wedge.

Apparatus: Lever with knife-edge and support, or metre stick and a wooden wedge to serve as fulcrum. Weight box, thread, etc.

Determine the C. G. by supporting the metre stick on the wedge.

Now place the wedge about 10 cm. away from the C. G., say, at the division 40. The bar tends to turn, or has a certain fixed turning tendency round the division 40.

Suspend a 200 gm. wt. by means of a cotton loop on the side away from the C. G., and adjust its distance so that the stick is horizontal. Read the position of the loop, correct to ·1 mm.

Similarly adjust 150, 100, 70, 50, 25 grammes in turn, and read the position when the stick is horizontal.

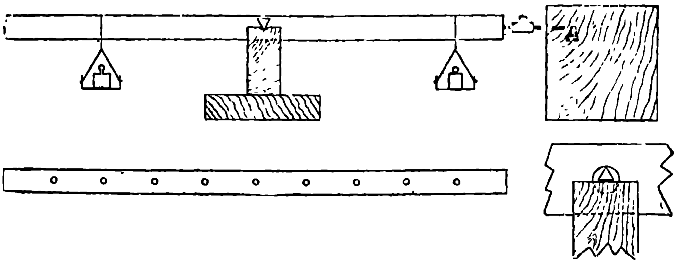


Fig 55 -Lever apparatus :--*Top*, metre stick with fixed central knife-edge, resting in the grooved metal top of a suitable wooden support *Bottom*, metre stick with separate knife-edge and equidistant holes, any one of which can be made the fulcrum (see *right*)

To see how the weights and the distances vary; multiply each weight by its distance from the fulcurm.

The products will be nearly equal.

We have in each case, neutralized a fixed turning tendency, *i. e.*, the moment of the weight of the bar round the fulcrum at 40 cm.

If these equal products $f \times d$ be held equal to this fixed moment, we should have weight of bar × distance of C. G. from fulcrum = mean $f \times d$.

To test if this is right, use the equation to calculate the weight of the bar (x), as below. In the end, weigh the rod by means of a spring balance.

Observations:

Position of C G of bar (1) 50·10 cm. (2) 50 05 cm. | Mean : ......

Position of Fulcrum : 40·00 cm

No.	WEIGHT USED f	BALANCING POINT	DISTANCE OF BALANCE POINT FROM FULCRUM d	$f \times d$
1	200		40 00 −... = ..	
2	150		= ...	
3	100		= ...	
4	70		= ...	
5	50		= ...	
6	30		= ...	

Mean $f \times d = \ldots$

$x \times$ (position of C. G.—40·00) = mean $f \times d$

$\therefore$ x = ... gm. (calculated)

Weight with spring balance = ... gm. (observed).

Record answers to the following:—

(1) How will you plot a graph to represent the relation between f and d ? Do you remember any experiment where a similar relation between two quantities existed ?

EXERCISE 40*a*.—**Arrange a number of weights on opposite sides of a bar supported at its C. G., and show that the sum of the clockwise moments is equal to the sum of the anti-clockwise moments.**

Lever apparatus. The only merit of the wedge and metre stick as a lever apparatus is its extreme simplicity and the ease with which any point can be made the fulcrum. The arrangement has, however, many drawbacks :

(1) The C. G. of the stick being *above* the support, the equilibrium of the unloaded beam is unstable.

(2) The stick is used flat, and is, hence, not a rigid lever.

(3) The fulcrum is liable to change easily as the stick slips on the wedge. This means annoyance and waste of time. A fairly simple, but much more efficient arrangement is the metre stick with a fixed central knife-edge (Fig. 55) The knife-edge rests on notches in two small pieces of brass plate on the top of a slotted

wood block. Another plan is to bore holes 10 cm. apart and line them with boot lace rings (Fig. 55) With a suitable knife edge any hole can serve as a fulcrum in this case. To provide for adjustment to horizontality, cut off the hundredth centimetre and attach a horizontal wire on which one-half of a wire connector slides. The wire can serve as a pointer against a pencil mark on the side of a plane wooden block.

For experiments (described below) on the steelyard and the balance, the fixed knife-edge arrangement is sufficient. The apparatus has the look of a balance, and work is quicker and pleasanter than with the plain metre stick and wedge.

EXPERIMENT 41.—**To find the mass of a body with a lever, using it (1) as a true common balance, (2) as a faulty balance, (3) as a steelyard.**

Apparatus : Lever apparatus, weight box, 2 small scale pans, bodies to be weighed (metal balls or glass stoppers), spring balance or common balance.

We have seen that when a lever is in a position of horizontal equilibrium, the moments of forces on one side equal the moments of forces on the other side. This principle is employed for comparing masses (through the moments of their weights) in the ordinary balance and the steelyard.

(1) **Common balance (true).** Suspend two light pans by means of cotton loops from the divisions 35 cm. and 65 cm. of the metre stick. With the fulcrum at 50 cm. adjust the lever horizontal, using sand or small bits of paper in one of the pans if necessary. The arrangement represents a *true* common balance with arms 15 cm. each. Now place the ball or stopper to be weighed in the left hand pan, and to the right one add weights from the weight box till lever is horizontal. Record the weights in this pan. Repeat with pans at 40 cm. from the fulcrum on both sides.

(2) Common balance (untrue). In a faulty balance the two arms are unequal, and the pans, too may have unequal masses. Weighings can, however, be carried out with such a balance.

Place one pan at 20 cm. from the 50 cm. mark and the other at 21 cm. Add paper or sand till lever is horizontal with the fulcrum at 50. The arrangement represents an apparently good balance which really is untrue. Now place the body (already weighed) in one pan, and add weights to the other till lever is horizontal. Record weights. Next weigh body by transferring it to the pan on the other side. Record the weights. Calculate the mean. Repeat with arms 40 and 41 cm.

Observations:

True balance		*Untrue balance*		
Length of arms	Weight of stopper observed.	Length of arms	Weight of stopper observed	Mean.
15 cm and 15 cm	...	20 cm and 21 cm.	1 ... 2 ...	..
40 cm. and 40 cm	..	40 cm and 41 cm.	1 ... 2 .	..
Mean weight:	...	Mean weight:	...	

Actual weight by spring balance or common balance = ...

Record answers to following :

Q. Which arrangement (15 cm. arms or 40 cm. arms) is more sensitive and why ?

Q. When weighing out one maund of sugar with a faulty balance in instalments of 5 seers, how will the honest shopkeeper weigh, and why ?

(3) **The steelyard** is a balance with unequal arms, and is used for weighing heavy luggage by means of small standard masses or "weights." Two methods are employed :

(*i*) To adjust small weights (on a hanger or in a pan) at a fixed distance from the fulcrum till the beam is horizontal.

(*ii*) To slide along a graduated beam a fixed standard mass (rider principle), different distances representing different weights.

In practice, both principles are combined in the same machine. In a doctor's weighing machine the stones are often represented by standard masses on a hanger while the final adjustment is effected by a sliding mass on a beam graduated in pounds and ounces.

We shall weigh the stopper or ball (already weighed) by means of our lever apparatus, using it as a steelyard in both the ways described above.

(*i*) *Adjusting weights at a fixed distance from fulcrum.* With the fulcrum at 50 cm., suspend one pan at the 40 cm mark (*10 cm. from fulcrum*) and the other at the 90 cm. mark (*40 cm. from fulcrum*). Adjust the stick horizontal, with shot or sand. Place the ball or stopper in the pan at 40. Adjust weights and small fractions in the other pan at 90 till the lever is horizontal. The mass of the ball is exactly 4 times the "weights" in the pan.

(*ii*) *Sliding small fixed weight along graduated arm:* With the fulcrum at 50 cm arrange a light scale pan at 40 cm. (10 cm. from fulcrum). Suspend on the opposite side of the fulcrum a 10 gm. weight by means of a loop of thread at some prominent division, say, 60, 65, or 70 cm. and adjust the beam horizontal by means of shot or sand in the scale pan. The position of the loop is the zero reading of the steelyard. The moment of a 1 gm. wt. in the pan is 10 This can be balanced by the 10 gm. weight sliding outward by 1 cm. Hence the arrangement is a convenient steelyard, each cm. beyond the 'zero' position representing 1 gm., and each millimetre ·1 gm.

Now place in the pan the stopper or ball used previously. Slide the 10 gm. weight till lever is horizontal. Read the position. The distance in cm. of the loop from the 'zero' reading gives the mass of the ball in grammes.

Observations :

Principle employed.	Fixed arms.	Fixed weight	Scale of instrument.	Value of mass when adjusted m	Distance of 10 gm weight from "zero reading" 60 65 or 70 cm. d	Calculated weight.
Adjustable mass at fixed distance	10 cm 40 cm	.	4 to 1	. gms		$m \times 4$ grammes.
Fixed mass at adjustable distance	10 cm.	10 gm	1 cm. = 1 gm.		.. cm	d grammes.

Mean: ......

Answer the following :

Which principle will you employ for weighing (1) very heavy bodies, (2) bodies of moderate mass? How are they combined in a railway platform weighing machine?

Parallel Forces. We have learnt that in a lever in equilibrium, the moment, round the fulcrum, of the forces on one side, equals the moment round the fulcrum of the forces on the other side. From this it is easy to deduce a rule for finding the resultant of two parallel forces.

Imagine a weightless lever on a wedge as shown. (Fig. 56). Two masses P and Q gms. are so adjusted that the lever is horizontal. Therefore $P \times AC = Q \times BC$. P and Q gm. wt are both downward forces, and yet the lever is in equilibrium. The upward reaction R' of the fulcrum must be therefore, neutralizing P and Q.

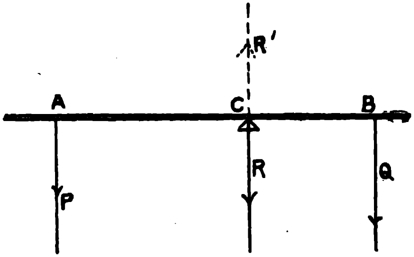


Fig. 56.—Resultant of parallel forces

To measure R' imagine the lever to be lifted by means of a spring balance at C. The reading will evidently be $P+Q$, as the balance must support the two masses P and Q. So $R' = P + Q$.

As R' neutralizes P and Q the resultant of P and Q must be a downward force equal to $P+Q$ acting at C. Call this resultant R.

Thus we can state that the resultant of two parallel forces P and Q at A and B, equals a force R at C, where

$$R = P + Q$$

$$\text{and } P.\, AC = Q.\, BC.$$

Let us verify this experimentally.

EXPERIMENT 42.—**To determine the magnitude and direction of the resultant of two parallel forces.**

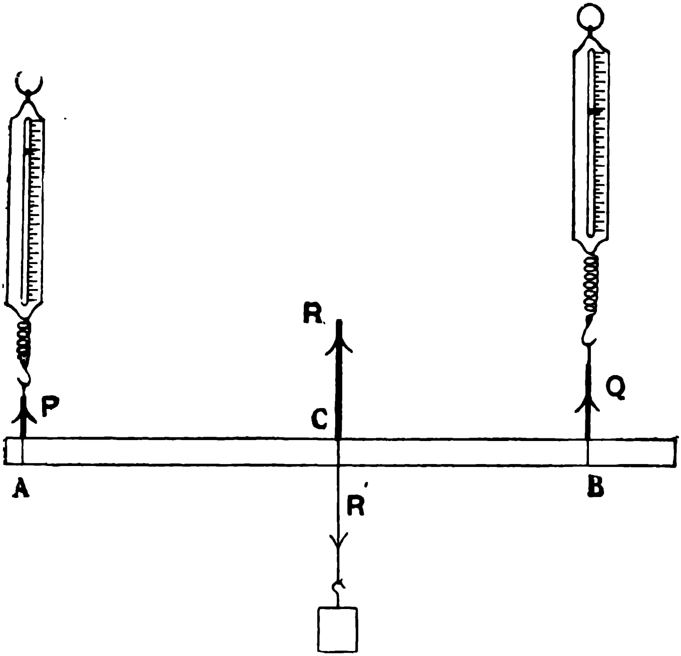


Fig. 57 – Determining the resultant of parallel forces

Apparatus:—Metre stick, wedge, two spring balances, clamps, weights, etc.

Determine the C.G. of the stick.

Weigh the metre stick with each balance and calculate the mean.

Support the metre stick *on edge* in thread loops attached to two balances (Fig. 57) whose rings are loosely slung on retort stand rods. Arrange the two balances vertically at different distances from the C.G. Suspend by means of a loop from the C.G. a ·100 gm. weight. Adujst the clamps so that the stick is horizontal, and the balances in the same vertical line with their loops.

Read the balances, and the position of their respective loops on the metre stick.

Change the middle weight at the C. G., as also the position of the balances and the stick. Adjust till stick is horizontal. Read the pointers and the positions of the balances again. Repeat once more.

The resultant R of P and Q at A and B is equal and opposite to the force R' applied at the C.G. (C).

***Enter observations thus*:**

Position of C.G : ..... cm.

Weight of stick : (1) ...... (2) . .. Mean : ..... .

Reading of balance at A	Distance	Reading of balance at B	Distance	Suspended mass + wt of stick			
(P)	AC	(Q)	BC	$R = R'$	$P \times$ AC	$Q \times$ BC	$P + Q$
	.		...	..			
	...		..	...			
...	...	...	.	...			
	...		...	...			
	..		...	...			
...	...	...	...	...			

Examine the last four columns to see if

$$R = P + Q$$

and $$P \times \mathrm{AC} = Q \times \mathrm{BC}.$$

Total Moment C for a Body at Rest is Zero. It is evident that if a body is at rest under the action of any number of forces parallel or otherwise, the resultant force on the body must be zero, and the total moment round any point whatever must likewise be 0.

EXPERIMENT 43.—**To verify that if a body is in equilibrium under the action of (*i*) 4 or 5 coplaner parallel forces (*ii*) three or four coplaner intersecting forces, the total moment round any point in the plane is zero.**

Apparatus : As in Expts. 42 and 39 (with an extra pulley).

Parallel forces : Determine the weight and C. G. of the stick. Suspend the stick from two balances as in Expt. 42 and attach two weights, say 50 and 100 grammes, at any two points. Adjust stick horizontal by raising or lowering the balances. Select any point O on the stick, and calculate the moment round this point of all the forces including the weight of the stick, calling the clockwise moments + and the counterclockwise moments –. Weigh the stick with a spring balance.

Find the algebraic sum of the moments.

Enter thus :

Force	Distance from O	Moment
P	...	*P* × ... = +
Q	...	*Q* × ... = –
W	...	*W* × ... = –
R	...	*R* × ... = +
S	...	*S* × ... = +
		Total = ...

Intersecting forces : Arrange with the apparatus used in Expt. 39 four forces as shown (Fig. 58). To represent the body, attach the four strings to a wire ring or piece of light tin-plate of any shape. The weight of this should be negligible. Draw the directions of the forces as before. Select any point O on the paper and from

O drop perpendiculars OP_1, OP_2, OP_3, OP_4 on the directions of the forces. Measure these perpendiculars The products of these perpendiculars with the respective forces are the moments. Draw a fairly large figure, and enter results in a table as for parallel forces.

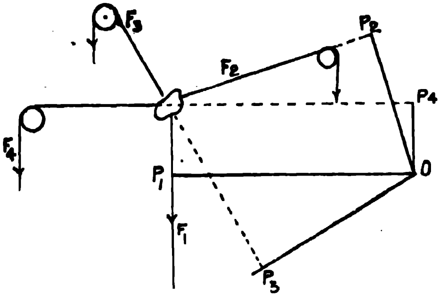


Fig. 58.—Moment of intersecting forces in equilibrium equals 0.

CHAPTER XIV

WORK AND POWER

FRICTION. FRICTION-BRAKE

Friction is the name given to the force that opposes sliding motion parallel to the surface of contact of two bodies. It is always *parallel* to the surface of contact, and *opposite* to the direction in which the body moves or tends to move.

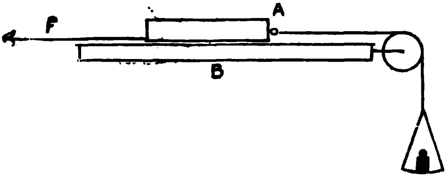


Fig. 59.—Determination of coefficient of friction.

Imagine a flat wooden block A with thread, scale pan, and smooth pulley as shown, resting on a table B. A 5 gm. wt. in the pan may not be able to move A. A 10 gm. wt. may also fail. The 5 gm. and 10 gm. pulls are neutralized by friction F, which must be equal to 5 gm. wt. in the first case, and 10 gm. wt. in the second case. **Friction** is, therefore, *up to a certain limit*, **a self-adjusting force.** If the mass in the pan is slowly increased, the block A will, at length, slide, and the pull that can just do this, evidently equals the maximum friction possible in this case. This amount is called "**limiting friction.**" Even if the body A moves, this force (but not more) continues to act against the direction of motion. "**Limiting friction" does not depend on area of contact, but only on the weight of A (which equals the normal reaction between A and (B) and the nature and condition of the surfaces.** As A may press on B on account of forces other than the weight of A, this pressure is generally called **the normal reaction between A and B.** If this is R, limiting friction F for two specified surfaces, is always μR, where μ is a constant called the **coefficient of friction.**

Imagine a block lying on a plane, one end of which is slowly raised. Let θ be the angle made by the plane with the horizontal when the block first slides.

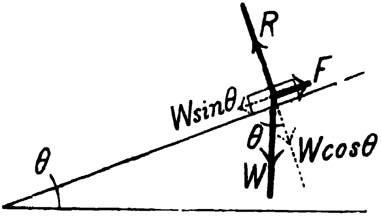


Fig. 60—The angle of friction.

The weight of the block, W, can be resolved into two parts, $W \sin \theta$ along the plane (refer to expt. 36) and $W \cos \theta$ perpendicular to the plane. The reaction R must equal $W \cos \theta$. As the block *just* slides. $W \sin \theta$ must equal the friction F.

$\therefore F = W \sin \theta$. But $F = \mu R = \mu W \cos \theta$.

$\therefore \mu W \cos \theta = W \sin \theta$ or $\mu = \tan \theta$.

The angle θ is called the **angle of repose,** or "**angle of friction,**" and we learn that μ equals $\tan \theta$ where θ is the angle of friction.

EXPERIMENT 44.—**To determine the coefficient of friction between two given surfaces.**

Apparatus : Inclined plane of the usual type (preferably glass top), wooden block hollowed out on top, or shallow wooden box with flat bottom, weight box, spring balance.

First of all, use the inclined plane as a horizontal table. Place a well planed wood block weighing about 200 gms. on the plane, and arrange a scale pan as in Fig. 59.

Adjust the load in the scale pan, so as to *just* move the block. Try three times.

Place on the block a 200 gm. weight and repeat. Repeat next with 400, 600, 1000 gms. Weigh the block with the spring balance.

Observations:

Mass of block :
Mass of pan :

Mass on block.	Reaction in gms. wt. = Mass of block + Mass on block R	Mass in pan.	Friction in gms. wt. = Mass in pan + Mass of pan F	Coefficient of friction $\mu = \frac{F}{R}$
0				
200				
400				
600				
1000				

Mean value of μ = ...

Next place the box or block on the plane. Raise plane slowly by pushing in under it a wedge or block bit by bit, till the body *just* slides. Measure by means of the plumb line as in Expt. 36, the height and the base (not hypotenuse) of the inclined plane. Take another reading. Repeat with different masses in the box or hollowed block, taking at least two readings in each case. Measure the base and height each time.

Observations :

Total mass on inclined plane.	Height of plane when mass *just* slides.	Base of plane when mass *just* slides.	$\mu = \tan\theta$ height/base.
200	...	...	...
	...	...	...
400	...	...	...
	...	...	...
600	...	...	...
	...	...	...

$$\text{Mean value of } \mu = \frac{\ldots\ldots}{6} = \ldots\ldots\ldots\ldots$$

Friction-brake. Imagine a smooth pulley with a band or string round it, to one end of which is attached a hanging weight W, and to the other a spring balance, the lower end of which is fixed to the floor.

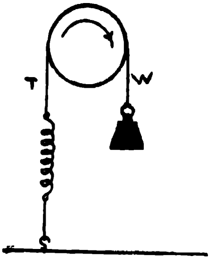


Fig. 61.—The Principle of the Friction-brake.

If the pulley is perfectly smooth running, the tension *T* of the spring balance will equal *W*. The slightest difference will cause the pulley to turn round, so as to equalise the tension on the two sides. Now imagine the pulley continuously turned in the direction of the arrow. As the spring balance is fixed, the pulley will slip past the string If the string and the pulley are perfectly smooth, *T* will not change, as the slipping brings into play no friction or other additional force. If, however, there is friction between the pulley and the string, the latter will be pulled in the direction of the arrow, and the tension *T* will, consequently, increase. The difference between *T* and *W*, $(T - W)$, will equal the friction brought into play. If the pulley pulls the string up, the string must be tending to pull the pulley back (Newton's 3rd law of motion), by an amount equal to $T - W$, and the agent moving the pulley has to perform work in overcoming this force. In one revolution of the pulley every point in its circumference has to be once pushed against this friction $T - W$. The work done in one revolution, therefore, equals

$$(T - W) \times \text{(circumference of pulley)}.$$

In the case of a bicycle retarded by its rubber brake the work done by the moving cycle against the friction in one revolution, equals *brake friction × circumference of wheel.* This principle is used in the construction of friction-brakes for measuring the H.P. of motors, steam engines, etc. The brakes consist of leather belts, which are kept pressed against the engine pulley by strong springs. The pulley slips past the belt, and the work done in one revolution is measured as explained above.

The student should read up the chapter on units of work, force, and power in some text book on Mechanics. The following table gives important data, that are of frequent use in measurement of force, work, energy, power, etc.

Name of unit	Physical quantity measured	System of units	Magnitude	Nature and remarks as to constancy, etc.	Conversion factor
Dyne	force	C.G.S.	Produces in 1 gm. an acceleration of 1 cm. per sec. per sec.	Absolute. No connection with 'g'.	1 milligramme weight (nearly).
Erg	work and energy	,,	1 dyne × 1 cm.	,, ,,	
Joule	,,	,,	10^7 ergs	,, ,,	
Watt	power, or rate of work	,,	1 joule or 10^7 ergs per second	,, ,,	$\frac{1}{746}$ H.P.
Kilowatt	,,	,,	1000 watts	,, ,,	1·34 H.P.
Poundal	force	F.P.S.	Produces in 1 lb an acceleration of 1 ft. per sec. per sec.	,, ,,	Weight of ½ oz. (nearly)
Pound weight	,,	,,	weight of 1 lb. = 'g' poundals	Gravitational. Varies with 'g' (about ·3% from equator to pole).	
Foot-poundal	work	,,	1 poundal × 1 ft.	Absolute. No connection with 'g'.	
Foot-pound	,,	,,	1 lb wt. × 1 ft = 'g' ft poundals	Gravitational. Varies with 'g'.	
H. P. or Horse-power	power or rate of work	,,	550 ft. pounds per sec. or 33000 ft. lbs. per minute.	,, ,,	746 watts, or $\frac{746}{1000}$ or nearly $\frac{3}{4}$ kilowatt.

EXPERIMENT 45.—**To determine the work that a motor or steam engine can perform in one revolution.**

Apparatus: Battery motor, fan motor, toy turbine or model steam engine. friction-brake consisting of two spring balances and string.

Two spring balances, with a rod passing through the top rings and supported by clamps, constitute the brake. A better arrangement is shown in the diagram. The rod supporting the balances can be moved up and down, and clamped by screws The string passes round the motor pulley as shown. Start the motor or steam engine. Increase the tension of the balances by moving the rod up, till the string is as tight as it can be made, without unduly slowing down the motor or engine. The speed should be about the normal for the machine. when at work. Read the balances. Measure the circumference of the pulley by wrapping fine thread round it two or three times, and measuring the length.

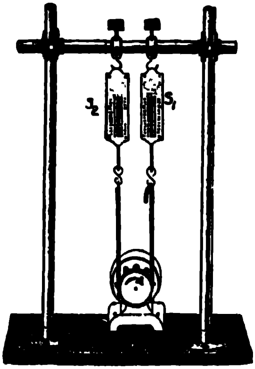


Fig. 62—Determining the work done by a motor in one revolution.

Observations:

Reading of balance S_1 : ... gms wt.

" " S_2 : ... gms. wt.

Friction : ... gms. wt. (1)

$$\text{Circumference of pulley} = \frac{\text{Length of thread}}{\text{No. of revolutions}} = \ldots\text{cm.} \quad (2)$$

Work done by engine in one revolution =(1) × (2) gm. wt. cm.

=(1) × (2) × 980 ergs.

= ...... ergs.

Note.—To measure the H.P. or watts, a counter is necessary for measuring the number of revolutions per minute (R. P. M.).

The counter can be pressed against the axle, or turned by it through a lever, and registers the number of revolutions automatically. The change in the figure on the counter, when pressed against the axle for one minute gives the R. P. M. In the case of a fan motor, the R. P. M. and voltage, etc are marked on the case.

$$\text{Work done by motor in a second} = \text{work in one revolution} \times \frac{\text{R.P.M.}}{60} \quad (3)$$

$$\text{Power in watts} = \frac{(3)}{10^7} = \ldots \quad (4)$$

$$\text{H. P.} = \frac{(4)}{746} = \underline{\underline{\ldots\ldots}}$$

CHAPTER XIV—*contd.*

WORK AND POWER

PULLEY BLOCKS. WHEEL AND AXLE

The student is, perhaps, already familiar with the fundamental fact about all machines that **what is gained in force (incorrectly 'power') is lost in speed.** This means that, in a perfect machine, the ratio of the **resistance** overcome to the force applied ($\frac{W}{P}$, called **mechanical advantage**) equals the ratio (called **velocity ratio**), of the distance through which the pulling force is applied, to the distance through which the **resistance** is overcome.

The **velocity ratio** depends on the design and dimensions of the machine, and **is not affected by friction. Friction,** however, **increases the force P necessary for a certain load W, so that the mechanical advantage $\frac{W}{P}$ is reduced.** The ratio of the actual mechanical advantage in any case to the maximum theoretical mechanical advantage (=velocity ratio) is called the **efficiency** of that machine Friction also reduces the **output,** or the useful work done by a machine. In a perfect machine, the **input,** or the work done by P equals the **output,** or the work done in raising W. Ordinarily, the output is less than the input. The ratio $\frac{\text{output}}{\text{input}}$ also measures the efficiency so that we have

$$\textit{Efficiency} = \frac{\textit{actual mechanical advantage}}{\textit{velocity ratio}} = \frac{\textit{output}}{\textit{input}}$$

EXPERIMENT 46.—**To measure the efficiency of a pair of triple pulley blocks.**

Apparatus : Two aluminium triple pulley blocks with pulleys arranged on a common axle, thread, clamps. Spring balance, weight box, scale pan, metre stick.

Weigh, with the spring balance, the pulley block that is to be the lower of the two. Arrange the blocks and thread (strong cotton) as follows :

Clamp the upper block one or two feet above the table, and so as to overhang its edge. Clamp temporarily the lower pulley block at a height of one or two feet above the floor.

Attach the thread to the lower hook of the upper block, and pass round lower pulley No. 1, then up and round upper pulley No. 1. Then down and round lower pulley No. 2, then up and round upper pulley No. 2 and so on. The free end from upper pulley No. 3 should have a scale pan attached. Now attach to the lower hook of the lower block a 200 gms. wt., as the load *W*. Release lower block from clamp, and by pulling at the free end make sure that all is well. Oil all the bearings. *If the lower block is released before a proper load is attached the block often overturns and the threads get entangled.*

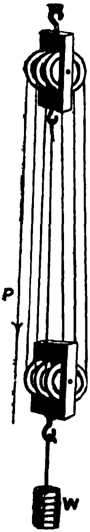


Fig 63.—A pair of triple pulley blocks.

To obtain the mechanical advantage. Load the scale pan attached to the free end till it just moves down. Try a second time. Record masses. The weight when the scale pan just moves up is *not* required, as the object is not to correct for friction, but to find out how it reduces the efficiency in actual use. Increase the load to 300 gms and find the corresponding value of *P*. Try 400 and 500 gms. in the same way. $\frac{W}{P}$ gives the actual mechanical advantage. To obtain the velocity ratio, count the number of threads supporting the lower block.

To measure $\frac{output}{input}$. Make or note a mark on the lower pulley block. Attach the free end of the thread to a clamp. Mark with pencil or a knot the thread where it just leaves upper pulley No. 3. Read the position of the mark on the lower block on a vertical metre scale held near it. Now pull the free end, so as to raise the lower block by about 30 cm. Clamp free end again. Make a second mark on the thread where it leaves upper pulley No. 3, and read the mark on the lower block again. Measure the distance between the 2 marks on the thread. This is the distance through which P has moved to raise W by about 30 cm.

Observations:

Weight of lower pulley block = ...
,, scale pan at free end = ...

Weight attached to lower block	Weight of block + weight attached = W	Weight in pan.	Weight in pan + weight of pan = P.	Actual mech. adv $\frac{W}{P}$	Theoretical mech. adv. or velocity ratio for a triple pulley block.
200	...	...	...	...	6
... Case No 3.	...	...	...	...	
...	...	...	...	...	
...	...	...	...	...	
...	...	...	...	...	

Mean value $\frac{W}{P}$ = ...

Efficiency $= \frac{W/P}{6} =$ ... or ... % I

Distance through which W rises : ... (1)
,, ,, ,, **P moves : ...** (2)

Output (take, say, case No. 3) = W × (1)
Input „ „ = P × (2)

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \dots \text{ or } \dots \% \qquad \text{II}$$

NOTE :—In actual engineering practice, the work done in raising the block will not be included in the output, as this work is not useful. On account of the large loads, however, this work is negligible. As we cannot conveniently use loads big enough to make the weight of the pulleys negligible, we regard this as a portion of the load, to get a correct idea of the efficiency.

I and II should agree fairly closely.

EXPERIMENT 46*a*. To determine the efficiency of the given wheel and axle.

Apparatus : Wheel and axle, weight box, pan, clamps, metre stick, thread, etc.

Hang up the machine against a wall. Arrange threads as shown, so that on pulling at the thread on the wheel, the load attached to the thread round the axle *rises*. Oil the bearings.

Determine P for various loads exactly as for the pulley blocks. Similarly measure the distance through which P moves in order to move W through about 30 cm. Measure the diameters of the wheel and axle by means of metre stick and vernier callipers respectively. The theoretical mechanical advantage or velocity ratio

$$= \frac{\text{Diameter of the wheel}}{\text{Diameter of the axle}}$$

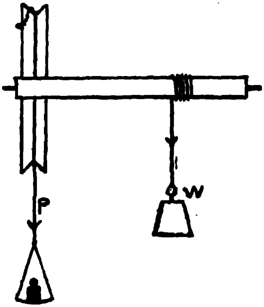


Fig.64.—Wheel and axle.

Tabulate observations as in experiment 46.

PART III

SOUND

CHAPER XV

MEASUREMENT OF FREQUENCY

The student has already performed one or two experiments with the simple pendulum. The terms **vibration** and **period**, defined in that connection, can be used for any periodic (or, as a special case, any regular to and fro) motion. A **tuning fork** when struck. or a wire on any stringed musical instrument when plucked, **vibrates**. The **vibrations** are communicated to the adjacent air, and **waves** of increased and decreased pressure **(condensation** and **rarefaction)** travelling in the air, carry the sound in all directions.

Period. The period has already been defined as *the time elapsing between two consecutive passages (in the same direction) of a vibrating body through its mean position.* The complete cycle of movements between two such consecutive passages is a complete vibration.

Frequency. This is the name given to **the number of complete vibrations executed by a body in one second**. If the period is t seconds, one vibration occupies t seconds, and the number of vibrations in one second (n), or the, frequency, is $\frac{1}{t}$. Hence **(frequency)** $\mathbf{n} = \frac{1}{\textbf{period (t)}}$ **or** $\mathbf{n} = \frac{\mathbf{1}}{\mathbf{t}}$ **and** $\mathbf{t} = \frac{\mathbf{1}}{\mathbf{n}}$.

Wave Length. Imagine a vibrating body (piano wire) in its extreme position, and producing at that instant, in the neighbouring air, a certain state of compression. This state of compression will travel forwards. In the meanwhile, the wire will be springing back, and transmitting a rarefaction, which will follow the condensation. After a full period, the wire will be in a position to produce exactly the same kind of condensation which it produced at first The first condensation will, by this time, have travelled away a certain distance.

This distance, which just separates the first condensation from the second one close to the wire, has been travelled by the wave in one period, and is called a wave length (λ). Therefore a

Wave length is *the distance between two nearest condensations (two nearest crests) or two nearest rarefactions (troughs)*. Also a

Wave length is *the distance travelled by a wave in one period*, or $\lambda = vt$, where λ, v and t are respectively wave length, velocity and period of the waves.

A **tuning fork** is a U-shaped bar of steel, with a straight handle at the bend of the U. When struck on a pad, or with a rubber or cork hammer, the fork vibrates, and gives out a musical note. The frequency of this note is the frequency of the fork. The greater this frequency, the higher the **pitch** of the note emitted.

If a short stiff bristle is attached to a fork, and the fork struck, the bristle, if passed rapidly over a smoked glass plate, will give a wavy trace. Each complete wave corresponds to a complete vibration of the fork. We will use such a wavy trace for determining the frequency of a fork.

EXPERIMENT 47.—**To determine the frequency of a tuning fork by means of a falling plate.**

Apparatus: Tuning fork and falling plate in wooden frame, hog's bristle, soft wax, metre stick, dividers.

The apparatus consists of a vertical frame, in which a slip of glass, about 30 cm. × 5 cm. (12″ × 2″), can freely slide. In front of the plate is a block to which the fork can be clamped.

Cover the slip of glass with soot by holding it over burning camphor, or a smoky Bunsen flame. Keep the glass moving to avoid breakage. Next attach to one prong of the fork, by means of a *very little* soft wax, a piece of hog's bristle about 1 inch or less in length. Clamp the fork firmly. Adjust the plate, so that it just touches the bristle, and clamp the frame in this position. Pull the plate up, strike the fork with a suitable hammer (a rubber stopper fixed on the end of a thin metal rod), and drop the plate. A wavy trace will be obtained. If the first attempt fails, repeat so as to get a suitable trace. This will consist of from 30 to 60 waves. Mark with a needle or pin 5 or 6 successive groups of 8 or 10 waves each. To do this mark the top of a wave towards one end, and then the 11th, 21st, 31st wave and so on. Measure, by dividers, the lengths of these consecutive groups. If these distances are d_1, d_2, d_3, etc., it will be found that, approximately, $d_1-d_2=d_2-d_3=\cdot\,\cdot$ Calculate these differences and their mean.

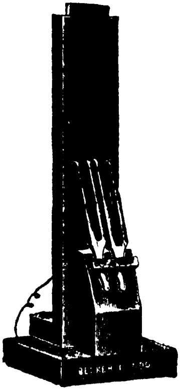


Fig 65
Frequencies compared by falling plate method

$$\text{The frequency } n = m\sqrt{\frac{g}{d_1-d_2}}$$

where m is the number of waves in each group.

The plate falling with acceleration 'g', covers the distance d_1 in the first interval of m vibrations (mt period), and d_2 in the next interval of m vibrations. The mean velocity of the plate in the first interval is $\frac{d_1}{mt}$ where and in the second interval $\frac{d_2}{mt}$. These are actually the velocities at the middle of these intervals. These middle instants are, themselves, separated by time mt. Hence, in time mt the velocity of the plate changes by $\frac{d_1}{mt} - \frac{d_2}{mt}$ or $\frac{d_1 - d_2}{mt}$, or with acceleration $\frac{d_1 - d_2}{(mt)^2}$. But this acceleration is 'g'; hence

$$'g' = \frac{d_1 - d_2}{(mt)^2}.$$

$$\text{or } \frac{1}{t^2} = m^2 \cdot \frac{g}{d_1 - d_2}$$

$$\therefore \quad n = \frac{1}{t} = m \sqrt{\frac{g}{d_1 - d_2}} \quad .$$

Enter thus:

					Differences
Length	1st group	of	10 waves	=.........	
	2nd	,,	,,	=.........	...
	3rd	,,	,,	=.........	...
	4th	,,	,,	=.........	...
	5th	,,	,,	=.........	...

$$\text{Mean difference} = \frac{\ldots\ldots}{4} = \ldots\ldots$$

$$\therefore \quad n = 10\sqrt{\frac{979}{\ldots\ldots\ldots}}$$

$$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Answer the following :

(1) What are the unavoidable sources of error in this experiment?

(2) Can you find 'g' approximately by this method?

The above is a commonly listed, but less accurate, form of apparatus as compared with one in which a heavy plate hangs from a thread passing round two nails, no groove or frame being used. The plate is released by burning the thread, and falls freely. A thick polished brass plate is more convenient than the fragile glass plate. The support can be made easily by any carpenter.

CHAPTER XVI

MEASUREMENT OF WAVE LENGTH AND VELOCITY OF SOUND

Resonance. A tuning fork readily excites loud sound **(resonance)** in an air column closed at one end. if the length of this column (l) equals, approximately. one quarter the wave length in air (λ) of the note produced by the fork. An air column of about three times this length is also readily excited by the same fork. Both of these lengths are, however, slightly shorter than the amounts stated $\left(\frac{\lambda}{4}\right)$ and $\left(\frac{3\lambda}{4}\right)$, by a small amount depending on the diameter of the air column. If this correction, called **open-end correction** or **end correction**, is x, it is found that the length of the shorter resonance column is

$$l_1 = \frac{\lambda}{4} - x \quad \ldots \quad (1)$$

and that of the second is

$$l_2 = \frac{3\lambda}{4} - x \quad \ldots \quad (2)$$

If l_1 and l_2 are measured experimentally, the values of λ and x can be obtained from equations (1) and (2). Subtracting (1) from (2),

$$l_2 - l_1 = \frac{\lambda}{2} \quad \text{or} \quad \underline{\lambda = 2\ (l_2 - l_1)}.$$

Multiplying (1) by three, and substracting

$$l_2 - 3l_1 = 2x \text{ or } \quad \underline{x = \frac{l_2 - 3l_1}{2}}.$$

The value x is known to be about $\cdot 3d$, where d is the diameter of the column. The resonance of air columns provides us, therefore, with a method of determining wave-length.

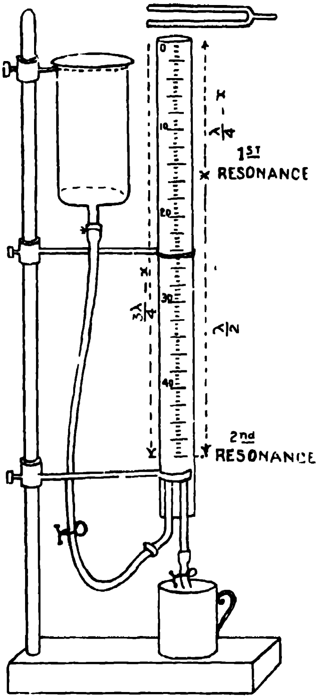


Fig 66. -Experimental determination of velocity of sound by resonance

Resonance apparatus. A common form of resonance apparatus consists of a glass tube 60 cm. long, and 3 to 4 cm. diameter. This tube communicates at bottom by means of rubber tubing, with a large glass or metal reservoir, which can be clamped at any height. By placing the reservoir above or below the resonance tube, water can be made to flow in either direction, and a resonance column of any length obtained. A pinchcock or screw-clip on the connecting tube regulates the flow.

In course of experiment, the position of maximum resonance has to be repeatedly measured. To do this, it is necessary to clamp the reservoir *alternately above and below* the resonance tube, after each trial Much time can be saved if a small discharge tube with a spring-clip is inserted into the bottom of the resonance tube. The reservoir may be *permanently* clamped at the same level as the open end of the resonance tube, and a tin can placed under the discharge tube The water level in the resonance tube can be *raised* by means of the pinchcock on the communicating tube, and *lowered* by means of the cock on the discharge tube. The can when full can be emptied into the reservoir.

By definition of wave length

$$\lambda = vt, \qquad (t = \text{period})$$

$$\text{or} \quad \lambda = v\,\frac{1}{n}. \qquad \left(t = \frac{1}{n} \text{ where } n \text{ is frequency}\right)$$

$$\text{or} \qquad \underline{n\,\lambda = v.}$$

If we determine λ experimentally, either n (frequency) or v (velocity of sound in air column) can be found, if the other is known.

EXPERIMENT 48.—**Given a fork of known frequency, to determine (1) wave length from the first position of resonance, assuming end correction = ·3 d, (2) wave length and end correction from the 1st and 2nd positions of resonance, (3) the velocity of sound at 0°C.**

Apparatus: Resonance tube connected to water reservoir, or other resonance apparatus. Fork, metre stick, leather pad.

Fix a mm. scale parallel to the tube, with the zero at the open end.

Nearly fill up the resonance tube. Hold with one hand the vibrating fork (strike on pad, not table) near the open end of the tube. and lower the level by means of the discharge tube. Close cock as soon as the column resounds strongly. This gives a rough idea of the position of maximum resonance. The more careful determinations are to be made in the same way, except that the water should flow at a slower rate. Raise or lower the level slowly by means of the cocks. Judge carefully. so as to obtain a position of *maximum* resonance. Measure correct to a millimetre (the determination cannot be made accurate enough to make estimation of tenths of any value). To do this ignore fractions less than ·5 mm., and count as 1 mm. those from ·5 upwards Repeat carefully, so as to obtain at least six readings. the position being fixed 3 times while the column is *increasing* in length and 3 times while *decreasing*. Calculate the mean. Measure the diameter of the tube

Obtain, in the same way. six readings for the second position of resonance. The column in this case will be very nearly three times the first length. Adjustments may, therefore, start with a column of approximately this length. Calculate the mean of the six readings.

Observations :

Diameter of resonance tube = ... cm.
Assumed end correction = ·3*d* = ... cm.
Temperature = ... °C.

Length column first resonance l_1	Corrected length. $l_1 + \cdot 3d$	Wave length $4(l_1 + \cdot 3d)$ λ	Length column second resonance. l_2	Wave length $2(l_2 - l_1)$ λ	Calculated end correction. $l_2 - 3l_1$ $\frac{l_2 - 3l_1}{2}$
Mean :			Mean :		
...			...		
...			...		
... $\frac{\dots}{6}$	...	...	... $\frac{\dots}{6}$	...	$\frac{\dots - \dots}{2}$
...			...		
... = ...			... = ...		= ...
...			...		

Mean value of λ = ... (1)
Frequency of fork = ... (2)
Velocity of sound at 25°C = (1) × (2)
= ... (3)
Velocity of sound at 0°C = (3) − 62 × 25 cm.
= ...

Record answers to the following :

(1) Has the calculated velocity of sound to be corrected for pressure ? Give reasons.

(2) Has the velocity to be corrected for the presence of water vapour in the air of the resonance column ?

(3) If given v, could you find n? Will this value be more reliable than that obtained by the falling plate?

(4) How could you compare the frequencies of two forks?

PART IV

HEAT

CHAPTER XVII

THERMOMETRY. EXPANSION

Unit of Temperature.

Temperature can be measured by measuring any one of the many effects that heat can produce, *e.g.* expansion, change of state, or change of electrical resistance. Expansion of mercury or alcohol is the effect commonly used for measuring temperature, the instruments constructed on this principle being called mercury or alcohol thermometers. For measuring temperature, we require a standard or "unit" temperature interval, just as for measuring length and time we have the centimetre and the second as the standard magnitudes or "units." A centimetre is the space between two fixed marks one rod ; similarly, the difference between the hotness or coldness, in other words **temperature,** of melting ice, and that of water boiling under normal pressure is our standard temperature interval

Scales of Temperature.

There are, however, different ways of subdividing and figuring the divisions of this interval. In the **Fahrenheit** and **centigrade scales,** these details are as follows :—

Scale	Melting ice	Boiling water	Degrees in interval	Relation
Centigrade	0	100	100	$1^\circ C = \frac{180}{100}$ or $\frac{9}{5}\,{}^\circ F$
Fahrenheit	32	212	180	

If F and C are the readings, on the Fahrenheit and centigrade scales, for the same temperature, $F = 9/5\ C + 32$ or $F = 1{\cdot}8\ C + 32$.

EXPERIMENT 49.—**To study graphically or otherwise, the relation between centigrade and Fahrenheit scales of temperature.**

Apparatus : A thermometer with both scales on the stem. Or centigrade and Fahrenheit thermometers, 250 c.c. beaker, wire gauze, tripod, burner, etc., clamps and stand, stirrer.

If a thermometer with combined scales is available, no experiment need be performed, as the object is merely to study the relation between the scales. Read the Fahrenheit scale opposite the principal divisions of the centigrade scale ,–10°, 0°, 10°, 20°, 30° ....100°C, and tabulate the two sets of data. With separate thermometers, the following experiment is necessary.

Support the thermometers vertically in a beaker of water on gauze and tripod. The bulbs should be well immersed. Heat the water to the boiling point. After the water has been boiling one or two minutes, and the mercury columns quite steady, read both the thermometers, estimating the tenths with care. Remove the burner, and allow the water to cool. When the temperature is about 90°C, stir thoroughly and read both thermometers carefully, as far as possible at the same time. Similarly read both the thermometers simultaneously at intervals of about 10°C. Stir the water every time you take a reading. Place your eye on a level with the mercury column, as when reading a burette. This is to avoid the error of parallax.

The relation between centigrade and Fahrenheit scales may be obtained *graphically* or by *arithmetical calculation.*

Graphic Method.* Plot a graph with the data obtained, *placing the zero for both scales at the origin, and using equal spaces, say 1 mm., for* 1° *of each scale.* Plot centigrade readings as abscissæ and Fahrenheit as ordinates. Examine the points plotted. They will be found to lie on a straight line. Draw the best line with the help of a thread stretched so that the points are either on it, or equally distributed on both sides of it. (Fig. 67.)

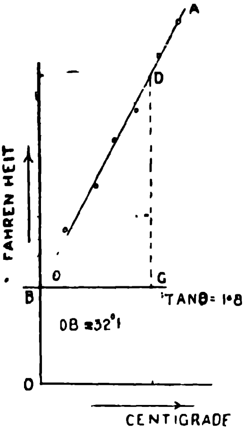


Fig. 67. –Graphical study of centigrade and Fahrenheit scales.

The equation for a straight line is $y=mx+c$, where m is the tangent of the angle between the straight line and the x axis, and c the intercept on the y axis. Read DG and BG on the squared paper. DG/BG is the tangent of the angle of slope and equals m. Read OB. This equals c. Substitute these values in the equation $y=mx+c$, writing °F for y and °C for x, and compare with the known relation °F=1·80×°C+32.

Record thus:

Centigrade readings.											
Fahrenheit readings.											

Graph drawn, *0 for both scales being at origin and 1°C and 1°F being represented by the same distance.* Result *straight line*, equation

---

*This graphical study of a simple relation already known to the student is specially instructive as straight line graphs are common in practical work, and the equation of a straight line is a part of the Intermediate Physics syllabus.

$$y = m\,x + c$$

$$m = \tan\theta = \frac{\cdots\cdots}{\cdots\cdots} = \ldots\ldots$$

Intercept $c = \ldots\ldots{}^\circ F$

Substituting, $^\circ F = \ldots\ldots\ {}^\circ C + \ldots\ldots$ I

Known relation : $^\circ F = 1{\cdot}80 \times {}^\circ C + 32$ II.

I and II will be identical.

Arithmetical Verification. *Record as follows* :

Centigrade readings (*observed*)							
Fahrenheit „ (*observed*)							
Fahrenheit readings *calculated* from known relation $F = 1{\cdot}8C + 32$							

Freezing point and Boiling point Corrections. The student will do well to remember the definitions and rules given on page 48 in connection with zero correction, etc.

Def. **Zero reading**=*actual reading with proper sign under conditions in which an ideal instrument would read zero.*

Def. **Zero correction** = *zero reading with sign reversed.*

Rule : **Corrected reading**=*observed reading*+*zero correction.* (Corrected for zero only).

Never use the term "zero error" as its significance is doubtful as to sign. "*Correction*" has *always* to be *algebraically added.* To do so, *merely write down the correction with its proper sign, immediately after the observed reading with its proper sign.*

EXPERIMENT 50.—**To determine the freezing and boiling point corrections of a thermometer, and to prepare a curve of corrections for intermediate temperatures.**

Apparatus : Ice, large funnel, hypsometer, barometer, clamps, stands, etc., heating arrangements.

Freezing point correction : Arrange a large funnel on a ring, with a beaker or tin can underneath to catch the drippings. Pound clear washed ice (about 1 lb.) in a clean duster. Support the thermometer with the bulb near the middle of the funnel, and pack it all round with the clean pounded ice. Press all round to ensure good contact with the bulb. Wait for half an hour, pressing the ice gently round the bulb from time to time, and, if necessary, adding more ice After half an hour, pull out the thermometer slowly so as to make the mercury surface just visible above the ice. Looking horizontally so as to avoid parallax error, read carefully the temperature registered. Note down this zero reading *with its proper sign*.

Boiling point correction : If the boiling point of your thermometer is correctly marked, it will read 100°·0C in steam, *provided the pressure is normal or 76 cm.* This, however, is rarely the case. Therefore, in determining the correction, allowance has to be made for pressure. *A change of 1 cm. in the barometric height corresponds to a change of 0°·36C in the boiling point*

Fix the thermometer in a cork fitted into the neck of an hypsometer. Push in as much of the thermometer as you can, as long as the graduations above 98° are visible. The quantity of water should be such that the bulb is about 2 inches above the surface. Make sure that there is some passage open for steam to escape, and heat the water to boiling. Read the thermometer from time to time. When the mercury has been stationary for at least five minutes, note down the reading.

Observations:

Reading of thermometer in pure ice = ... (1)

Freezing point or zero *correction* = – (1) =... I.

Reading of thermometer in steam = ... (2)
Barometric height = ... (3)
True boiling point for this height = ... (4)
Boiling point *correction* = (4) – (2)
= ... II.

Curve of Corrections : Suppose the freezing point correction to be + 1°C, and the boiling point correction to be — 2°C. It is evident that the correction for intermediate readings on the scale decreases at first,

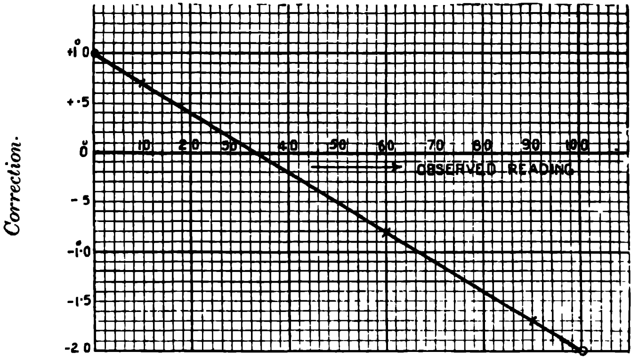


Fig. 68.—Curve of corrections for a thermometer

as we travel from zero upwards. At some point between 0° and 100°C it will amount to zero. After that the correction is negative, and increases numerically till, at the boiling point, it amounts to —2°C. To calculate the correction for intermediate, readings requires a certain amount of arithmetic. A simple graph, however, can supply the correction for any reading at a glance. To draw the graph for the above case, lay a scale of abscissæ from 0° to 100°. Above 0° plot the freezing point correction + 1° and below (as correction is —) 100° the boiling point correction —2°. The straight line joining these two points (Fig. 68) is the " curve of corrections " for the thermometer. The corrections for observed readings 10°, 60° and 90°C can be read at a glance as $+0^\circ{\cdot}7$C, $-0^\circ{\cdot}8$C and $-1^\circ{\cdot}7$C respectively.

Draw a graph of corrections, and ascertain the corrections for the boiling point of ether and the melting point of wax, observed as $34^\circ{\cdot}9$C and $55^\circ{\cdot}2$C.

We have assumed that the correction changes uniformly along the scale. This is the case if the tube is of uniform bore. For very accurate work, the corrections at 10°C, 20°C, 30°C.........100°C are individually determined and a " curve of correction " plotted with these values. The discussion of this is beyond the scope of this book.

Coefficient of Linear Expansion. The **increase in length of a bar or tube, 1 cm. long, per degree** *rise of temperature, is called the* **coefficient of linear expansion** of the material of the rod or tube. The value of this coefficient can be found by measuring the increase in length of a bar or tube about 80 cm. long, when heated from ordinary temperature to the tempereture of steam. Bars can be heated by being enclosed in glass or metal jackets, through which water or steam can be passed. Tubes require no jackets, the water or steam being passed through the tube itself.

A very serviceable and accurate laboratory apparatus for the purpose can be made in the college workshop, if a micrometer screw of the kind shown in Fig 69 is available The micrometer screw has a pitch scale and a head scale like a spherometer, but is mounted on a small metal bracket with a binding screw for a wire connection.

The materials are used in the form of tubes. Copper, brass and iron tubes about 80 cm. long and 1·5 to 2 cm. diameter are provided with side tubes as shown (Fig. 69). A side tube in the middle (not shown in the diagram) for the thermometer is desirable. One end should have a thin steel disc *d* soldered on, as otherwise the point of the micrometer screw is liable to form pits in the brass or copper. These tubes should be provided with permanent flannel coats, except for the end faces and small lengths near these, where the bare tube rests on the supports.

The support for the tubes consists of a wooden base 100 cm. × 10 cm. × 3 cm. Two massive blocks *bb* of the shape shown are *firmly* screwed on to the base with long screws from below, so as to be *absolutely rigid.* An ordinary iron bolt provided with a pointed glass rod *g* (fixed into the cup *c* with plaster of Paris) can be passed through the side of one block, and made fast with two nuts *nn*. The micrometer screw *m* is firmly screwed to the other block. A thin board *ss* screens the base from radiation from the tube. The inner raised ends of the blocks have V-cuts in which the tube rests, one of the cuts being lined with brass plate *p* so as to be in metallic connection with the tube. A binding screw is soldered to a continuation of this brass plate. The support should be painted white so as to absorb as little heat from the tube as possible.

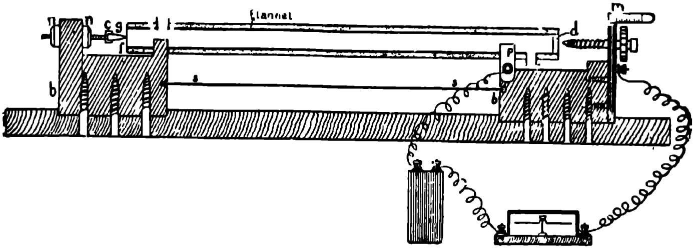


Fig. 69.—Linear expansion apparatus

EXPERIMENT 51.—**To determine the coefficient of linear expansion of brass.**

Apparatus: Brass tube 80 cm. long, with side tubes, linear expansion apparatus, steam boiler, thermometer, metre stick, Leclanche or dry cell, simple galvanoscope.

Start heating the steam boiler. Place the tube so that it rests firmly on the V-grooves, the bare portion pressing well on the brass plate *p*. The face *f* should press against the glass rod *g*. Screw back the micrometer, so as to leave a gap of 3 or 4 mm. between the steel disc *d* and the point of the screw. Connect the battery and galvanoscope as shown. Connect the brass tube to the boiler, so as to pass a rapid current of steam through it. After the steam has been passing 5 minutes, screw the micrometer forwards, at first quickly, later on slowly. As soon as the needle is deflected, read the disc scale. Do not record this reading. Screw back a little bit, and try again after

2 or 3 minutes, approaching the tube very slowly and carefully. Repeat reading every 2 or 3 minutes, pressing the tube gently against the glass rod every time, to be sure that it has not moved. When the successive readings differ only by one division or less, record three or four readings at intervals of two or three minutes. The simple electrical device used is sensitive and convenient. The temperature of the tube may be noted, if a thermometer has been fixed in a side tube in the middle of the main tube. In its absence take the steam temperature in the boiler, or calculate it from the barometric height.

Disconnect tube from boiler, and pass a current of air or water at ordinary temperature. The tube will contract. Press tube against *g*, and screw forward the micrometer till the point touches the tube, counting the complete revolutions and the divisions in the necessary incomplete revolutions as explained under spherometer. Record three or four readings of the disc scale as soon as they are constant within a division or so. Read the thermometer fixed in the middle of the main tube or take the temperature of the tap water, or that of the air, by means of a thermometer whose bulb is held in the air or water current from the tube. Remove the tube from its support, and measure its length (correct to a mm.) between the two faces by means of a metre stick. If necessary read the barometer. *The vernier need not be read.*

Observations :

See next page.

Enter observations thus :

Pitch of screw = ·5 mm.
Divisions on disc = 100
Least count of screw = ·005 mm.

Disc reading (when steady) steam passed *divisions.*	Details of screwing down to steady position, air or water passed.	Disc reading (when steady) air or water passed *divisions.*	Expansion.
... ... Mean : 53·4 Read fractions of a mm. directly (not *divisions*) if the disc is so graduated. Check graduations once for all by counting the divisions.	*First incomplete revolution*: From mean value 53·4 *downwards* through 50, 40 etc. to 0 or 53·4 div. *Complete revolutions*: THREE. *Last incomplete revolution*: From 0 or 100 *downwards* through 90 to mean value (see next column) 84·7 or 100-84·7 or 15·3 div.	... ... Mean : 84·7	Total divisions in two incomplete revolutions 53·4 + 15·3 or 68·7 Expansion = 3 × ·5 + 68·7 × ·005 mm. = 1·5 + ·3435 mm. = 1·8435 mm. = ·1844 cm.

Temperature of tube when hot, t_1 = ...°C
" " cold, t_2 = ...
Difference, $t_1 - t_2$ = ...

Length of tube, l = ...cm.

Coefficient of linear expansion $= \dfrac{\text{Expansion}}{l \times (t_1 - t_2)} = \text{........}$

CHAPTER XVIII

CHANGE OF STATE

MELTING POINT. BOILING POINT
VAPOUR PRESSURE. HUMIDITY

Melting. Most solids, if heated sufficiently, assume the liquid condition. The liquid, if cooled, can be reconverted to a solid. These changes of state obey two laws:

(1). *For a given substance, liquefaction and solidification take place at a fixed temperature (not materially affected by atmospheric pressure) called its* **melting point.**

(2). *Liquefaction (or solidification) of 1 gramme of the substance is always accompanied by absorption (or evolution) of the same fixed amount of heat called* **latent heat**. The absorption or evolution is not accompanied by change of temperature, if the substance is kept well stirred.

Both of the above laws can be applied to determine the melting point. The first law is applied in the capillary tube method, in which the temperature at which a small amount of the solid just melts is noted. The second law is applied in the cooling curve method, in which the temperature at which the latent heat is evolved by a slowly cooling molten solid is noted. This temperature is evidently the melting point.

EXPERIMENT 52.—**To determine the melting point of naphthalene by (*i*) capillary tube method, (*ii*) cooling curve method.**

Apparatus: Piece of thin-walled capillary tubing, beaker, tripod, thermometer, test tube, stirrers, clamp, ordinary watch with seconds hand.

Capillary tube method: Heat a 4-inch piece of thin-walled glass tubing, 4 or 5 mm. diameter, at the middle point over a large bunsen burner, and when soft, draw out to a capillary.

Break the drawn out portion in the middle so as to have two capillary tubes, with portions of the original tubing at the ends, to serve as cups Place a little powdered naphthalene in the cups and shake so as to have some of the powder in the capillary portions. Carefully seal up the bottoms, without exposing the naphthalene to the flame. Attach one of the capillary tubes so prepared to a centigrade thermometer, by thread or rings cut from rubber tubing, so that the substance is near the bulb.

Preliminary Experiment. A preliminary experiment to obtain a rough idea of the melting point will save much time. Place the thermometer and capillary tube in water in a *small* beaker. Heat and observe the temperature when the substance melts. In the determination proper, the water may be heated quickly to a temperature about 5° less than this, *and then heated very slowly.* Without the preliminary experiment, the bath has to be heated slowly from the very beginning which means unnecessary waste of time, and one feels uncertain all the time as to when the substance will melt with the result that sometimes the whole of the substance is found to have melted before the thermometer is read

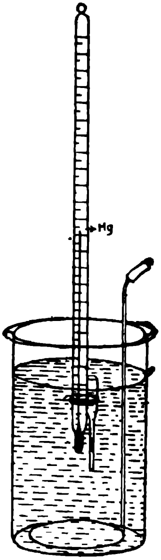


Fig. 70.—Determination of melting point: capillary tube method

Final Experiment. Attach the *second* (reject the first) capillary tube to the thermometer, placing the two in water in a *large* beaker. Heat quickly to within 5°C of the approximate melting point. Then lower the flame, and keep stirring the liquid and watching the thermometer all along. Arrange, if possible, to have a rise of 1°C in one minute. *Stirring is important*. Read the thermometer *as soon as there is the slightest sign of melting.* This temperature is the melting point.

A reading of the point of solidification by allowing the beaker to cool is often advised. This procedure is of doubtful value, and in some cases actually misleading. Many substances, especially organic, change slightly if once melted, and will not solidify at the same temperature. The solidification is also difficult to detect in many cases (*e.g*, wax), as the solidified substance often remains transparent or translucent instead of becoming quite opaque. The safest way is to use a *thin* capillary, *to heat slowly*, and to read the thermometer at the *first* sign of liquefaction. (With naphthalene, however, the solidifying point is quite marked).

The cooling curve method: Place enough powdered naphthalene in a test tube to cover, when melted, the bulb and one or two inches of the stem of a thermometer supported in a cork in the mouth of the tube. The cork should be a loose fit to keep the pressures inside and outside equal. A small stirrer is also necessary. Place the test tube in boiling water (*not* over a flame, or the naphthalene may get overheated and crack the thermometer) in a *small* beaker. When the naphthalene is all melted, remove the burner and stir. Record temperatures every minute, stirring all the while. Continue stirring and recording during solidification, and for a few minutes after it. The temperature will fall regularly at first, then keep stationary, and then fall again. All the three stages should be observed.

In the stationary stage, heat is being evolved (because the substance is hotter than the surroundings) and yet there is no change of temperature.

The heat evolved is the *latent heat*, and the *steady* temperature the *melting point.*

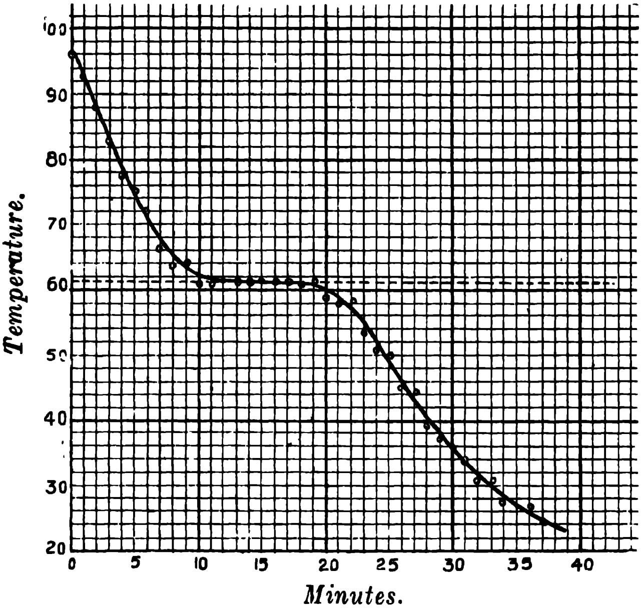


Fig. 71.—Determination of melting point: cooling curve method. The horizontal portion corresponds to 61°·4, which is the melting point

Draw a curve with temperature as ordinates and times as abscissæ. The horizontal portion corresponds to the melting point. The test tube is cooled much too rapidly if supported in air. A large beaker, on the other hand, makes the cooling tediously slow. Cooling in a *small* beaker is on the whole best.

Evaporation and Ebullition. These are the processes by which a liquid is converted to vapour. **Evaporation** is the slow conversion of a liquid into vapour, and takes place on the liquid surface at all temperatures. **Ebullition** or boiling is rapid conversion of a liquid, throughout its body, into vapour, which rises through the liquid in the form of bubbles. Ebullition is governed by the following laws:

(1) **Ebullition** *or* **boiling** *takes place at a definite temperature (called boiling point) depending on the liquid and the atmospheric pressure. Increase of pressure increases the boiling point and vice versa.*

(2) *For converting 1 gramme of liquid at its boiling point to vapour at the same temperature a definite amount of* "**latent heat**" *is necessary.*

(3) *The* **vapour pressure** *of a liquid* **at** *its* **boiling point equals** *the* "**atmospheric pressure**" *(corresponding to that boiling point).*

Laws (1) and (3) can be applied to determine the boiling point of a liquid.

EXPERIMENT 53*a*.—**To determine the boiling point of a pure liquid (not a solution).**

Apparatus: Boiling tube, liquid (alcohol), thermometer, cork, delivery tube, gauze, burner, etc.

Bore two holes in the cork, and into these fit a thermometer and a long straight delivery tube. Fill two to three inches of the tube with the pure liquid, add a few glass beads or bits of porcelain, and arrange the thermometer bulb about an inch above the liquid surface. Surround the tube with sand in a metal dish, and heat the sand over burner and tripod. The liquid will boil. Arrange that the boiling liquid does not touch the bulb. After a couple of minutes the reading will be found to be stationary. Wait another couple of minutes, and read. This temperature is the boiling point. If the temperature goes on rising slowly, the liquid is probably a mixture of liquids having different boiling points. The long delivery tube reduces this difficulty by partially returning the condensed vapour to the tube, so that the composition remains more or less unchanged. The long delivery tube is also of use in keeping inflammable vapour away from the burner.

NOTE :—Fires are frequently caused by inflammable vapour lighting up, or by boiling tubes breaking. The conflagration in the latter case may be serious The box of sand in the laboratory corner is important. Never heat the tube over a flame. Besides risk of fire, the flame, if large, may touch the tube above the liquid, and give a wrong reading by overheating the vapour.

When determining the boiling point of a solution (e. g. *salt sloution*) *the bulb should be immersed in the boiling solution.*

The Vapour Pressure or Bent Tube Method. The method of determining boiling points described below has many advantages over the boiling tube method. It is equally applicable to all solutions and pure liquids, inflammable or otherwise, that do not act on mercury. A small quantity of the liquid is sufficient and, in addition, the boiling point for pressures 8 or 10 cm. *above* or *below* the atmospheric, can be readily found. It is also free from all risk of fire. The principle used is given in law (3) of ebullition ; **vapour pressure at boiling point equals atmospheric pressure** ; *conversely, boiling point is the temperature at which the vapour pressure of a liquid equals the atmospheric pressure.*

EXPERIMENT 53.—**To determine (1) the boiling point of a liquid (alcohol or water) and (2) the change in boiling point per cm. rise of pressure.**

Apparatus: Bent tube, mercury, steel scale, clamp, stand, burner, *large* beaker, stirrer.

Prepare a bent tube (use a fish-tail burner) of the shape shown

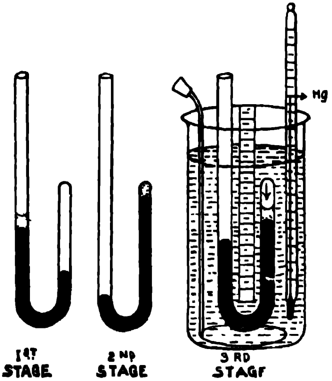


Fig. 72.—Determination of boiling point for pressures, 66—86 cm. of mercury.

(Fig. 72). The following dimensions are recommended. Bore of tube 6 to 8 mm., closed limb 10 cm., open limb 20 to 25 cm. This is sufficient for determining the boiling point for pressures from 66 cm. to 86 cm. of mercury.

Put enough mercury in the tube to fill the closed limb and the whole of the bend. Add a cubic centimetre of the given liquid, and by tilting the tube, transfer it to the top of the closed end.

If the liquid has become very dirty by coming in contact with mercury, transfer it to the open limb, absorb in cotton wool or filter paper fixed to the end of a knitting-needle or wire, and add a fresh supply. Manipulate *till the liquid is free from all air bubbles.* Absorb liquid left in the open limb with filter paper. A *small* quantity does no harm ; in fact, it may be an advantage, as it balances the hydrostatic pressure of the liquid in the closed limb. The liquid in the closed end should occupy about 5 or 6 mm. of the tube. If the required boiling point is considerably *below* 100°C, arrange the bent tube, prepared as above, vertically in a *large* beaker filled with *water.* If the boiling point is nearly 100°C or *above*, the beaker should be filled with *saturated salt solution.*

If only part (I) of the experiment is to be performed, follow directions under part (I). For performing both parts proceed at once to directions for complete experiment given later on.

DIRECTIONS FOR PART I. A steel scale between the two limbs is recommended, as it helps in judging when the two mercury surfaces are on the same level. Heat the beaker carefully on gauze and tripod. Keep stirring As soon as the mercury in the closed limb begins to fall, lower the flame *so that the fall is gradual.* Remove the flame altogether when the mercury in the closed tube is two or three millimetres *higher* than the mercury in the open tube The mercury will continue to descend. *Keep stirring throughout.* Read the thermometer when the two mercury surfaces are ***on the same level.*** This temperature is the boiling point. Heat a little so that the mercury in the closed tube is a little ***below*** the other mercury surface. Remove the burner, and allow the liquid to cool, stirring all the while. The mercury in the closed tube will now ***rise,*** Read the temperature when the two surfaces are again level. The mean of the two readings gives the boiling point at atmospheric pressure.

Record as follows :

Boiling point. (temperature *ascending*) = ...°C
Boiling point, (temperature *descending*) = ...°C
Mean boiling point = ...°C
Barometric height, correct to 1 mm. = ...mm.

DIRECTIONS FOR COMPLETE EXPERIMENT. Arrange a steel scale between the two limbs of the mercury tube as shown in Fig. 72. Heat the beaker carefully on gauze and tripod. The mercury surface in the closed tube will fall. Continue heating slowly, and when the mercury in the closed tube has ***nearly*** reached the curved portion of the bend, remove the burner. ***Keep stirring.*** As soon as the mercury surfaces are stationary read their positions on the cm. scale as also the temperature. Allow the liquid to cool. ***Keep stirring.*** The mercury in the closed tube will rise slowly. When it has risen 1 cm. or so, repeat readings of scale and thermometer. Obtain 6 or 7 such sets of readings (stirring all the while) ***before*** the mercury reaches the top of the closed tube. A ***large*** beaker is necessary, as in a small one the fall of temperature is too rapid, and accurate readings are impossible.

Read the barometric height correct to 1 mm. ***The vernier need not be read.***

Observations:

Liquid used : water. Bath : salt solution.
Barometric height H : 73·3 cm.

Mercury closed limb.	Mercury open limb.	Open limb reading-closed limb reading p	Total pressure on liquid $(H+p$ P	Boiling point t°	Mean change in t per cm. of pressure
cm.	cm.	cm.	cm.		
22 69	30 22	+ 7·53 (+7·5)	73·3 + 7·5 = 80·8	...	Divide total difference in t by total difference in P.
...				...	
...				...	
...				...	
...				...	
...				...	
30·65	22·24	− 8 41 (− 8 4).	73 3 − 8·4 = 64·9	...	

Draw a *smooth* graph with P as abscissæ and t as ordinates. The boiling point for the existing atmospheric pressure can be read from the graph over the proper value on the axis of pressures.

Relative Humidity and Dew-point. The condition of the atmosphere as regards the contained moisture is determined by **hygrometers.** Some of them determine the **'dew-point'** or the temperature at which the dew just begins to form. The contained moisture is sufficient to saturate the atmosphere at the temperature of the dew-point. The pressure of the vapour actually present, therefore, equals the saturation pressure for the dew-point. The vapour necessary for saturating the atmosphere at its actual temperature is more, and *the ratio of the contained moisture to the amount* *necessary for saturation is called the* **humidity** *or* **relative humidity**. Therefore.

Relative humidity

$$= \frac{\text{contained moisture}}{\text{moisture necessary for saturation at room temperature}}$$

$$= \frac{\textit{saturation pressure at dew-point}}{\textit{saturation pressure at room temperature}}.$$

In **dew-point hygrometers,** a suitable surface is slowly cooled, and the temperature at which this surface begins to get dulled on account of the deposition of moisture is noted. This is the dew-point. In another form, **the wet and dry bulb hygrometer,** the dew-point and humidity are determined indirectly from the temperature of a wet surface from which evaporation is going on. The lower the humidity, the lower is the temperature of the wet surface. The relation between the vapour pressure, dew point or humidity and this temperature is, however, **empirical,** and in any case, complicated. Tables specially prepared for the purpose are, therefore, always used to obtain these from the wet and dry bulb readings.

EXPERIMENT 54.—**To determine the dew-point and the relative humidity of the air by means of the wet and dry bulb hygrometer.**

Apparatus : Wet and dry bulb hygrometer, cotton rag, strong solution caustic soda.

Well wash a cotton rag with strong caustic soda. Rinse thoroughly with water. Tie one end all round one of the bulbs, and let the other dip in water in the reservoir.

Watch the thermometers from time to time. When steady, record the temperatures. Obtain the difference between the wet and dry bulb readings.

Refer to humidity tables at the end of the book.

Against the proper wet bulb reading, and in the proper difference column, read the pressure of the vapour actually present in the atmosphere. Find for what temperature this pressure is the saturation pressure. To do this, look up this pressure in the difference column 0. The wet bulb reading opposite this is the dew point, because when the air is saturated, wet and dry bulb readings are both equal (difference 0), and identical with the dew-point. The pressure in the 0 column against the dry bulb reading in the first column is the saturation pressure for that temperature. Actual pressure divided by this saturation pressure is the humidity.

Centigrade readings should be converted to Fahrenheit. The tables supply figures at intervals of 2°F. For intermediate temperatures use proportionate values. The % humidities can also be read off direct, below the vapour pressure, and are useful for checking calculations.

The determination of the various factors from the tables involves a bit of calculation. The record may be somewhat as follows :

Observations:

Dry bulb : 69°·1 F.
Wet bulb : 60°·5 F.

Difference : 8°·6 F.

Determination of actual vapour pressure:

Against	Read under 8°	Read under 10°	Calculate difference for 2°	Calculate difference for ·6	Calculate p for 8°·6
60°	·41	·38	·03	$\frac{6}{2} \times \cdot 03 = \cdot 01$	·41 − ·01 = ·40
62°	·45	·42	·03	·01	·45 − ·01 = ·44

Difference in p for 2° = ·04

$60^\circ\cdot5 = 60^\circ + 0^\circ\cdot5$; " " for $0^\circ\cdot5$ = ·01

∴ Pressure for $60^\circ\cdot5$ and difference $8^\circ\cdot6$ = ·40 + ·01

= **·41**

Vapour pressure.

Dew point : Pressure : ·41.

Look up for 41 or near figures, in 0 column

We read against ·39 52°

" " ·42 54°

∴ ·03 stands for 2°

41 = (39 + ·02); or ·02 " $\frac{2}{3} \times 2^\circ$ or $1^\circ\cdot3$

∴ Against ·41 we have $52^\circ + 1^\circ\cdot3 =$ **$\underline{53^\circ\cdot3}$**

Dew-point.

Saturation pressure for $69^\circ\ 1$.

Read against 68° in 0 column ·68

" " 70° " 0 column ·73

∴ 2° correspond to ·05

69·1 = 68 + 1 1 : ∴ $1^\circ 1$ " 1·1/2 × 05

or ·03

∴ Sat. pressure for $69^\circ\ 1$ equals 68 + 03

= **·71**

Saturation pressure.

% Humidity.

$$\text{This equals : } \frac{\text{actual pressure}}{\text{sat. pressure}} \times 100 = \frac{\cdot41}{\cdot71} \times 100$$

= **58**

Humidity

NOTE—The repeated application of proportional parts in the above calculation is admittedly tedious. The graphical method given below can be more easily applied, and gives accurate results with very little trouble.

Lay a scale of abscissæ (Fig 73) for differences 8° to 10° on squared paper. Above 8° and 10° plot the four pressures ·45, 42, ·41, ·38 for 62° and 60° as shown. AB and AB are the graphs for 62° and 60°. CC above $8^\circ\cdot6$ is the locus of pressures for wet bulb temperatures 60° to 62° for a difference of 8° 6. We have to locate on *CC* the point corresponding to 60° 5. To do so erect any where near *CC* a long vertical line *DD* to represent temperatures from 60° to 62°. *E* on this line corresponds to 60°·5. Join *DC*, *DC* and complete triangle *DCFCD*. Join FE to cut *CC* in *G*. *G* corresponds to 60°·5 and 8°·6. The pressure corresponding to *G* can be seen to be ·411.

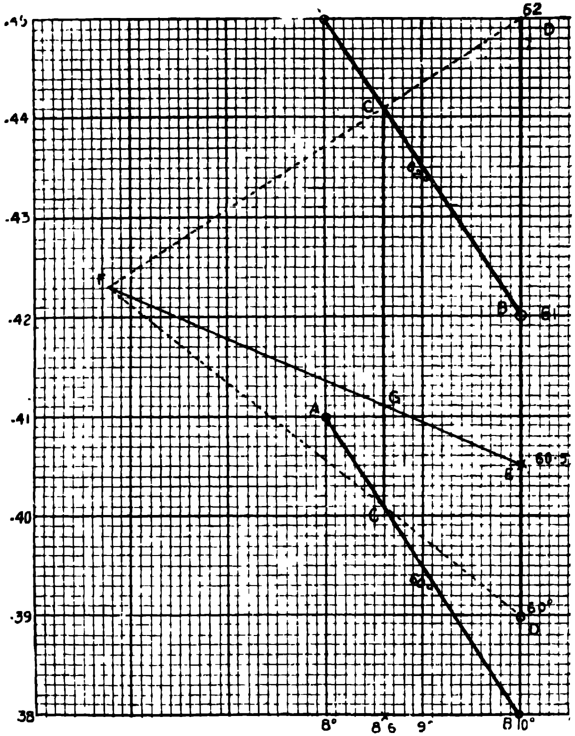


Fig. 73 —Vapour pressure for wet and dry bulb readings involving fractions of a degree, obtained graphically.

The graphical constructions for saturation pressure and dew-point, etc. are simpler, and are left as exercises to the student.

CHAPTER XIX

CALORIMETRY

SPECIFIC HEAT

The Method of Mixtures. The commonest method of determining specific heat, latent heat, etc. consists in mixing hot solids, liquids or steam etc., with water contained in a light cylindrical copper vessel called a **calorimeter**, and measuring the temperature rise of the water by means of a thermometer. With simple apparatus the accuracy of this method, called **the method of mixtures** is not high, but the experiments are ideal exercises in intelligent thinking and smart handling, and with care, results accurate within 3 to 5% can be obtained.

The Standard Thermal Equation. The student is already familiar with the standard thermal equation :

Total heat gained = Total heat lost.

The gain and loss of heat is obtained by *multiplying a rise or fall of temperature with a mass* (of water, solid or steam, etc.). Of the two factors, temperature and mass, one or the other is decidedly a *weak spot* in these experiments (see page 8), and should, therefore, be measured with the maximum accuracy possible with the apparatus provided. In some cases, both factors may be "weak spots," and all possible care has to be exercised to secure a decent result. The weak spot of each experiment is discussed under that experiment.

Calorimeters. The accuracy of calorimetric work can be appreciably increased by using *thin* calorimeters, provided with outer jackets, or sliding into felt-lined holes in wooden blocks. Fig. 75 shows such a calorimeter. Untidy flannel wrappings are unnecessary, and the wooden block can be handled without communicating heat to the calorimeter. The fall of temperature due to radiation in such a calorimeter is inappreciable.

Covers. Light metal covers (without holes) may be used to prevent evaporation, when calorimeters containing alcohol or warm water are being weighed, or otherwise standing idle previous to the calorimetric measurement proper. They should be *removed* a minute or so before the transfer, the thermometer introduced and read, and the substance tipped in. The form of cover with holes for the thermometer and the stirrer increases, considerably, the difficulty of manipulation at the time of introduction of the substances, and is undesirable. The utility of a cardboard or wooden cover may be, sometimes, actually *negative*, on account of the water absorbed from the wet stirrer, passing up and down the hole (*e. g.* latent heat of ice experiment). In the experiment on the latent heat of steam a cardboard cover is useful.

Stirrers are indispensable. A stirrer of plain copper wire (horizontal ring and vertical handle with a non-conducting wood or rubber tube tip), should form a part of, and be always weighed with, the calorimeter.

Thermometers. Half-degree thermometers are the least sensitive that can be employed for temperature measurements. Thermometers divided to fifths should, if possible, be employed to minimise the dangers of this **"weak spot"**.

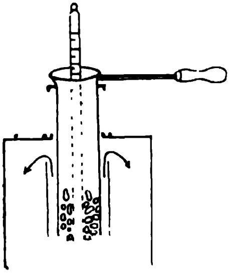


Fig. 74.—An easily pulled out heating tube for specific heat work.

Heaters. The usual brass or copper tube fitted into the top of an hypsometer is often pulled out with difficulty Even a careful and intelligent student will sometimes scald his hands, and meet at the critical moment of transfer, with a mishap *which is not entirely his own fault* A wooden handle to this tube is a necessary aid, and instead of being tightly fitted into a cork, it should have round the mouth a flange 3 mm. wide which rests lightly on the hypsometer hole, and authomatically seals this hole all round the tube (see Fig. 74).

The heater shown in Fig. 76 is very convenient. Small *roundish* lumps of the substance are placed in the double heating tube in the centre, and can be transferred to the calorimeter, by placing the latter under the side tube and pulling upwards the inner tube with the oblique end. There is practically no loss of heat during. transfer which is very quick. The extra tube with a side hole can be used for testing the upper fixed points of thermometers Copper heaters are the most economical, and tin heaters the most expensive *in the long run.*

Substances. The solids, used in calorimetric work by the method of mixtures, should be in *fragments*, as large solid balls and cylinders do not give up their heat quickly. For lead, lead short is best. Other metals should be stocked in the form of 3 to 4 mm. pieces cut from 3 mm. wire Long or flat chips do not roll out of the heater easily.

Determination of Specific Heat by the Method of Mixtures. The expression for specific heat in this method is

$$\frac{(\textit{Weight of water}+\textit{water-equivalent of calorimeter}) \times \textit{Rise of temperature of water}}{\textit{Weight of solid}\times\textit{fall of temperature of solid}}.$$

The masses concerned are *large*, say 50 to 100 grammes, and can be easily measured correct to at least 4 figures. The rise of temperature is between 5° and 10°C and the fall of the solid something like 70°. With a thermometer graduated in single degrees, the rise will be registered up to 0°·1C (as 6°·7 or 5°·3), or correct to two figures only. With a half-degree thermometer a zero or a 5 alone can appear as the third figure, so that the third figure may be wrong by 2 or 3. It is, therefore, essential that this rise be measured with great care, as it constitutes the **"weak spot"** of the experiment. An error of 0°·1C in the fall of 70° is not so important. But if different thermometers are used for the calorimeter and the heater, they *may* differ by more than 1°C, and the fall may be wrong by 1·5% or more. A single thermometer for both calorimeter and heater is however an inconvenient arrangement.

Before proceeding to experiments on specific heat the following exercise should be mastered.

EXERCISE 55*r*.—**Locating defects in apparatus and method in heat experiments.**

Before proceeding to experiments, answer in your note book the following questions:

(1). For a specific heat experiment, one student weighs carefully in half an hour 5·369 grammes of marble, and the other weighs out the calorimeter empty, and two-thirds filled with water, recording the weights 20·1 and 115·2 grammes, the smaller fractions being ignored. The rise of temperature measured with a ½ degree thermometer is 1°·15C. The result is extremely disappointing, being double the actual value. Which of the two students is to blame, and why ?

(2). The above experiment is repeated, the calorimeter thd water being weighed correct to ·1 gramme as before. The temperature rise in this case, measured with a degree thermometer, is 6°·3C, a larger amount of marble having been used. The result is better, though not quite satisfactory (10% too high). What has caused the improvement ? Carelessness in which part of the work is probably responsible for the 10% error ?

EXPERIMENT 55.—**To determine the specific heat of a solid (fragments.)**

Apparatus: Heater, calorimeter. stirrer, ½° thermometer, balance and weights, pipette, spring balance, beaker, tripod.

As a temperature rise of less than 5° is very unsatisfactory it is important to estimate the amount of solid required. For 100 grammes of water in the calorimeter, a rise of 6° to 7° can be obtained by roughly

100 gms copper, brass or zinc.
80 gms. iron.
300 gms. lead or mercury.
40 gms. aluminium or marble.

In the case of an unknown substance, a rough preliminary experiment, rapidly made, is essential.

Preliminary experiment: Deliver from a burette or pipette a measured volume of water so as to fill two-thirds of the calorimeter. Weight about an equal mass of solid quickly, correct to within 5 grammes, with an ordinary or spring balance. Place the solid in a glass or metal tube, and immerse

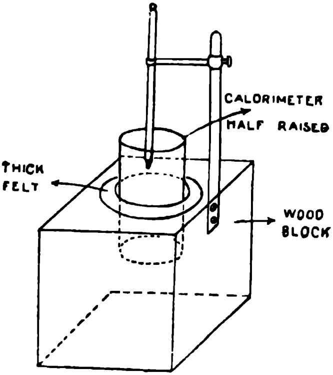


Fig. 75.—Calorimeter.

this tube in water boiling in a beaker, for 5 to 10 minutes. Read the thermometer in the calorimeter before and after introducing the heated solid. This gives a rough idea of the rise of temperature. If this is too small, or above 8° or 9°, a larger or smaller amount of the solid should be weighed out for the final experiment, so that the estimated rise of temperature is about 6 or 7 degrees.

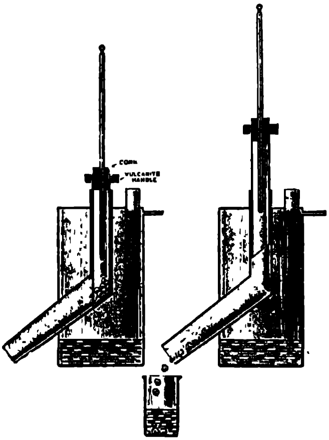


Fig. 76—A heater in which the "transfer" need cause no anxiety.

Final experiment : Make sure that there is water in the heater, which should be about half filled. Begin heating. Weigh the calorimeter and stirrer (i) empty (ii) two-thirds full of water (correct to ·1 gm. or 3 figures if half degree thermometer and to ·01 gm. if $\frac{1}{5}°$ thermometer is used. Why ?). Next weigh the estimated amount of solid to the same degree of accuracy as the calorimeter. Place *part of* the solid in the heater tube, insert the thermometer and add the rest of the solid. In this way, the delicate bulb can be surrounded by the solid without screwing or pushing into the hard fargments. In time steam will issue freely from the heater. Read the thermometer from time to time. The temperature will rise, quickly at first, but very slowly later on. In the plains, the temperature will rise to about 99°C. *When the temperature of the heated solid has been stationary for at least* 5 *minutes*, the calorimeter may be attended to.

The calorimeter shown in Fig. 75 is very satisfactory. If a plain single calorimeter is employed, it should be placed on a cork or wood block, and flannel (absolutely dry) wrapped round it. Record the stationary temperature of the hot solid. Next read the temperature of the water in the calorimeter as carefully as you can, stirring all the while. The thermometer bulb should not touch the bottom or the sides of the calorimeter. As soon as the thermometer has been satisfactorily read, the calorimeter may be placed under the side tube of the heater, the inner tube pulled up, and the solid transferred. *If some of the substance sticks or rolls away, do not wait or worry ; proceed straight to stirring and watching the thermometer.* Note the highest *steady* temperature. If it is suspected that some of the solid has not gone into the calorimeter, weigh the calorimeter again, and obtain the amount introduced by substraction.

Radiation correction : With fragments the steady temperature is very soon reached, and the radiation correction is inappreciable.

Observations :

Weight of calorimeter and stirrer = ... (1)
" " " + water = ... (2)
Weight of solid (fragments) = ... (3)
Temperature of hot solid = ... (4)
Initial " of calorimeter = .. (5)
Final " " " = ... (6)
Weight of water taken = (2) − (1)
= ... (7)
Water equivalent of calorimeter = (1) × ·094
= ... (8)

Total water-value of calorimeter + water = (7) + (8)
= .. (9)
Temperature rise of calorimeter = (6) − (5)
= ... (10)
Temperature fall of fragments = (4) − (6)
= ... (11)
Heat lost by fragments = *Heat gained by calorimeter and water.*

(3) × (11) × x (specific heat required) = (9) × (10).
Log (9) = ...
Log (10) = ...
...
Log (3) = ..
...
Log (11) = ...
Antilog ..
= ..

$$\therefore x = \frac{(9) \times (10)}{(3) \times (11)}$$
$$= \;\ldots\ldots$$

Questions :

Why is it necessary to see that the temperature shown by the thermometer in the heater has been constant for 5 minutes? When the thermometer records 90°·0C, why should you not transfer, assuming the fragments to be at 90°·0C also?

EXPERIMENT 56.—**To determine the specific heat of a liquid with a solid (fragments) of known specific heat.**

Apparatus : Same as for previous experiment. In addition, kerosene or methylated spirits.

As the specific heat of the solid is known, the amount reqiured per 100 gms. of water can be found from the rough data on page 210. But as we are substituing the unknown liquid water, the estimate has to be modified. For all ordinary oils, alchohols, benzene, glycerine, petroleum, etc. [sp. heat ·4 to ·6] take half the amount necessary for water. For aqueous solutions of salts, take the same amount as for water. A preliminary experiment is required only if the nature of the liquid supplied is ***absolutely*** unknown.

Substitute for water the given liquid, and proceed exactly as in the previous experiment. The precautions as to stirring, reading of thermometers, etc., are exactly the same. The calorimeter may be kept covered (on account of the volatile nature of alcohols etc.,) but the *cover may be discarded just before the transfer is made.*

Observations:

Weight of calorimeter and stirrer = ... (1)
" " + alcohol = ... (2)
Weight of fragments = ... (3)
Temp. of fragments = ... (4)
Initial temp. of calorimeter = ... (5)
Final " " " = ... (6)
Weight of alcohol = (2)—(1)
= ... (7)
Water-value of calorimeter = (1) × ·094
= ... (8)
Temperature rise " = (6) − (5)
= ... (9)
" fall of fragments = (4) − (6)
= ... (10)
Specific heat of " = ... (11)

Heat gained = *Heat lost.*

$$(7) \times x \times (9) + (8) \times (9) = (3) \times (11) \times (10)$$

$$(7) \times (9) \times x = (3) \times (11) \times (10) - (8) \times (9)$$

$$= \ldots\ldots\ldots\ldots\ldots \quad (12)$$

$$\therefore x = \frac{12}{(7) \times (9)}$$

Log (12) = ...
Log (7) = ...
...
Log (9) = ...
Antilog ...
= ...

$$= \underline{\ldots\ldots\ldots}$$

CHAPTER XIX—*contd.*

CALORIMETRY

LATENT HEAT

EXPERIMENT 57.—**To determine the latent heat of water.**

Apprartus : Calorimeter, thermometer, balance and weights, ice in a shallow dish, filter or blotting paper, stirrer with *open mesh* gauze round ring (Fig. 77).

IMPORTANT NOTE :—In determining the latent heat of water, the mass of ice used is not very large (about 15 grammes for 100 c.c. calorimeter), and is obtained by subtraction from weighings of the clorimeter before and after the introduction of the ice. These weighings should, therefore, be carried out correct to ·01 gramme, with as much care as is necessary in temperature measurements. In addition, as the melting of ice takes more time than the cooling of fragments of metal the radiation correction is of some importance. To apply this conveniently, use warm water 6° to 7° *above* the atmosphere, and add enough ice to cool it an equal amount *below* the atmosphere. The *loss* of heat in the first half of the interval equals approximately the *gain* during the second half. In very hot weather, the further heating of water may not be desirable on account of serious loss by evaporation. The radiation correction is best applied in this case, by noting the time (t) taken by the calorimeter after the addition of ice, to attain the steady temperature and determining, after this steady temperature is reached, the rise in *half* the time ($\frac{t}{2}$). This rise *subtracted* from the steady temperature recorded, gives the corrected minimum steady temperature.

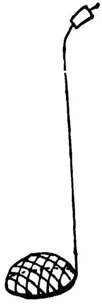


Fig. 77.—Wire-gauze stirrer for ice and water mixture.

As a rough preliminary exeperiment, add a moderate-sized lump of ice to the calorimeter filled two-thirds with water. No weighings are necessary. From the fall of temperature produced, estimate the amount of ice necessary for a fall of about 10° to 15°C. Weigh carefully the calorimteter (i) empty, (ii) one-half to two-thirds filled with warm water 6 to 7 degrees above room temperature In very wet weather, use hotter water and avoid a big fall below the atmosphere, which may result in the deposition of dew. *Dew is dangerous on account of the large latent heat* given up by the vapour when condensing.

Have ready one or two lumps of ice of suitable size. Stir the water and read the thermometer carefully. Next dry the lump or lumps (not more than two, as a large number of pieces are likely to carry too much *water* with them) with filter paper, wrap them in same, and raising the special stirrer so as to allow no drops from it to fall outside the calorimeter, introduce the dried lump or lumps, press under stirrer, and go on stirring *gently*, and watching the thermometer. Note the lowest *steady* temperature. Remove the thermometer, taking care to let the bulb rest for a moment against the calorimeter above the water surface, so that as little of it as possible is carried away by the thermometer. Finally weigh the calorimeter carefully.

Observations :

Weight of empty calorimeter and stirrer = .. (1)
" " calorimeter + water = ... (2)
Initial temperature of water = ... (3)
Final " " " = .. (4)
Final weight of calorimeter = ... (5)
Weight of water used = (2) − (1) = ... (6)
Water-value of calorimeter = (1) × ·094 = ... (7)
Total water-value of calorimeter = (6) + (7) = ... (8)
Weight of ice = (5)—(2) = ... (9)
Fall of temperature = (3)—(4) = ... (10)

Heat gained = *Heat lost*
(9) × [L + (4)] = (8) × (10)

$$\therefore L = \frac{(8) \times (10)}{(9)} - (4)$$

$$= \underline{\underline{\ldots\ldots\ldots}}$$

log (8) = ...
log (10) = ...
....
log (9) = ...
antilog ....
= ....

Latent Heat of Steam. In determining the latent heat of steam, the mass of steam used is only 4 or 5 grammes on account of the lage latent heat evolved, and is obtained as the *difference* of much larger weights, of the order of 100 grammes. A small percentage error in these will cause a large percentage error in the weight of steam (see page 9). The weighing should, therefore, be carried out with special care, and all precautions observed to prevent loss of water by evaporation and spilling, as also the addition of extraneous water carried over with the steam.

A good plan is to obtain the weight of the steam from two weighings of a light condenser, instead of from two weighings of the calorimeter. If such a condenser is used, a little spilling or splashing from the calorimeter is *not* fatal, as it does *not* affect the weight of the steam, which is condensed apart from the water in the calorimeter. Moreover, there is no spurting, as there is no bubbling of the steam inside the water.

EXERCISE 58*a*.—**Answer the following questions:**

(1). The latent heat of steam is determined without a condenser with the usual apparatus. The calorimeter is weighed before and after condensation of steam (97·2 and 100·4 grammes), the weight of steam being 3·2 grammes. The rise of temperature is carefully measured with a ½ degree thermometer as 18°·65C (from 11°·60 to 30°·25). The value found is 420. Where does the trouble most probably lie ?

(2). The student repeats the above exeperiment, weighing as before but arranging a larger rise of temperature 77°·10 (12°·00 to 89°·10). The weight of steam obtained by difference is now 6·3 grammes. The value turns out to be 815. What has happened ?

EXPERIMENT 58.—To determine the latent heat of steam.

Apparatus: Calorimeter with condenser as shown, boiler, steam-trap arrangement, thermometer, weight box.

[IMPORTANT NOTE.—The determination of the weight of steam is the " **weak spot** " of this experiment. The effects on this weight of evaporation and spurting from the calorimeter are avoided by weighing the steam separately in the condenser. Water carried over with the steam cannot be entirely kept out, and is a serious source of error. A large number of devices and 'traps' are described by various authors, and their use is attended with varying degrees of success. Part of the water is due to the con-, densation of steam in the delivery tube This tube should, therefore, be as short as possible, and should be thickly wrapped over with flannel. The other part is water mechanically carried over from the boiler by the steam as a spray. This can be caught in a "trap."

Experiments by the author's students with the boiler and entirely enclosed all-metal steam-trap arrangement shown in Fig. 78, seem to show that the apparatus possesses marked advantages over the external steam-trap. The trap is completely surrounded by steam and the total length of tube outside the boiler is only 4½". This is completely covered over with rubber tubing, 1 inch of which projects beyond the metal end. The tube is further insulated by flannel or cotton wool. The rubber tube is of a size into which the condenser *loosely* fits, so that connection and disconnection of condenser are easy. The metal tube of the condenser does not come into contact with the metal end of the delivery tube, but remains half an inch below it. Conduction of heat along the metal is thus prevented. On account of the delivery tube being short and heavily lagged, the condensation is small, and a drop at the end of the tube takes a long time to form. The fall of drops may be timed and the number received by the condenser, while connected with the delivery tube, calculated. This number is rarely more than one. If the weight of a drop is 05 gramme (20 to a c c.) a correction may be applied to the weight of steam obtained from weighings of the condenser. The boiler is supported on a metal bracket from a vertical board on a wooden base. The board acts also as a screen. The error rarely exceeds 3%.]

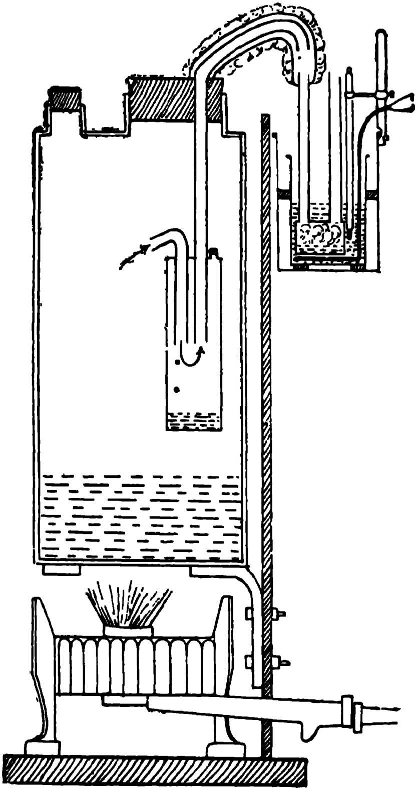


Fig. 78.—Latent heat of steam apparatus with compeletely enclosed steam-trap, short delivery tube, and separate steam condenser.

Start heating the boiler. Weigh the condenser alone.

Next weigh the calorimeter, stirrer and condenser together.

Repeat this weighing with enough cold water (if possible, 5° to 7° below the atmosphere) in the calorimeter to well cover the condenser. Arrange an unweighed cardboard or thin cork cover with 4 holes (2 condenser, 1 stirrer, 1 thermometer), and a half degree thermometer in the calorimeter. When steam is issuing freely from the delivery tube, read the temperature of the calorimeter carefully, and after removing any water drop sticking to the end of the delivery tube with blotting paper, slip one end of the condenser into the rubber end of the delivery tube. Steam will thus pass into the condenser. The non-conducting cover prevents steam from accidentally blowing into the calorimeter, or imparting heat to it. Keep stirring. When the temperature has risen 15° to 20°, disconnect the condenser, stir, and carefully read the highest *steady* temperature of the calorimeter. Remove condenser, dry exterior with filter paper carefully, and weigh accurately. Read the barometer correct to a millimetre. *The vernier need not be read.*

Observations :

Weight of condenser = ... (1)
Weight of calorimeter, stirrer and condenser = ... (2)
" " " + water = ... (3)
Initial temp. of calorimeter = ... (4)
Final temp. of " = ... (5)
Weight of condenser with condensed water = ... (6)
Barometric height = ... (7)
Temperature of steam corresponding to (7) = ... (8)
Weight of steam condensed = (6) − (1) = ... (9)
Total water-value of calorimeter and contents = (3) − (2) + (2) × ·094 = ... (10)
Temperature rise of calorimeter = (5) − (4) = ... (11)

Temperature fall of condensed water $= (8) - (5)$

$= \dots \quad (12)$

Heat lost = *Heat gained*

$$(9) \times L + (9) \times (12) = (10) \times (11)$$

$$\therefore\ L = \frac{(10) \times (11)}{(9)} - (12)$$

$\text{Log}\,(10) = \dots \qquad = \dots\dots$

$\text{Log}\,(11) = \dots$

etc., etc.

CHAPTER XX

RADIATION

NEWTON'S LAW OF COOLING. SPECIFIC HEATS BY THE METHOD OF COOLING. EMISSIVITY

Rate of cooling. If you remove the burner from under a beaker of boiling water, and watch the temperature, you may find that, in the first minute, the temperature falls through 10°C. In such a case, the fall in temperature in the second minute may be only 7° to 8°C. After the temperature has fallen to 40°C, the fall in a minute may be only one or two degrees. The rate of fall becomes smaller and smaller, and finally becomes zero, when the temperature of the water is the same as that of the surroundings.

It can be shown that the *fall of temperature per minute varies as the* **excess** *of the temperature of the radiating surface above the temperature of the surrounding air*, provided *that this excess is not large* (above 20°). As a fall of 1°C means in a given case a definite amount of heat, we can state the above law as follows:

Newton's Law of Cooling. *The heat radiated by a given surface per unit time is proportional to the* **excess of temperature** *of the radiating surface above the temperature of the surroundings.*

EXPERIMENT 59.—**To verify Newton's law of cooling.**

Apparatus: Calorimeter, stirrer, thermometer watch with seconds-hand, hot water.

Heat water to boiling in a beaker or metal vessel. Arrange the calorimeter with the thermometer and stirrer but without jacket etc., on a plain wooden block, so as to expose it to the air on all sides. Read the air or room temperature. Pour the hot water into the calorimeter, so as to fill two-thirds. When the temperature is about 70° to 80°, begin reading the temperature and noting the time. Record temperatures every minute, and later on, every two or three minutes. Stop when the temperature is a few degrees above that of the atmosphere. Plot a *smooth* curve with times as abscissæ and temperatures as ordinates. From the curve read out the temperatures at the beginning and end of a number of two or three-minute intervals. Calculate the mean temperature for each interval, as also the excess of this mean temperature above that of the surroundings. Divide the fall in each interval by the mean excess for that interval. The quotients will be found nearly constant for all cases where the mean excess is less than 20°.

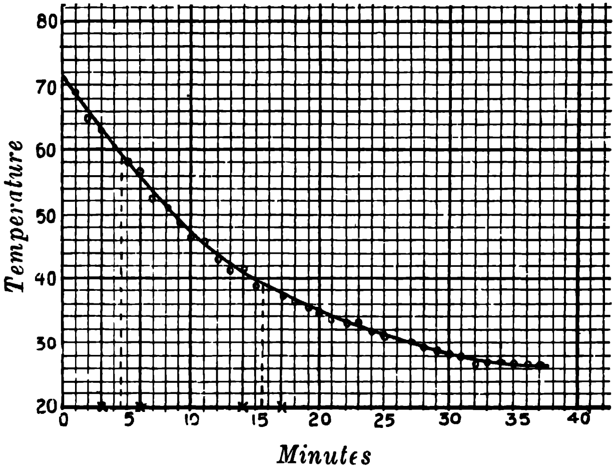


Fig. 79 —Newton's law of cooling verified by measurements on a cooling curve.

The data necessary for calculating the ratio $\frac{\text{fall in any interval}}{\text{mean excess for interval}}$ could have been found from the original readings of time and temperature. These readings, however, are apt to be considerably wrong, as *temperatures corresponding to particular instants in the case of a body which is rapidly cooling cannot be accurately determined.* The smooth graph, however, hides up the small individual errors of observation, and the readings from this curve being of the nature of *mean* values are more regular and consistent than the actual readings.

Observations :

Time in minutes																		
Temp																		

Curve drawn. Data from curve obtained and tabulated :
Temperature of air : ....

Equal intervals 3 mt. or 4 mt.	Temperature θ			Fall during interval $d\theta$	Mean excess above air e	$\frac{d\theta}{e}$
	Initial	Final	Mean			
0—3 mt.						
3—6 mt.						
6—9 mt.						
...						
...						

The values in the last column will be equal for all values of e less than 20°.

Determination of Specific Heat of Liquids by Cooling. If a calorimeter containing water takes t_1 minutes to cool from 50° to 48°, the heat lost during the t_1 minutes is $t_1 R$, where R is the heat radiated per minute. If the *same* calorimeter when containing hot kerosene takes t_2 minutes to cool from 50° to 48° (the same temperatures as before), the heat lost is $t_2 R$, because the heat radiated per minute by the calorimeter surface is still R, as the mean temperature and mean excess are the same in the two cases. But

$t_1 R$ = *Thermal capacity of calorimeter and contained water*) × (*fall of temperature*=2°), and

$t_2 R$ = (*Thermal capacity of calorimeter and contained kerosene*) × (*fall of temperature*=2°).

$$\therefore \frac{t_1}{t_2} = \frac{\textbf{Thermal capacity of calorimeter and water}}{\textbf{Thermal capacity of calorimeter and liquid}}.$$

This conclusion is independent of any law of cooling, as we deal in both cases with the *same initial and final temperatures*, as also with the *same mean excess of temperature.* All that is necessary is that the other conditions affecting radiation should be the same in the two cases. This can be secured by using the *same* calorimeter in succession for the two liquids, instead of using simultaneously two *apparently similar* calorimeters. The volumes of liquids used should also be nearly equal, so that the temperature distribution over the surface of the calorimeter may be the same in the two cases.

EXPERIMENT 60.—**To determine the specific heat of a liquid by the method of cooling.**

Apparatus : Calorimeter, stirrer, thermometer, vessels for heating water and the liquid, water bath.

Petroleum, alcohol, etc., should not be carried near or placed over flames. They should be heated by immersing tubes containing them in boiling water, or in beakers over a water bath.

1st method: Deliver with a pipette a measured quantity of hot water into the calorimeter, arranged as in the previous experiment on Newton's law of cooling. Note the time t_1 taken to cool from 70° to 40°.

Weigh the calorimeter with its contained water.

Repeat with the same volume of hot liquid, noting again the time of fall t_2 from 70° to 40°. Weigh the calorimeter and contents. Finally weigh the empty calorimeter and stirrer.

Enter observations thus:

Volume of liquids taken : ... c c.

Time t_1 in seconds for water to cool from 70° to 40° = ... (1)
,, t_2 for kerosene ,, ,, = ... (2)
Weight of calorimeter + water = ... (3)
,, ,, + kerosene = ... (4)
,, empty calorimeter = ... (5)
Weight of water = (3) - (5)
= ... (6)
,, ,, kerosene = (4) - (5)
= ... (7)

If R units of heat are radiated per second in each case,
Heat given up by cal. + water = $t_1R = 30^\circ \times$ *thermal capacity of cal. + water*
,, ,, kerosene = $t_2R = 30^\circ \times$ *thermal capacity of cal. + kerosene.*

$$\therefore \frac{t_1}{t_2} = \frac{\textit{thermal capacity of (cal. + water)}}{\textit{thermal capacity of (cal. + kerosene)}}$$

$$= \frac{(5) \times 094 + (6)}{(5) \times \cdot 094 + (7) \times x} = \ldots$$

Log... = ...
Log... = ...

$\therefore x =$ ......   Antilog ...
= ...

2nd method: Record temperatures every minute or every two minutes for both liquids, and draw cooling curves for both on the same sheet of squared paper. From the graphs obtain the times (t_1, t_2)

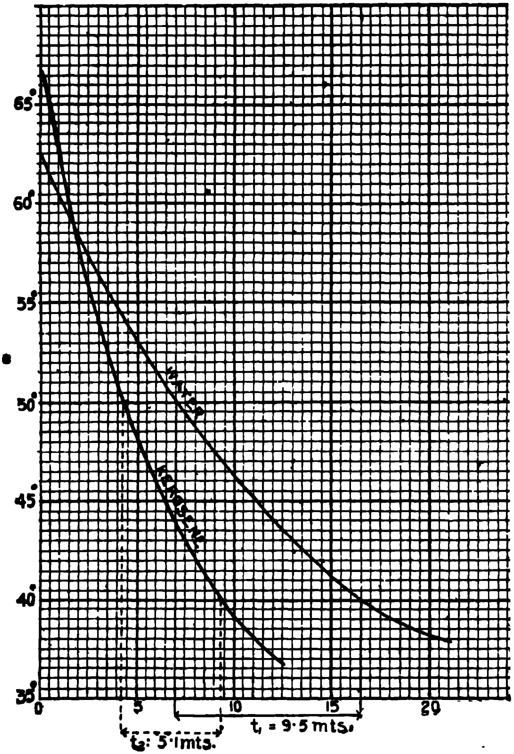


Fig. 80.—Determination of the specific heat of a liquid by cooling. Graphical method.

taken in each case to cool from 70° to 60°, 60° to 50°, 50° to 40°. The three ratios t_1/t_2 will be nearly equal. The mean value may be used for calculating the specific heat as in the 1st method. The advantage of this method is that it is not necessary to start observations at any definite temperature. If readings start with 70°C in one case, and the second liquid found below 70° on being poured into the calorimeter, readings can be taken and the specific heat calculated from the graphs all the same.

Observations :

Time in minutes	0	2	4	6	8	10	12	14	16	18	20
Temperature of water											
Temperature of liquid											

Curves drawn and t_1, t_2 for various intervals read from the graphs :

For fall 70° to 60° : $t_1 = \dots$ $t_2 = \dots$ $\frac{t_1}{t_2} = \dots\dots\dots$

" 60° to 50° : $t_1 = \dots$ $t_2 = \dots$ $\frac{t_1}{t_2} = \dots\dots\dots$

" 50° to 40° : $t_1 = \dots$ $t_2 = \dots$ $\frac{t_1}{t_2} = \dots\dots\dots$

Mean $\frac{t_1}{t_2} = \dots\dots\dots$

$$\frac{t_1}{t_2} = \frac{\text{weight of calorimeter} \times \cdot 094 + \text{weight of water}}{\text{weight of calorimeter} \times \cdot 094 + \text{weight of liquid} \times x}$$

$\therefore \quad x = \dots\dots\dots$

Comparison of Emissivities. Emissivities of dull-black and polished metallic surfaces can be compared by determining the time taken by each to radiate a definite amount of heat. Imagine a dull-black calorimeter containing 100 grammes of water at 80°. If the temperature falls to 40° in t_1 seconds, the rate of radiation is

$$\frac{(80-40) \times (\textit{water equivalent of calorimeter} + 100)}{t_1}$$

calories per second. If the same calorimeter, when polished, but with the same quantity of water (100 grammes), takes t_2 seconds to cool from 80° to 40°, the rate of radiation is

$$\frac{(80-40) \times (\textit{water equivalent of calorimeter} + 100)}{t_2}$$

calories per second. The emissivities are, therefore, inversely as the respective times of cooling, because other conditions affecting radiation, *e.g.*, initial and final temperatures and mean excess of temperature above surroundings, have been the same in the case of both the surfaces.

EXPERIMENT 61.—**To compare the emissivities of dull-black and polished copper surfaces.**

Apparatus: Calorimeter, dull-black paint, thermometer, watch, sandpaper coarse and fine.

Scratch a mark along the inside of the calorimeter. Polish the surface with coarse and fine sandpaper. Fill up to scratch with hot water (80°), and record times and temperatures as for a cooling curve (expt. 59), till the temperature is about 40°.

Next coat the outside of the calorimeter with a thin coating of lampblack rubbed up with *weak* shellac varnish. This will dry up quickly; if necessary, hasten the drying with hot air or by means of a flame. Arrange calorimeter as before. Fill up to the mark with hot water again, and repeat observations as taken for the polished surface. Draw cooling curves for both surfaces on the same sheet of squared paper, and obtain the times for cooling from 80° to 70° 70° to 60°, 60° to 50°, 50° to 40° from the two curves.

The emissivities are inversely as the times taken by the two surfaces for cooling through identical temperature intervals.

Record observations exactly as for determining specific heats by the method of cooling. Obtain values of $\frac{t_1}{t_2}$ from the curves, and calculate the mean. $\frac{e_1}{e_2}=\frac{t_2}{t_1}$, where e_1, e_2 are the emissivities of the surfaces, and t_1, t_2 the times taken, respectively, to cool through identical temperature intervals.

PART V

LIGHT

CHAPTER XXI

PHOTOMETRY

Illuminating Power or Luminosity. The total amount of luminous energy (L), radiated per second by a source of light in all directions, is called the **illuminating power** or **luminosity** of the source.

Intensity of Illumination of a Surface or Screen. The amount of luminous energy (I) incident on 1 square centimetre of a screen in a second is called its **intensity of illumination**.

The Inverse Square Law.—The illumination of a screen depends on the luminosity of the source and the distance of the screen from the source. At a distance r from the source, the luminous energy L radiated by it is distributed over the surface of a sphere of radius r. As this surface is $4\pi r^2$ the amount of energy received per cm.^{2} is $I=\frac{L}{4\pi r^2}$. The *intensity of illumination varies, therefore,* **inversely as the square of the distance** *from the source*

Comparison of Luminosity or Illuminating Power. Luminosities (L_1, L_2) of two light sources (S_1, S_2) are compared by adjusting the sources to distances where they illuminate a screen equally. Suppose S_1 and S_2 are at distances d_1 and d_2 from the same screen which both illuminate equally. Then $\frac{L_1}{4\pi d_1^2} = \frac{L_2}{4\pi d_2^2}$ or $\frac{L_1}{L_2} = \frac{d_1^2}{d_2^2}$. In other words, *luminosities are as squares of respective distances from which the sources illuminate a screen equally.*

Photometers. Photometers are used to determine if two sources S_1 and S_2 are illuminating a screen equally. This determination is made in a number of ways. If it can be arranged that two neighbouring areas of a screen are illuminated, one by S_1 and the other by S_2, the eye can judge if the two areas are equally illuminated. This can be done by comparing adjacent patches of light thrown by the two sources through a hole in front of the screen. In the **Jolly paraffin-block photometer,** two small adjacent blocks of paraffin, with a tinfoil partition between the two, are illuminated, one by one source, and the other by the other, and the distances adjusted so that the blocks are equally bright. In the **Rumford photometer** the two sources jointly illuminate the same screen. Each source, however, casts the shadow of a rod held in front of the screen. If the shadows are equally dark, the illumination contributed by S_1 (missing from the shadow cast by S_1) must equal the contribution of S_2 (missing from the shadow cast by S_2). In the **Bunsen's or grease spot photometer** a paper disc with a grease spot in the middle is employed. The appearance of the grease spot indicates whether the opposite sides of the disc are equally illuminated or otherwise. Seen from the *less* illuminated side the spot appears *brighter* than the rest of the paper. From the other side it appears dark on a bright ground. If the opposite sides are equally illuminated, the spot becomes very nearly indistinguishable from the rest of the paper. Hold up the Bunsen disc between yourself and the window. The spot appears *bright* on a dark ground, as you are looking at it from the less illuminated side. If you look at the same disc with your back towards the window, the spot will appear dark on a light ground. The paraffin-block photometer is the best of the simple photometers mentioned. The Bunsen's disc is very inconvenient to use.

Differently coloured lights cannot be compared by means of the above photometers. With such lights, the shadows of the rod in the Rumford photometer are differently coloured. The Bunsen disc also fails to disappear. For laboratory experiments, the various light sources employed are electric lamps, gas flames and candles. Candles, oil flames and small gas flames can be compared with one another or with the carbon filament lamp, as all of these emit yellowish light. Metal filament lamps can be compared with one another or with incandescent gas flames from special burners.

EXPERIMENT 62*a*.—**To verify the inverse square law of distance for illumination of a screen.**

Apparatus: Two holders, for one or more candles, on uprights sliding on a graduated wooden bench at least 6 ft. long, paraffin-block photometer.

A dark room with walls and tables painted a dead black is necessary.

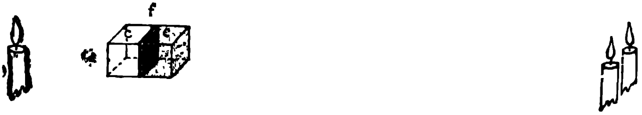


Fig. 81.—The paraffin-block photometer.

The photometer consists of two half-inch cubes *cc* of clear paraffin wax, with opaque tinfoil between the adjacent faces. The cubes are in the same line as the sources (Fig. 81). They are best mounted in a small blackened box with the front and sides removed.

Arrange a single lighted candle, at about 50 cm.

from the photometer on one side. Place the second holder, with two lighted candles, on the opposite side. Adjust the distance till the blocks appear equally bright. Record distances correct to half a centimetre or so A millimetre or two does not matter. Repeat two or three times with both sources at different distances.

Next arrange three candles on one side, and one on the other. Adjust three times at different distances, and record as before.

Repeat with four candles, and one candle, on opposite sides.

If the inverse square law is true, the intensities of illumination of the two blocks should, respectively, be proportional to $\frac{L_1}{d_1^2}$ and $\frac{L_2}{d_2^2}$. But as the blocks are equally illuminated $\frac{L_1}{d_1^2} = \frac{L_2}{d_2^2}$ or $\frac{L_1}{L_2}$ should in every case equal $\frac{d_1^2}{d_2^2}$. In this experiment $\frac{L_1}{L_2}$ is 2, 3 and 4 respectively, in the three cases.

Record as follows:

S_1	S_2	$\frac{L_1}{L_2}$	d_1	d_2	$\left(\frac{d_1}{d_2}\right)^2$	Mean $\left(\frac{d_1}{d_2}\right)^2$
2 candles	1 candle	2	.. ..	.. ..	. ...	....
3 ,,	1 .,	3	....	. . ..	. ...	......
,,	1	4	... .	....	. . ..	...

EXPERIMENT 62.—**To compare the luminosities of two light sources.**

Apparatus : Rumford photometer, paraffin-block photometer, or Bunsen's photometer (preferably between inclined mirrors for simultaneous observation of both sides), Osram lamps 60, 30 and 20 watts, carbon filament lamp, candle.

The above photometers can be used for comparing the illuminating powers of sources that emit light of the *same* colour. A large metal filament electric lamp can be easily compared with a similar lamp of less candle power, as both emit the same kind of light. A carbon filament lamp or a smoky gas flame may be compared with a candle, as both give yellowish light.

The amount of electric power, or the watts consumed by an electric lamp is generally stamped on the bulb or the brass cap. If the candle power is measured by a photometer, the number of watts consumed per candle can be calculated. We shall do this for a carbon lamp. The power consumed by a metal filament lamp is generally taken as 1·2 watts per candle.

Perform the following two experiments with any one of the photometers provided : (1) Comparison of a large 50 or 60 watt metal filament lamp (Osram) with a similar 20 or 30 watt lamp. (2) Comparison of a 30—50 watt carbon lamp with a single candle.

Rumford Photometer.

Place the two sources (the stronger source at the greater distance) in front of the photometer, and adjust their distances till the two shadows of the rod are equally deep. The shadows should be close to one another. When the adjustment is correct, measure the distances d_1 and d_2 of the sources from the ***screen*** (not the rod) Repeat twice, using different distances each time. Record as shown below.

Paraffin-block Photometer.

Place the two lamps one or two metres apart and place the photometer between them. Adjust the position of the photometer till the blocks appear equally bright. Measure d_1 and d_2. Repeat twice, using different distances each time.

Bunsen's Photometer.

Place the grease spot between the two sources, and adjust their distances, so that the grease spot either disappears or appears to be of the same shade, when viewed from either side. If the grease spot is mounted between two inclined mirrors, the two images of the two sides can be seen at the same time, and the adjustment can be made with great convenience. Without mirrors, the best plan is to adjust the position of the grease spot till it disappears when viewed from one side. Read this position or mark it by means of a chalk mark on the table. Next look from the opposite side and adjust till the spot again disappears. Read or mark this position as before. Calculate or mark the mean position. The distances of the sources from this mean position are d_1 and d_2. Repeat twice using different distances.

Records as follows :—

S_1	S_2	d_1	d_2	$\left(\frac{d_1}{d_2}\right)^2 = \frac{L_1}{L_2}$	C.P.	Watts per C. P.
Osram 60 W	Osram 30 W	... ...	... ...	... Mean:		The candle power of a 30 watt lamp is approximately 25, so that the watts per candle equal 1·2.
Carbon lamp	1 candle	... ...	... ...	... Mean:	$\left(\frac{d_1}{d_2}\right)$	$\frac{\text{Watts as } \textit{marked}}{\textit{Calculated} \text{ C. P.}} = 3 \text{ to } 5.$

Q. Which is more economical to use, the carbon lamp or the metal filament lamp?

EXERCISE.—**Measure the candle power of an electric lamp with (1) filament vertical (2) filament horizontal in plane of bench, (3) filament at 45° to horizontal in plane of bench**, and state in which plane an electric lamp gives out most light.

CHAPTER XXII

REFLECTION FROM A PLANE MIRROR

Angle of Incidence equals Angle of Reflection. When light is reflected from a polished surface, plane or curved, *the angles made by the incident and the reflected rays, with the normal to the surface at the point of incidence, are equal.* As a consequence of this,

(1) *All rays diverging from a point P appear, after reflection from a plane mirror, to be diverging from a single point P′ behind the mirror.*

(2) *The point P′ (called the virtual image of P) is on the normal to the mirror through P, and is as much behind the mirror as P is in front of it.*

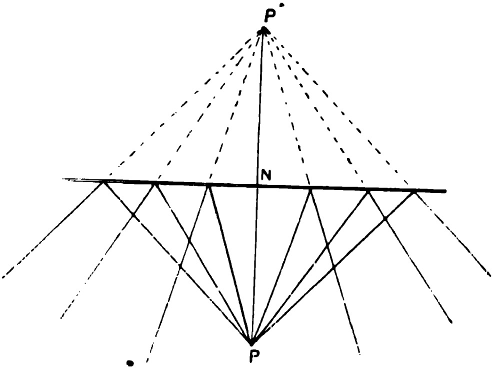


Fig. 82.—Reflection from a plane mirror.

Rules about the Use of Pins. Pins fixed *vertically* on paper on a drawing board are often used to represent points on incident and reflected or refracted rays. *Thin* one inch pins should generally be employed. When fixing pins adjust so that the *points* (not heads) are in a line as required *because it is the points that mark the ray on the paper.* Two pins representing a ray should be at least 4 inches or 10 cm. apart If nearer, the thickness (supposed negligible) of one pin may subtend an appreciable angle at the other. *Before* pins are removed, the pin-pricks should be surrounded by small circles, and suitably marked or numbered to avoid confusion. Lines should be drawn with a *sharply pointed pencil* accurately through the pin-pricks.

The Method of Parallax. Let three students A,B,C, stand in a line. To A, the *farther* student C appears covered by B. If A (or his eye) moves to the *right* to A_R the *farther* student C becomes visible to A to the *right* of B. If A (or his eye) moves to the *left* to A_L the *farther* student C now appears on the *left* of B.

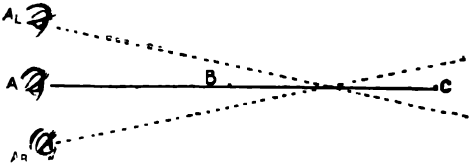


Fig 83.—The principle of the parallax method.

CONCLUSION: *Of two objects in a line with the eye, the one that appears to the* **right** *of the other on the eye moving to the* **right**, *or the* **left** *of the other on the eye moving to the* **left**, *is the* **farther** *of the two.*

Evidently if B and C are at the same point, or actually **coincident**, *they will appear coincident* **in all positions** *of the eye.*

In light experiments, pins are often fixed by trial so as to appear coincident with images in all positions of the eye. When so adjusted in the no-parallax position, the pin-prick marks the position of the image. Virtual images that cannot be received on a screen are often located in this way. The method is applicable also to real images.

EXPERIMENT 63.—**To verify the equality of the angles of incidence and reflection, and to measure the distance of the virtual image formed by a plane mirror.**

Apparatus: Paper, pins, drawing board, protractor, plane mirror 1 inch × 6 inches.

Metallic mirrors with the *front* surface polished are very accurate. They, however, tarnish soon. Use strips from *good* quality *thin* silvered plate glass. Mirrors giving distorted images are useless. Mirrors should be supported *accurately vertical* with one end fitted into a slit (exactly perpendicular) in a block of wood 2″×1″×1″.

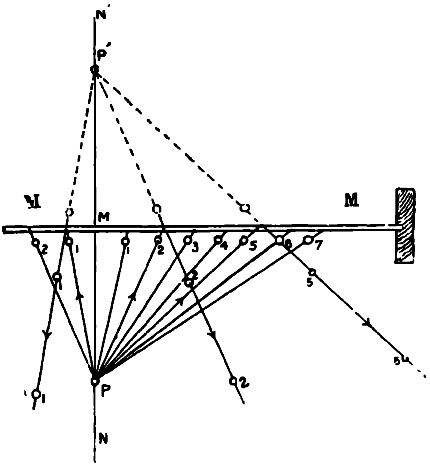


Fig. 84.—Verification of the equality of the angles of incidence and reflection.

Draw MMM and NMN′ (Fig. 84) perpendicular to one another.

On the normal NMN′ fix a point P, 10 to 15 cm. from the mirror. Draw from P five or six equally spaced rays, by drawing lines from P to equidistant points on MM.

Draw two lines in the same way on the other side of the normal. Fix a pin at P and one pin on each of the lines drawn. For neatness, put these pins in a line parallel to the mirror and about 1 cm. in front of it. Number these pins 1,2,3...as shown.

Place the mirror with the *silvered back* on MM. The reflected rays corresponding to the incident rays are to be fixed each by a pair of pins. Fix a pin in a line with the images of P and 1. Similarly fix a second pin. Number them 1, 1. Similarly fix two pins 2, 2 in a line with the images of P and 2. Fix in this way all the reflected rays. Draw lines through similarly numbered pin-pricks. P111, P222, P333 are the paths of the various rays. The rays will meet on MM. Produce the reflected rays behind the mirror. They should intersect in one point P′ on the normal. PM should equal P′M. Draw normals to the mirror at the various points of incidence, and measure *i* and *r* for each ray. These should be equal in every case.

Record thus :—

i	*r*	*i*	*r*
...	...	...	...
...	...	...	...
...	...	. .	...

PM = ... cm.

P′M = ... cm.

Distance of P′ from normal NN′ = ... cm. (*e. g.* ·1 cm.)

Total Internal Reflection. It can be shown experimentally that *the angles of incidence and reflection are equal even in case of* **total internal reflection.** See Expt. 68 part *b*.

EXPERIMENT 63*a*.—**To prove that, for a fixed incident ray, if the mirror rotates through an angle θ, the reflected ray rotates through an angle 2θ.**

Apparatus : Same as in the previous experiment.

Draw a line MM (Fig. 86) as usual, as also a number of equally spaced lines M_1M_1, M_2M_2 intersecting at O.

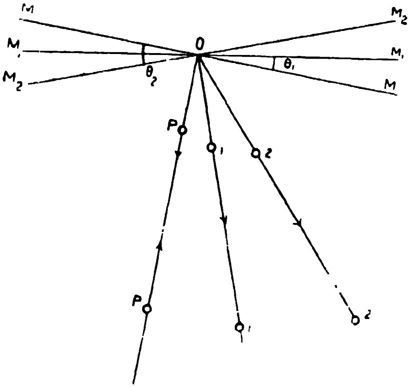


Fig. 86.—The reflected ray turns through twice the angle through which the mirror rotates.

These lines mark the various positions of the mirror Place the silvered surface of the mirror on MM and fix two pins P, P so that they are in a line with their own images. PP, therefore, represents a normal incident ray reflected along its own path. Shift silvered surface to M_1M_1. Fix pins 11 in a line with images of P,P. Shift mirror to M_2M_2, and fix pins 22 in a line with images of P,P. Do this for 4 positions in all. Draw the reflected rays. Measure angles $\theta_1, \theta_2, \theta_3$ (MOM_1, MOM_2, MOM_3). Also measure the angles turned through by the reflected rays (PO1, PO2, PO3, etc.),

Tabulate as follows :

Angle through which mirror rotates θ	Angle through which reflected ray rotates	29
..	...	...
...	...	...
...	...	...
...	...	...

The numbers in the last two columns in the same row will be equal.

EXPERIMENT 64.—**To locate by parallax method the virtual image formed by a plane mirror.**

Apparatus: Same as in previous experiment, with the addition of some pins longer than usual, $1\frac{1}{4}''$ to $1\frac{1}{2}''$.

Fix the mirror with the silvered surface accurately on a line MM′. Fix 4 ordinary pins P_1, P_2, P_3, P_4 at various distances in front of the mirror (Fig. 85).

Place the eye about 25 cm. behind P_1. Looking in a line with P_1 and its image fix a *tall* pin P_1' behind the mirror so that the top of P_1', seen above the mirror, coincides with the image of P_1 in all positions of the eye.

Do not place eye close to P_1. On moving your eye to the *right*, if the top is to the *right* of the image, the tall pin is fixed *too far*, and *vice versa*. Locate the images of P_2, P_3, P_4 by tall pins P_2', P_3', P_4', fixed in no-parallax positions. Draw lines $P_1 P_1'$, $P_2 P_2'$, etc. Measure distances of P_1, P_1', P_2, P_2' etc., from the mirror, recording these on the respective lines. Measure angles marked (θ_1, θ_2......) to see if they are right angles.

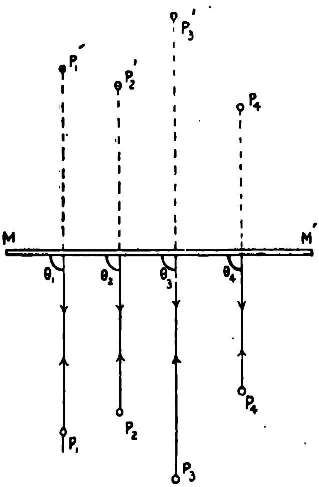


Fig. 85.—To verify that the image and object are equally distant from a plane mirror.

Record observations thus:

Distance of object P	Distance of image P′	Angle between PP′ and MM′.
...	...	...
...	...	...
...	...	...
...	...	...

CHAPTER XXIII

REFLECTION FROM A SPHERICAL MIRROR

Spherical Mirrors. A portion of a spherical reflecting surface is called a spherical mirror. The reflecting surface should form only a *small* portion of the sphere, in other words the " *aperture* " should be small. Mirrors of *large* aperture give *indistinct* and *distorted* images, and the usual formulæ do not apply to them. If the side towards the centre is the reflecting surface, the mirror is concave; if the outer side or the side away from the centre is the reflecting side, the mirror is convex.

Images by Spherical Mirrors. Light rays diverging from a point P may, after reflection from a **concave spherical mirror** of *small* aperture

(1) come to a real point focus P′ (called **real image**) in front of the mirror (Fig. 87*a*), or

(2) appear to diverge from a virtual point focus P′ **(virtual image)** behind the mirror (Fig. 87*b*).

(3) In case of a **convex mirror,** the focus P′ is *always* **virtual,** and behind the mirror (Fig. 87*c*).

The Formula for Mirrors. Rays from a very distant point (parallel rays) come to a focus at a point midway between the mirror and the centre of curvature or at a distance **(= focal length, f)** equal to half the radius ($\frac{r}{2} = f$). For objects at other distances, the formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ is used, where u and v are, respectively, distances from mirror of object and image. This formula applies to concave and convex mirrors for both real and virtual images, provided we bear in

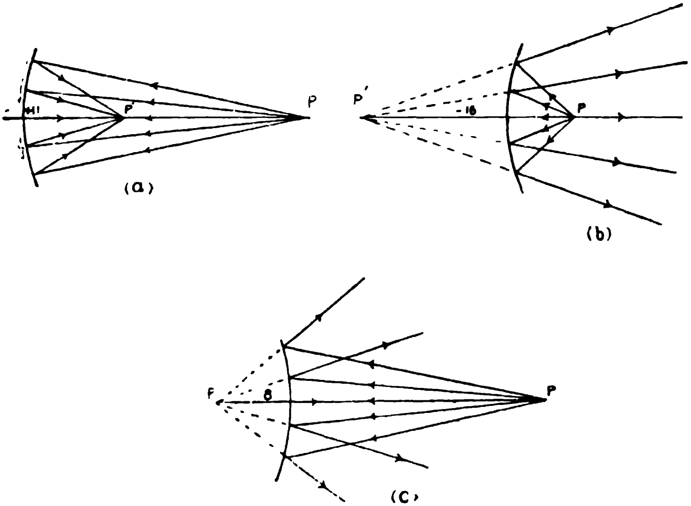


Fig. 87.—Real and virtual images formed by concave and convex mirrors.

mind the following

Rule of Signs. *Distance of image or principal focus is* **positive** *if such focus or image is* **in front** *of the mirror or on the same side as the object, and* **negative** when **behind** *the mirror or on the side of the mirror away from the object.* In Fig. 87 the distances from mirror of the image P′ in *a*, *b*, *c* are +11 cm.,—16 cm. and —8 cm. (wrongly marked 8 in figure), respectively. In mirrors, v for real images is always +. The focal length and radius of curvature are + for concave and − for convex mirrors; u is always + for both concave and convex mirrrors.

The Optical Bench. Measurements of u, v, f, etc., can be carried out conveniently on an optical bench. The bench consists of a long (2 metres) wooden base 4 or 5 inches broad and 1 inch thick with a raised edge along one side, against which the bases of the various uprights rest. Metre sticks are screwed on to the top of this raised edge. The mirrors, candles, needles, etc., are supported in suitable holders mounted on rectangular bases provided with sharp lines or arrowheads for reading the position of the upright on the centimetre scale fixed to the bench.

Tall uprights with holders sliding up and down are often unreliable. The sliding rods are apt to stick, or to move too loose. The form of bench described by Watson, modified as given below, is inexpensive, rigid, and accurate, and meets all Intermediate needs, if all mirrors used are of the same diameter (3 inches diameter is a standard size) The modification consists (*i*) in making the uprights somewhat higher, for convenience in parallax experiments and (*ii*) in providing the needle holders and uprights with a small up and down and cross motion. Semicircular screens, and semicircular wire-gauze pieces are also necessary, as they allow passage of light through the other half of the circle. Fig. 88 shows the bench adjusted for receiving diminished and enlarged images of the full and half wire-gauzes on the half and full screens. If more elaborate benches are employed, the uprights should not be very tall, should have a cross motion, and in any case be **perfectly rigid and stable.** An over-all height of 8 inches with a vertical play of 2 to 3 inches is suitable.

The Candle and Screen Method and the Parallax Method. There are two distinct methods of experimenting with mirrors and lenses In one, the image of an illuminated object (generally wire-gauze) is caught on a screen. The distance of the gauze from the mirror is u and that of the screen from the mirror v. This method is rapid, convenient and accurate, but requires at least a partly darkened room, as magnified images are often not visible in a well lighted laboratory. In the other method, a metal needle is the object, and the image is located by adjusting another needle, so that there is no parallax between it and the image of the first needle. The method requires very little apparatus, but is slow, and tiring, especially because *it is necessary to use the mean of a number of readings for the position of an image.*

Possible Accuracy. Reading small fractions of a millimetre on simple benches is generally meaningless. *If the uprights are very rigid*, and the indices on the bases *sharp lines*, such fractions may be read. Index correction should always be applied in such cases, and the thickness of lens or mirror allowed for. In any case, *the final result cannot be expected to be correct up to 1 mm.* It should always be expressed correct to a millimetre, ·5 mm. or above being considered 1 mm., and smaller fractions ignored. **Unsteady uprights should never be used.**

EXPERIMENT 65.—**To determine the focal length of a concave mirror by means of real images on a screen.**

Apparatus: Optical bench, wire-gauze, screen, candle, screen with central cross-wires, index correction rod.

The experiment consists in measuring u and v for a number of images in different positions and applying the formula $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$. The object is *wire-gauze*, the *candle* behind *only an illuminant.* The light from the candle should pass through gauze to mirror, so that *candle and mirror are always on opposite side of the gauze.* The image is focussed when the *wire-gauze* is distinctly imaged; *the candle flame should not be looked for in the image*, u is the distance from mirror to *gauze*, not candle.

Obtain a rough idea of the focal length by casting on a screen images of distant trees, etc. Without this knowledge a student may often waste much time looking for an image in impossible positions. The gauze or screen should *never* be placed at a distance from the mirror *less* than the focal length roughly determined as above.

Index Correction. The *real distance* between a mirror and a screen is often somewhat more or less than the *apparent distance* as read from the marks on the uprights.

The difficulty arises on account of the fact that it is impossible to fit mirrors, screens etc. so that they are, in every case, vertically above these marks or indices. An **index correction** may, therefore, be necessary. To determine this correction take a rod of known

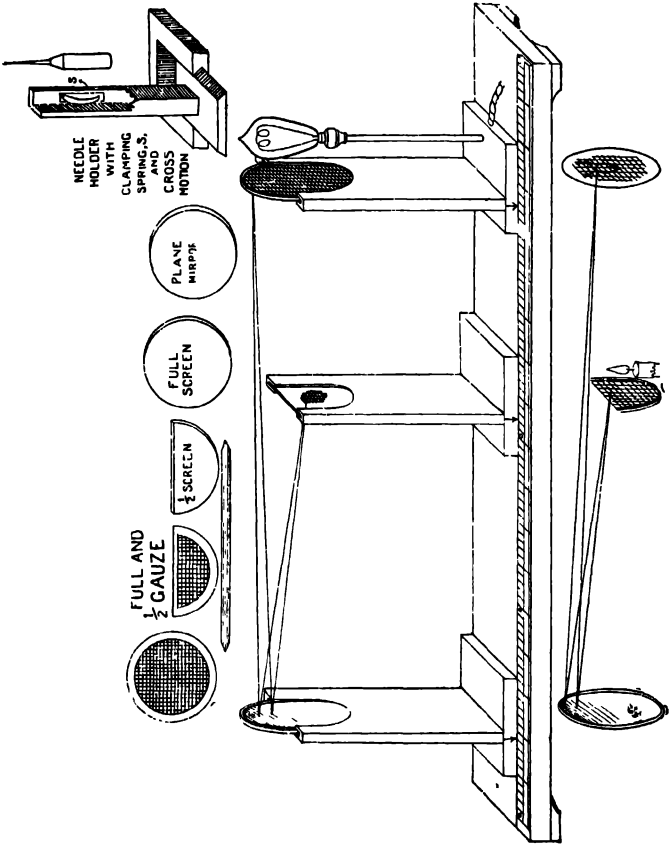


Fig. 88.—A simple, rigid and inexpensive optical bench. Fittings are interchangeable, if standardized. 3″ mirrors and 2″ lenses in semicircular grooves of corresponding diameter make automatic centring in a horizontal line possible. The special holder shown provides the parallax needle with a small up and down motion, as also with the indispensable cross motion. Observe how semicircular screens, etc., allow passage of light above them.

length, say, 20·00 cm. Holding the rod parallel to the bench (Fig. 89) push the uprights concerned so that the mirror touches *one* and the screen (or needle) the other end of this rod. The *real distance* is now 20·00 cm. Read the uprights in this position, and get the *apparent* distance by subtraction. If this is 19·70 cm., the index correction is evidently positive and equal to 20·00—19·70 or +·30 cm This correction applies to this pair of uprights only, and for the particular fittings carried by these uprights. If the uprights or the mirror and screen, etc are changed, the index correction must be redetermined.

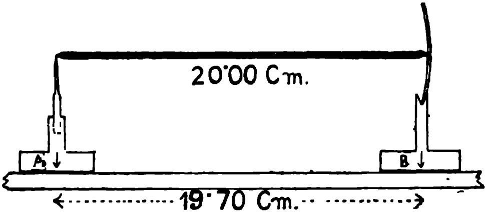


Fig. 89 —Determination of index correction: + 30 cm

If the index correction rod is carried on an upright provided with an index, the other uprights need not have any indices All distances can be determined directly by means of the mounted rod alone For instance, the real distance between a mirror and a screen can be found by placing the index rod between them so that one end touches the mirror and reading the index of the rod upright Another reading is taken when *the other end* of the rod touches the screen. The difference between the readings added to the length of the rod gives the real distance from mirror to screen. If the screen and mirror are so close to one another that the rod cannot be placed between them, the distance can be found by making the *same* end of the rod touch the mirror and the screen in succession. The difference between the readings gives the real distance from mirror to screen With this method, the determination of separate index corrections for various pairs of uprights is unnecessary.

Next proceed to determine u and v as follows :

(*a*) $u=v=r=2f$. Use screen with central cross-wires Place it at about twice the roughly determined focal length, and adjust till image of cross-wires is close to the wires and quite distinct. Read indices. Repeat once. Determine and add index correction.

(*b*) Place gauze and mirror on opposite ends of bench and arrange the semicircular screen (Fig. 88) near the roughly known principal focus. Focus the small image; measure u and v. Decrease u to nearly half the first value and measure u and v. Next decrease u by a smaller amount, and measure u and v. Proceed to cases where $v > u$. Use semicircular gauze and full screen for the magnified image. Determine index correction for uprights : gauze to mirror, and screen to mirror (Fig. 89).

Observations :

Images of distant trees at about : ... cm
Index correction mirror to gauze = 20·00 − (difference between mirror and gauze readings)
= ...
" " " to screen = 20·00 − (difference between mirror and screen readings).
= ...

To each correction *add algebraically* the thickness of mirror.

Observed u	Observed v	Corrected u	Corrected v	$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$	f
1 ... − ...	... − ...	... − ...	... − ...	... + ...	
=	=	=	=	=	
2				Use tables of reciprocals in manner explained. (Appendix)	
3					
4					

$$\text{Mean value of } f = \frac{\ldots + \ldots + \ldots + \ldots}{\ldots\ldots} = \ldots \text{ cm.}$$

Optician's Lens Tester. Check value by means of an optician's lens gauge for measuring the power of a lens (see chap. 26). If the reading is D, the radius $= \frac{-52}{D}$.

EXPERIMENT 66.—**To determine the focal length of a concave mirror by parallax method.**

Apparatus : Optical bench with mirror and two needles on uprights, index correction rod.

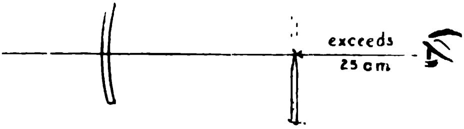


Fig. 90.—Illustrates the position, $u = v = r = 2f$.

In this experiment measurements similar to those in the previous experiment are necessary, one needle serving as object, and the other locating the image formed, by the method of parallax.

Determine the focal length roughly with the help of distant objects.

(1) $u = v = r = 2f$. Use one needle only. Adjust distance till there is no parallax between needle and its real inverted image. Adjust height of needle so that the *tips* of needle and image touch. Read index. Keep mirror fixed. Move needle and readjust. *The eye should be always about 25 cm. behind* the *needle which is locating the image. It should not be placed close to it* (Fig. 90). Use the mean of two or three readings; f equals half the corrected distance.

(2) Obtain a number of values of u and v by means of two needles. Adjust till the **image** of the needle nearest your eye coincides with the needle near the mirror. When so adjusted, each needle will be found coincident with the image of the other, so that the distance of either may be called u or v (Fig. 91).

Obtain index corrections, and record observations

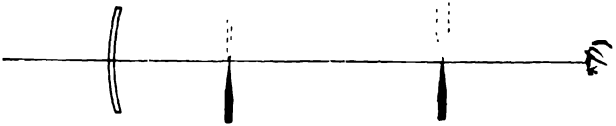


Fig 91.—Two needles in conjugate positions.

exactly as for the previous experiment.

Size of Object and Image. For mirrors (and lenses).

$$\frac{\textbf{Size of Image}}{\textbf{Size of Object}} \text{ or } \frac{i}{o} = \frac{u}{v}.$$

EXPERIMENT 66*a*.—**To verify the relation $\frac{i}{o} = \frac{v}{u}$ for real images on a screen.**

Apparatus: Optical bench, screen, gas flame or electric lamp, scale on glass, wire-gauze or screen with cross-wires.

For object use a scale ruled on glass illuminated by a *broad* gas flame. With a candle flame, the image of only a few millimetres can be observed.

An electric lamp can be used. Interpose *very thin* paper between lamp and scale, as otherwise the filaments may be focussed along with the scale. This can be avoided also if the lamp is placed at some distance (5 cm. or more) from the scale instead of close to it. A milkglass electric bulb or a *broad* flame from a fish-tail burner is very effective. If a glass scale is not available, rule with black ink two lines 1 cm. apart on a slip of glass, or use a *thin* metal screen with a hole 1 cm. square and cross-wires. In the absence of above, employ ordinary wire gauze, using the distance covered by, say, ten meshes, as the object. This distance may be measured with the help of dividers Focus the image Measure its size on a paper mm. scale pasted on the screen, or by means of dividers and scale, *using as large a portion of the image as can be clearly seen.* Tabulate u, v, i, o, $\frac{v}{u}$ and $\frac{i}{o}$ for a number of images magnified as well as diminished. Index corrections, if small (1 mm. or less), can be ignored, as verification of the formula beyond two significant digits is not possible with simple apparatus.

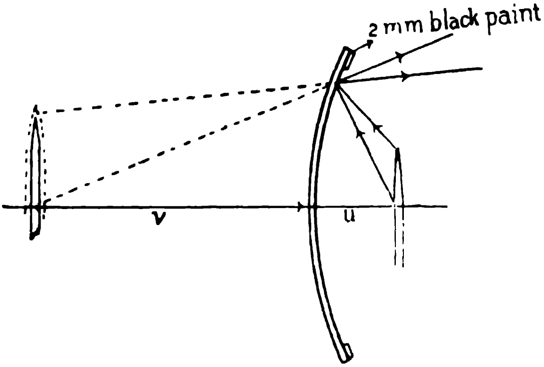


Fig. 92. Locating a virtual image by parallax

Virtual Images. The student has already learnt how to locate, by the method of parallax, the virtual image behind a plane mirror. He is, therefore, in a position to experiment on virtual images formed by concave or convex mirrors. The focal length can also be determined. Remember that *v for virtual images formed by mirrors (concave or convex) is always negative, as such images are always behind the mirror*

EXPERIMENT 66*b*.—**To locate, by parallax, virtual images formed by spherical mirrors, and to measure *f*.**

Apparatus: Concave and convex mirrors, two tall needles (thin knitting needles) on uprights, one sewing needle, optical bench.

Concave mirror : Use for object a fine sewing needle, tied to the ordinary needle. The virtual image is magnified, hence the object has to be fine to avoid the image being very thick. Locate erect virtual image by a tall needle behind mirror. The mirror should be taken out of rings or frames, etc., for this. The portion of the image near the edge of the mirror is often distorted. To remedy this, paint a narrow ring 2 mm. broad round the circumference, with black paint (black japan). Repeat 4 times with different distances. Measure u and v as usual.

Convex mirror : Proceed exactly as above, except that the fine sewing needle is not required. Two knitting needles are sufficient.

These adjustments can also be made by viewing the back needle through a hole in the centre of the mirror, or through a transparent patch from which the silvering has been removed. This method is, however, inferior, as unless the hole is large, the range over which the eye can move is too small to allow accurate adjustment.

The following figures are from students' note-books:

CONCAVE MIRROR			CONVEX MIRROR		
u (cm.)	v (cm.)	f (cm.)	u (cm.)	v (cm.)	f (cm.)
21·3	−70·5	+30·6	84·5	−32·4	−32·4
18·4	−45·4	+30·9	45·5	−19·4	−33·8
18·6	−45·1	+31·6	30·9	−15·9	−32·8
23·3	−92·7	+30·6	22·7	−13·5	−33·3
		Mean : +30·9			Mean : −33·1

☞ The signs of u and v should be duly considered when calculating f.

For calculation of focal lengths use tables of reciprocals (see Appendix).

CHAPTER XXIV

REFRACTION AT A PLANE SURFACE

Refraction. The bending of a ray at the interface between two optically different media is called **refraction.** If the ray passes from medium A into medium B, the portion in A is called the **incident ray,** the portion in B the **refracted ray,** and the ray after leaving B the **emergent ray.** The angles made by these rays with the normals to the interfaces concerned at their respective points of refraction, are called the **angles of incidence, refraction and emergence,** respectively, and we shall always denote them by i, r and e. The amount of bending at the point of refraction or the total bending at one or two successive refractions is called **deviation** and we shall denote this angle always by δ.

Apparatus Used. *Solid refracting media* are used in the form of blocks of simple geometrical shape. The commonly employed forms are, in decreasing order of simplicity of operation :

(1) **Semicircular discs**, with which *one* refraction at a single surface can be conveniently observed.

(2) **Rectangular blocks and cubes.** With these a single refraction at one surface is observed less conveniently than with (1). *Two* refractions generally take place.

(3) **Prisms** (generally equilateral or rectangular isosceles). In these, refraction phenomena are complicated by the angles between the faces, and a separate chapter is therefore devoted to their discussion.

Liquid refracting media are placed in **hollow cubes** or prisms with glass sides, or on **concave mirrors.** The upper surface of liquids is plane (except close to the edges) and optically perfect.

The pin method is the one generally employed for tracing the rays. Images are located by parallax.

EXPERIMENT 67.—**To study the relation between i and r with a semicircular glass disc.**

Apparatus: Semicircular disc 8 to 10 cm. diameter, pins, paper, protractor, drawing materials.

Fix a sheet of paper on the drawing board and draw a circle of ten cm. radius (Fig. 93). Draw two perpendicular diameters. NM is the normal and RS the refracting surface. In quadrant NS mark 1, 2, 3,...at an angular distance from N of 15°, 30°, 45°, 60°, 75°. Fix pins at the centre P and at 1, 2, 3, ... 6. 1P, 2P, 3P are the incident rays. Place the half disc with its diameter on RS, and its centre exactly at P.

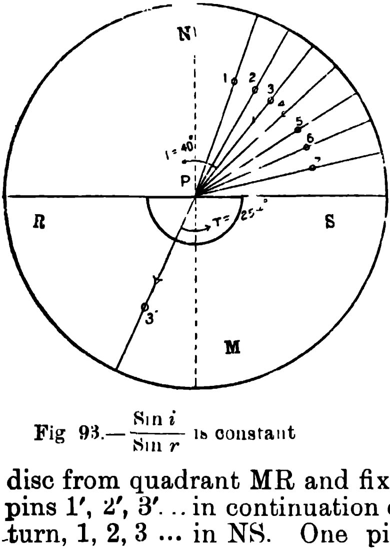


Fig 93.—$\frac{\text{Sin } i}{\text{Sin } r}$ is constant

Look through the disc from quadrant MR and fix near the circumference pins 1′, 2′, 3′... in continuation of the image of P and in turn, 1, 2, 3 ... in NS. One pin is sufficient for each ray, as there is no refraction at the curved surface, the refracted and emergent rays being normal to it. Connect these pins 1′, 2′, 3′ ... in MR to P. These lines are the refracted emergent rays. Measure r_1, r_2, r_3, or angles made by these rays with PM. Prepare a table with 5 columns, and in the first two record i and r.

Relation between i and r: Probably you know already the relation that exists between i and r. But that is because you were told. If you were the first experimenter on refraction, you would feel on looking at the values that as *i* and *r* increase and decrease together, *r* may be $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$ or some such fraction of *i*. If you found this not to be so, you might try other relations between the angles. You might next think of trigonometrical ratios and light on $\frac{\sin i}{\sin r} = \mu$, but there is no reason to believe that you would do so at the very first attempt. As a matter of fact, the actual relation was discovered many years after the fact of bending was discovered. To illustrate how formulæ are discovered and as an exercise in the use of logarithmic sines, etc., tabulate $\log \frac{i}{r}$, $\log \frac{\tan i}{\tan r}$ and $\log \frac{\sin i}{\sin r}$. The quotients need not be calculated. Simply calculate log (quotient) from tables of logarithmic sines and tangents. Calculate the quotient only for the column in which the logarithms are reasonably constant.

Observations:

i	r	$\log \frac{i}{r}$	$\log \frac{\tan i}{\tan r}$	$\log \frac{\sin i}{\sin r}$
20°	13° 2	1 3010 1 1206 1804	$\bar{1}$ 5611 $\bar{1}$ 3702 ·1909	$\bar{1}$·5341 $\bar{1}$ 3586 1755
40°	25°·4	1 6021 1 4048 ·1973	$\bar{1}$·9238 $\bar{1}$ 6765 2473	$\bar{1}$ 8081 $\bar{1}$ 6324 1757
60°	35° 3	1·7782 1 5478 ·2304	2386 $\bar{1}$·8501 ·3885	$\bar{1}$·9375 $\bar{1}$ 7618 ·1757
80°	41° 0	1 9031 1 6128 2903	·7537 $\bar{1}$ 9392 8145	$\bar{1}$·9934 $\bar{1}$·8169 ·1765

The numbers in the last column are reasonably constant, the mean value being ·17 $\frac{234}{4}$ or ·1759. $\frac{\sin i}{\sin r}$ therefore equals antilog ·1759 = 1·499, say 1·50.

Hence $\frac{\sin i}{\sin r} = \mu = 1{\cdot}50$ (the constant μ is the index of refraction from air to glass).

Five values of *i* (15°, 30°, 45°, 60°, 75°) would be suitable.

Record answers to the following:

(1) Besides this algebraical method of studying the relation between *i* and *r* what other method could you have applied?

(2) How would the result for *sin i* and *sin r* differ from that for the other functions? Plot a graph between *sin i* and *sin r*, and obtain from it the *mean* refractive index.

EXPERIMENT 68.—**With a rectangular glass block or cube.**

(*a*) **To trace the path of refracted rays, and to determine the index of refraction.**

(*b*) **To trace the path of rays suffering total internal reflection and to verify for such rays the equality of the angles of incidence and reflection.**

Apparatus: The same as in the last experiment, but with rectangular glass block 12×8×3 cm. or 3 to 5 cm. cube, instead of semicircular disc.

There are a number of ways of arranging rays at different angles, and of fixing the direction of the refracted or emergent rays. *Single pins*, adjusted by trial and *fixed in contact with the glass block, are liable to displace the block. Two pins at a distance are better.* Rays incident at *one* point at different angles lead to better diagrams and easier calculations than those diverging from a point and incident at various points of the block.

(*a*) Block 12 × 8 × 3 cm. about.

Method 1. See diagram (Fig. 94).

Diameter of circle 16 to 20 cm. Fix pins P and 1, 2, 3....on circumference beforehand Pins 11, 22, 33, below block are fixed by trial so as to be in line with images of P1, P2, P3, etc., as seen through block.

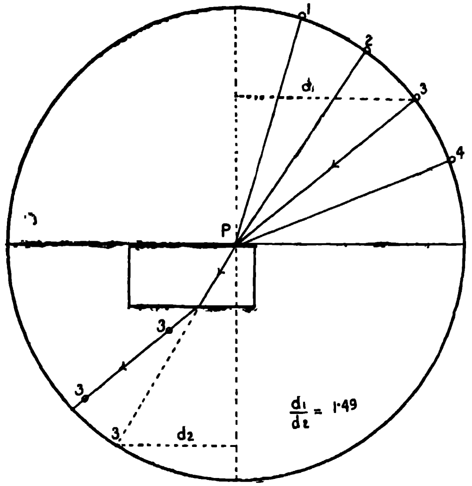


Fig. 94.—Determination of refractive index.

Join 11, 22, 33, and produce to lower surface of block. Connect these points, where the rays meet the lower surface, to P. These lines are the refracted rays. Produce these along dotted lines to meet the circumference. Measure angles i and r at P, and obtain sines from tables; or drop perpendiculars on the normal as shown for the ray P3. As the hypotenuses are all equal, these perpendiculars are proportional to $\sin i_1$, $\sin i_2$, *and* $\sin r_1$, $\sin r_2$, etc.

Method 2. See diagram (Fig. 95).

Fix the pin P in contact with the upper and pins 1, 2, 3...5 in contact with the lower surface of the block beforehand. Pins 1, 2, 3, may be about 1 cm. apart. Looking from the side of these pins through the block, fix pins 1′, 2′, 3′ on the upper side of the block so that they appear in a line with P1, P2, P3, etc. Join 1′, 2′, 3′, to P. These lines are incident rays corresponding to our fixed refracted rays P1, P2, P3, etc. Measure the angles and read the sines ; or draw

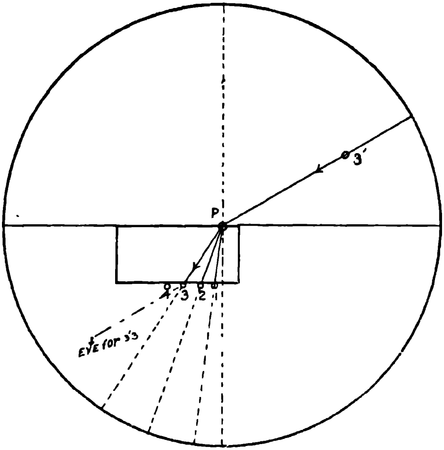


Fig 95.—Determination of refractive index.

as before a large circle with P as centre. Drop perpendiculars on normal as shown in Fig. 94. Measure these perpendiculars.

Record thus :

	r	$\sin i$	$\sin r$	$\mu = \frac{\sin i}{\sin r}$

Mean μ = ...

or

Perpendicular for i (d_1)	Perpendicular for r (d_2)	$\frac{d_1}{d_2} = \mu$

Mean μ = ...

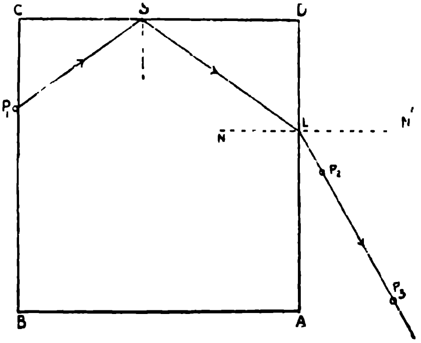


Fig. 96—Equality of the angles of incidence and reflection for an internally reflected ray.

(*b*) See diagram (Fig. 96). Make a small ink dot S near the bottom on one of the vertical faces of a rectangular or cubical block in about the position shown. Fix a pin P_1 in contact with the block so that the angle C S P_1 is 30° to 35°. The pin P_1 as also the dot S will be visible if you look into DA towards CD. Fix two pins P_2, P_3 in line with the image of S and the image of P_1 reflected from CD and refracted through DA. Produce the line P_3 P_2 to meet the face AD in L. Draw lines P_1S, SL, LP_2 P_3 as shown. These give the path of the ray through and out of the block. At S draw a normal to the face CD. Measure the angles between this normal and the incident and reflected rays P_1S and SL. These will be equal. Draw a normal through L, and measure the angles $N'LP_2$ and SLN. The ratio of the sines of these angles equals μ for the block.

A hollow cube filled with liquid can also be used for the above experiment.

Refractive Index of a Liquid. This can be found by placing the liquid in a rectangular or cubical hollow vessel or hollow prism with plate glass sides, and treating the vessel so filled as a transparent solid.

EXPERIMENT 68*a*.—**To determine the refractive index of water.**

Apparatus: Hollow cube or hollow prism. Pins, paper, etc.

Place water in a hollow cube. Trace refracted rays as in experiment 68 (*a*). Calculate $\frac{\sin i}{\sin r}$ for the various rays. When a hollow prism is employed, determine angle of prism and minimum deviation as in Expt. 73 and apply the appropriate formula.

[NOTE. For calculation of refractive indices, use table of logarithmic sines.

For fractions of a degree use proportionate values.

Angles of incidence and refraction less than 10° or more than 80° should be avoided, as results based on them are decidedly inaccurate.

Examples of use –

μ for glass $= \frac{\sin i}{\sin r} = \frac{\sin 30^\circ}{\sin 19^\circ 5}$; $\log \mu = \log \sin 30^\circ - \log \sin 19^\circ{\cdot}5 = \bar{1}{\cdot}6990 - \bar{1}{\cdot}5234 = {\cdot}1756$;

μ for glass = antilog $\cdot 1756 = 1{\cdot}498$.]

Refraction at nearly Normal Incidence. Objects under water, when observed along the normal, appear to be at $1/\mu$ or $\frac{3}{4}$ of their real depth. Conversely, a point in air if viewed by an observer in water would appear at $\frac{4}{3}$ of its real distance from the surface. These facts can be utilized for determining the refractive index of a liquid or solid.

The measurement of apparent depth can be carried out in the following ways :

(1) By means of a **concave mirror** on which a thin layer of the liquid is placed. This method is simple, convenient, and accurate, and can be used with alcohol and other expensive liquids, as only a small quantity is required.

(2) By fixing the position of the image of an object under glass or water as seen by refraction at normal incidence, by parallax with a needle held outside the liquid a little above the object, or with the ***image*** of such needle formed by reflection at the water surface when held above it. This method is even

moderately satisfactory only if large quantities of liquid are available, and *deep* layers (15 - 20 cm. thick) employed. It can be applied to solids in the form of *thick* blocks. Thin layers give excellent results only if a travelling microscope is used to measure the apparent depth.

EXPERIMENT 69.—**To determine the refractive index of (1) kerosene by concave mirror, (2) glass by parallax with a needle, (3) water by parallax with the reflected image of a needle.**

Apparatus: (1) Concave mirror on base of retort stand, with a horizontal needle sliding up and down the rod, (2) tall glass vessel, preferably with plane sides (failing that a tumbler or beaker), (3) glass block.

(1) Place the mirror on the base of the stand on a low table or stool, so that you can conveniently look down into the mirror. Move the needle up and down till there is no parallax between the *tip* of the needle and the *tip* of its own inverted image (Fig. 97 *left*). This will be the case when the needle is at C, the centre of curvature of the mirror. Measure the distance d_1 of the needle above the base. Repeat adjustment three times. Calculate mean d_1. Repeat the experiment with liquid on the mirror (Fig. 97 *right*). The needle will have to be pushed down to coincide with its own image. Measure the diminished distance d_2. The path of one of the rays is OAB,

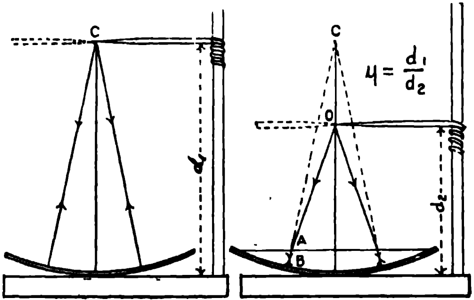


Fig 97.—Determination of the refractive index of a liquid with a concave mirror.

then BAO, the reflection at B being normal. As BA is normal to mirror, it must meet, when produced, the axis at C. In other words, C is the image of O by refraction at A. Hence $\frac{d_1}{d_2} = \mu$. Adjust the needle and measure d_2 three or four times.

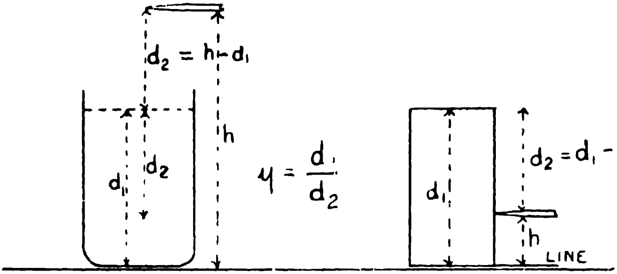


Fig. 98.—Determination of refractive index by measurement of apparent depth.

(2) Place the glass block (Fig. 98 *right*) centrally on a pencil line on a sheet of white paper, the *longest* side of the block being *vertical*, and the *shortest* side parallel to the line. Slide up and down along the side, a needle arranged parallel to the pencil line, and adjust its position so that there is no parallax between the needle point seen directly with one eye and the refracted image of the line *seen through the block* with the other. Measure h the height of the needle point above the pencil line. Repeat four times. Calculate mean h. Lastly measure d_1 the height of the block. The apparent distance of the line is $d_2 = d_1 - h$.

(3) Place a *tall* beaker, gas jar or tumbler full of water (Fig. 98 *left*) on a *black* surface. Drop a pin into the water to serve as an object, arranging it as much as possible along a diameter. Place the horizontal needle parallel to the pin *above* the water surface. A dark image of this needle formed by reflection at the water surface will be easily seen, if

sky. Light from a window reflected downwards by a sheet of paper held suitably above the needle will also help. Adjust the needle up and down till the reflected image of the needle point coincides with the pin in all positions of the eye. Measure h the height of the needle above the pin. This equals its height above the table provided the thickness of the glass bottom is negligible. Repeat four times. Calculate mean h. Lastly measure d_1 the height of the water surface above the pin. The apparent depth $d_2 = h - d_1$, because the reflected image is as much below the water as the needle is above it.

Record observations thus:

EXPT. 1	HEIGHT OF NEEDLE WITHOUT LIQUID d_1	HEIGHT OF NEEDLE WITH LIQUID ON MIRROR d_2	REFRACTIVE INDEX $\frac{d_1}{d_2}$
	cm.	cm.	
Refractive index of kerosene by means of a concave mirror	... ...	... ...	$\frac{\ldots}{\ldots} = \ldots$
	...	...	
	Mean :	Mean :	

EXPERIMENTS 2, 3	HEIGHT OF NEEDLE h	REAL DEPTH d_1	APPARENT DEPTH d_2	REFRACTIVE INDEX $\frac{d_1}{d_2}$
Refractive index of glass block by needle seen directly	... ... Mean : ...	...	$(d_1 - h)$ $= \dots$	$\frac{\dots}{\dots} = \dots$
Refractive index of water by reflected image of needle	... ... Mean : ...	...	$(h - d_1)$ $= \dots$	$\frac{\dots}{\dots} = \dots$

CHAPTER XXV

REFRACTION THROUGH A PRISM

Prisms. Triangular prisms alone are employed for ordinary work. One of the three faces is sometimes blackened or ground. This is an advantage as undesired images obtained by internal reflections are eliminated. If such images have to be studied, all the three faces should be clear.

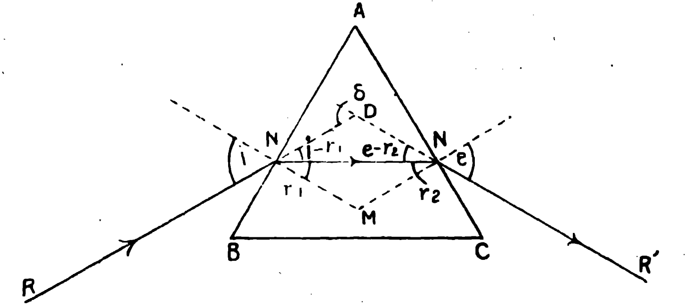


Fig. 99.—Refraction through a prism.

Refraction through a Prism. Fig. 99 shows a prism and a typical ray path. i and e are the angles of incidence and emergence. r_1 and r_2 are the angles with the normals, inside the prism. The angle of the prism is called a. The deviation is δ. In the quadrilateral ANMN angles at N and N are right angles (NM, NM are normals). Hence

$$a + \angle NMN = 180^\circ$$

But $r_1 + r_2 + \angle NMN = 180^\circ$ also : $\therefore a = \mathbf{r_1 + r_2}$ ... **(1)**

Also

$$\delta = (i - r_1) + (e - r_2) \text{......(sum of interior angles)}$$

$$= i + e - (r_1 + r_2)$$

$$= i + e - a \qquad \therefore a = i + e - \delta \text{ ...}(2)$$

In case $i = e$, r_1 will equal r_2. Call both of these r and the deviation for this case δm.

(1) and (2) become $a = 2r$

$$\text{and } a = 2i - \delta m$$

$$\text{hence } i = \frac{a + \delta m}{2}$$

$$\text{and } r = \frac{a}{2} \qquad \therefore \mu = \frac{\sin i}{\sin r} = \frac{\sin\frac{(a + \delta m)}{2}}{\sin\frac{a}{2}} \quad \ldots(3)$$

We shall prove that δm is the minimum or least value δ can ave for the prism.

Practical Work with Prisms. Tracing of rays through prisms presents no difficulty. The typical ray path (Fig. 99) is obtained by fixing two pins on RN, and fixing NR′ by two pins adjusted in a line with the images of the two fixed on RN, as seen through the prism. N and N are obtained by prolonging the lines to meet the prism faces. RNNR′ is the complete path.

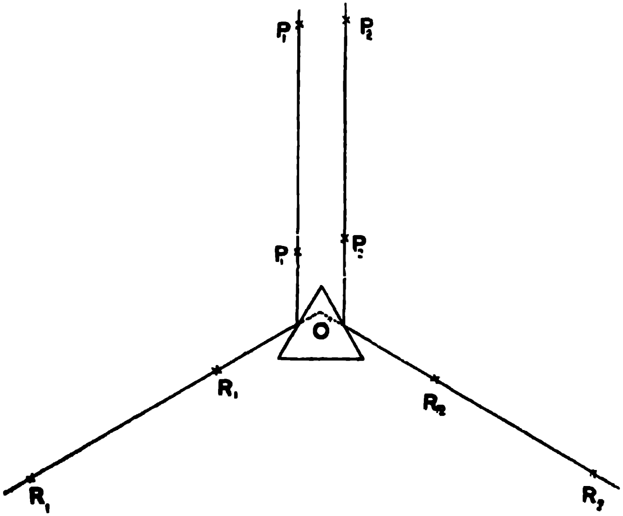


Fig. 100—Measurement of the angle of a prism by reflection from the two faces.

EXPERIMENT 70*a*.—To measure the angle of a prism by reflection from the two faces.

Apparatus: Prism, pins, paper, drawing materials, etc. See diagram (Fig. 100). P_1, P_1, and P_2, P_2 are 2 pairs of pins on parallel lines to represent incident rays R_1, R_1 are fixed in a line with the images of P_1, P_1 formed by reflection in the left face R_2, R_2 are similarly fixed in line with images of P_2, P_2 in the right face. Complete and produce R_1R_1 and R_2R_2 to intersect at O. Measure the angle R_1OR_2. One half of this angle equals the angle of the prism. Mark the position of the prism on paper and verify by measuring a with a protractor Determine the three angles and show that they add up to 180°.

EXPERIMENT 70.—(1) To verify the relation $a = i + e - \delta$.
(2) To determine μ and δm (minimum deviation).
(3) To show that for minimum deviation $i = e$.

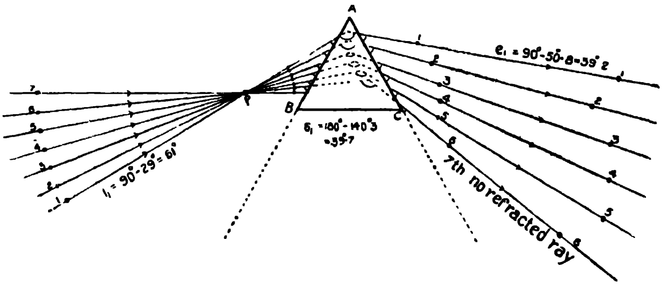


Fig. 101 —Six rays refracted through a prism.

A very instructive and accurate experiment can be performed by tracing through a prism the course of six or seven rays at various angles of incidence, i_1, i_2, i_3...etc. The corresponding angles of emergence and deviation are also measured, and tabulated. Two graphs on the *same* sheet of paper are constructed, one for i and e, the other for i and δ. δm and the values of i and e corresponding to δm can be obtained accurately from these graphs. The work can be done conveniently in two instalments: tracing and drawing, 1st stage; measuring, tabulating and plotting, 2nd stage. The experiment covers nearly all work on prisms.

The diagram (Fig. 101), table and graph (Fig. 102) are from an actual experiment.

Diagram:

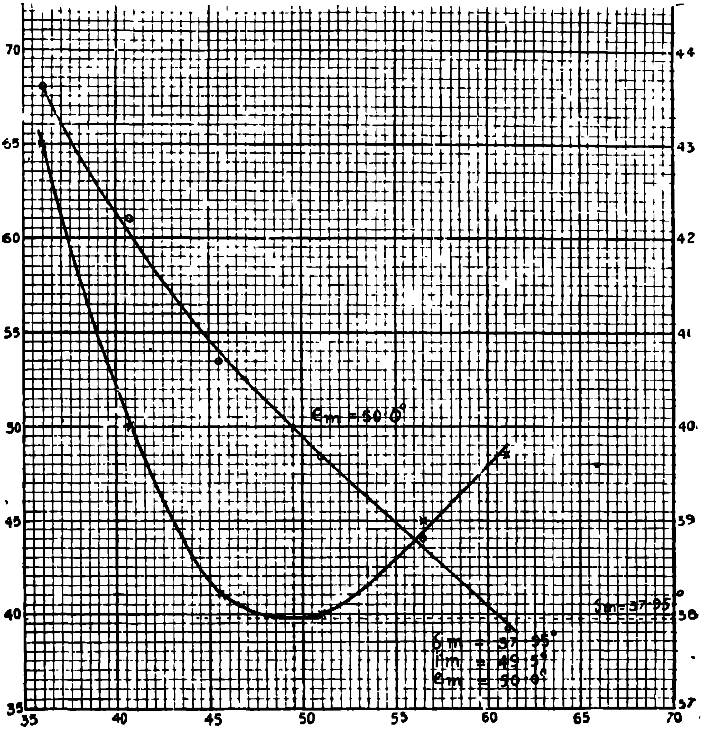
Draw 6 or 7 lines P1. P2, P3... arranging the angles with a protractor to be about 5° each. Stick pins P, 1, 2,...7. Place the prism ($1\frac{1}{2}$ inch size is convenient) as shown, adjusting the position so that on looking into the face AC, P can be seen *in a line with any of the pins 1, 2, 3...7.* Mark this position by passing a sharply pointed pencil round the prism. The position can thus be recovered in case of displacement after part of the work is done. Trace one by one the seven emergent rays corresponding to incident rays 1P, 2P, 3P, by pairs of pins 11, 22, 33,...77 on the right. Tracing becomes easier if all pins are removed except the four that are being used. For instance when tracing 2P22, remove all pins except 2P22, and so on for other rays. Complete the diagram as shown. Measure the angles between the incident rays and face AB, as also those between the emergent rays and the face AC. Complements of these are i_1, i_2,...i_7 and e_1, e_2......e_7.

Measure the obtuse angles formed at the intersection of the emergent and incident rays. The supplements of these are equal to δ_1, δ_2 δ_3...δ_7. Calculate and record all these angles on the diagram, as shown for one of the rays. Measure a, the angle of the prism.

Record as follows :

i	e	δ	$i+e-\delta=a$
°	°	°	°
61·0	39·2	39·7	100·2 − 39·7 = 60·5
56·5	44·0	39·0	100·5 − 39·0 = 61·5
51·0	48·8	38·0	99·8 − 38·0 = 61·8
45·5	53·5	38·2	99·0 − 38·2 = 60·8
40·7	61·0	40·0	101·7 − 40·0 = 61·7
36·0	68·0	43·0	104·0 − 43·0 = 61·0
...	...	...	Mean = 61°·2

a directly measured = 61°·2.



Graphs : (1) Plot i as abscissæ (scale from 35 to 70 or 35 to 65).

(2) Plot δ as ordinates against i (right scale from 37 to 44).

(3) Plot e as ordinates against i (left scale from 35 to 70).

(4) Draw smooth curves *through* points or *with points equally distributed on both sides.*

(5) Mark the lowest point on the i, δ graph.

(6) Through this point draw lines parallel to the two axes to cut the axes and the i, e graph. The points of intersection give i_m, e_m, δ_m. Record these on the graphs as shown. The actual values for the experiment were $i_m = 49^\circ{\cdot}5$, $e_m = 50^\circ{\cdot}0$, $\delta_m = 37^\circ{\cdot}9$

Refractive index : Calculate μ from formula

$$\mu = \frac{\sin \frac{1}{2}(a + \delta_m)}{\sin \frac{1}{2} a}.$$

Experimental Determination of the Position of Minimum Deviation. This is a difficult matter. Near the position of minimum deviation, the rate of change of deviation with change of angle of incidence* is very small. In consequence, for a change of 2 or 3 degrees in i, δ changes very little, and appears to be a minimum for positions of the prism differing by 2° or 3°. Minimum deviation itself (because it changes little) can, therefore, be accurately measured, and an exact value of μ obtained, but it is not so easy to show that $i = e$ for this value of the deviation †.

The usual method is to look through a prism at two pins in a line in the ordinary way, and to turn the prism round a vertical axis, till the images are as near the actual position of the pins as possible. This position of the prism is regarded as the δ_m position, and the corresponding emergent ray traced. *i* and *e* can be measured and compared. The following method* gives good results.

---

* In the position of minimum deviation, $\frac{d\delta}{di} = 0$.

† This can be shown easily with a spectrometer.

EXPERIMENT 71.—**To place a prism in the minimum deviation position, to show that $i=e$ for this position, and to obtain δ_m and μ.**

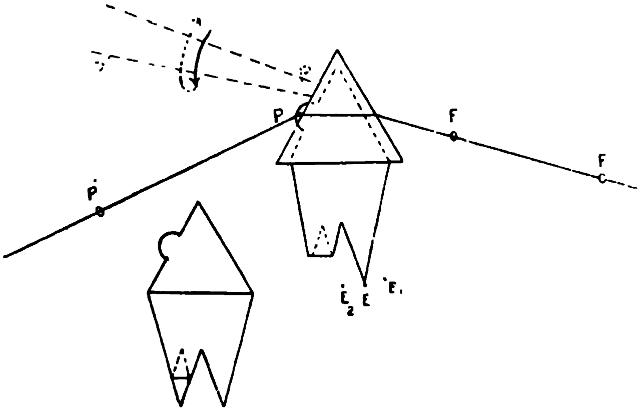


Fig. 103 —Experimental determination of the position of minimum deviation.

Cut out of thick paper a piece of the shape shown in Fig. 103. This is to form a revolving platform for the prism.

Bend up one of the legs into the position shown by the dotted line, to form a handle. The other is to serve as a pointer.

Place the prism on the platform on a sheet of paper, and fix a pin P through the paper into the board, to serve as a pivot. This pin should be close to the left face. Fix another pin P′ at a distance of 8 to 10 cm. On looking into the right face, the pins P, P′ will appear along the dotted line as shown. The angle between PP′ and the dotted line PP′ is δ. Rotate the prism so as to cause the dotted line to move in the direction of the full arrow, that is, into positions of *less* deviation. If you continue rotating the prism in order to reduce the deviation, the dotted line will at a certain stage appear stationary and then begin to go *back* as shown by the dotted arrow. As soon as this happens make a dot (E_1) opposite the pointer. Rotate backwards, till the deviation, after slightly decreasing, again begins to increase. Make a second dot E_2 opposite the pointer. The δm position is very nearly midway between E_1 and E_2 Bisect $E_1 E_2$ at E. Place the pointer at E. The prism is now in the position of minimum deviation. Trace the position of the prism on the paper, and trace in this position also the emergent ray FF′ by two pins. Measure i, e, δ_m, a, and calculate μ. i and e will not differ much.

---

* After Stewart, De and Chetan Anand.

Dependence of Refractive Index on the Colour of Light used. That the refractive index of glass for blue or green light is more than for red light is easily shown by means of very simple apparatus.

EXPERIMENT 72.—**To show with a prism that the refractive index and deviation are more for blue light than for red.**

Apparatus : *Black* paper, red and blue paper or red and blue (deep but bright) silk thread Prism.

Cut out of stiff *black* paper (not white) a piece 2 inches by 1 inch. Bend up at right angles along the middle so that a piece 1 inch square can stand vertically on the table (Fig. 104). Along the middle of the vertical portion paste, vertically in a line with one another, two pieces half an inch long, of thread or paper (1 mm. wide), the upper half red, and the lower half blue or green. Red and blue pencil marks will also do. Place a prism about 30 cm. from the card, and adjust so that the red and blue marks are seen by *refraction through two faces* as usual (not by internal reflection). The blue or green image will be farther removed from the actual position of the card than the red image, showing that the deviation (and hence the refractive index) is greater for blue than for red light. Sketch on paper the actual and apparent positions of the marks, as also the positions of the prism and the eye. Draw also the paths of the rays to show the unequal deviation. Mark the two angles of deviation. If red and blue marks on *white* paper are employed, the results are very confusing on account of the white light from the paper producing its own spectrum.

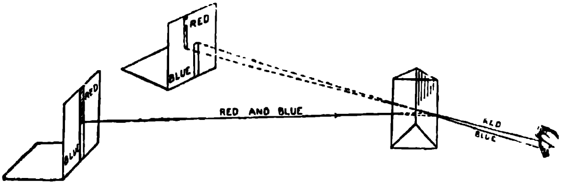


Fig 104.- Refractive index varies with the colour or wave length of the light employed.

Total Reflection in a Prism.--If r_2 exceeds the critical angle ϕ for the material of the prism, the ray cannot emerge. Whether r_2 exceeds ϕ or not depends on r_1 and a because

$$r_1 + r_2 = a$$
$$\text{or} \qquad r_2 = a - r_1.$$

r_1 is always less than ϕ. Therefore, if a exceed 2ϕ, r_2 will always exceed ϕ. **Hence a ray cannot be refracted in the ordinary way through a prism whose angle exceeds twice the critical angle of the material.** The ray will be *totally* reflected, and emerge from the 3rd face.

Total reflection will take place even in an ordinary equilateral prism (although a is less than 2ϕ), provided r_1 is made small enough, because if r_1 is small, r_2 $(= a - r_1)$ may exceed ϕ.

EXPERIMENT 73.—To study total reflection in a prism.

Apparatus: Right-angled and ordinary prisms solid glass cube, hollow right-angled prism.

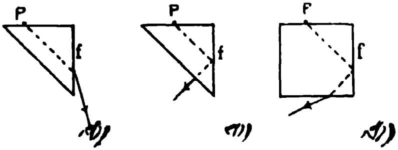


Fig. 105.—The absence of an emergent ray when the refracting angle exceeds twice the critical angle.

Fill the hollow right angled prism with water. Arrange the water prism, as also the solid right angled prism and the cube as shown (Fig. 105). Stick a pin P close to one of the sides (not the hypotenuse) of the prisms and the cube as shown. Try to see the pin through the prisms or cube from the adjacent face *f*. The pin cannot be seen through the cube and the solid prism, while it *can* be seen through the water prism. Why? (The refracting angle is 90° in each case. The critical angle is 42° for glass and 49° for water. See previous page.)

To study total internal reflection in an ordinary equilateral prism ABC, make a small ink dot D near the bottom of one face AC. Place the prism on a sheet of paper and pass a pencil all round to mark its position. Look into the face BC. *The dot D will be clearly seen on a silver bright back ground.* Stick a pin P opposite the face AB so that a *bright*

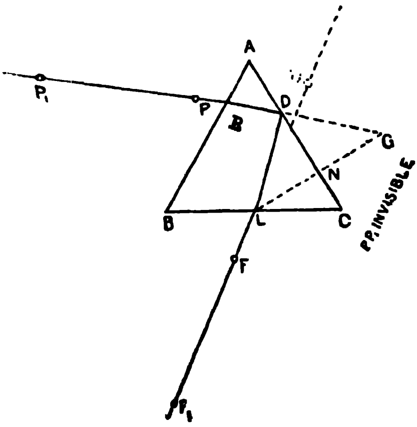


Fig. 106.—Total reflection in a prism.

image of P on a *silver bright back ground* appears in a line with the dot D Stick similarly a second pin P_1, so that *clear bright* images of P, P_1 and D as seen through BC appear in one line (shown dotted). Dim images seen by *ordinary* (not *total*) internal reflection should not be used. Stick pins F and F_1 also in a line with the *bright* images of P_1, P and D. P_1P and FF_1 are the incident and the emergent rays. Produce these to meet the faces AB and BC in R and L. Make a dot on the paper exactly under D, and complete the path RDL inside the prism. Draw a normal to AC at D, and measure the angles made with it by the incident ray RD and the reflected ray DL. They will be equal. Record these angles and compare them with the critical angle for glass (42°).

If we are asked to trace the path of a *given* incident ray P_1P through a prism, *the prism has to be moved about* till the images of P_1P and a dot D are in a line. This is not difficult. The pins F and F_1 are afterwards fixed and the path $P_1PRDLFF_1$ completed.

If in the above case, we *assume* that the angles of incidence and reflection are equal for total internal reflection, the path RDL can be found without the dot D. To do this drop a perpendicular LN on AC and produce it to C so that LN = NG. Draw GR to cut AC in D. RDL the path inside the prism is thus found.

CHAPTER XXVI

REFRACTION THROUGH A LENS

Lenses. A portion of a refracting medium enclosed between two spherical surfaces is called a **lens.** If the diameter of the lens is small as compared with the radii of curvature of the surfaces, the lens is said to be of **small aperture.** Parallel rays falling normally on such a lens come to a **real or virtual point focus,** *whose distance (f) from the lens is not necessarily one-half of the radius of curvature of either surface.* As the **focal length** (f) of a lens depends also on the refractive index of its material, the connection between the focal length and the radii of curvature of the surfaces is not as simple as in the case of a spherical mirror. If, instead of parallel rays, rays from an object at a distance u from the lens fall on the lens, the image is formed at a distance v, such that

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

This formula applies to all lenses, convex or concave, and for all images, real or virtual, provided we observe the

Rule of Signs. The rule of signs for distances measured from the lens is the same as for spherical mirrors. **Distances of points on the same side of the lens as the object are regarded +, while distances on the other side are regarded –(see Fig. 107).**

Convex Lens. According to the above rule of signs, the focal length of a convex lens is negative. Fig. 107 (top left) show a convex lens with principal focus on the side *away* from the light source, hence $f = -10$ cm. (*negative*)

Fig. 107 (top right) shows the same lens with a virtual image on the *same* side as the object.
Hence $u=+6$ cm., $v=+15$ cm. (*positive*).

Fig. 107 (bottom) shows the same lens with a real image on the side *away* from the object.
Hence $u=+25$ cm. $v=-16{\cdot}6$ cm. (*negative*).

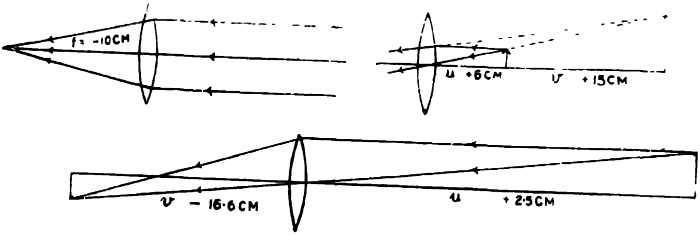


Fig. 107.—The rule of signs illustrated.

Determination of the Focal Length of a Convex Lens. The focal length of a convex lens can be measured in a number of simple ways :

I. By means of an optician's lens-gauge or dioptrometer, which, on being pressed on the two surfaces of a lens (convex or concave), gives its power in dioptres (*see below*). The focal length is obtained from this in a minute.

II. By measuring the distance at which parallel rays come to a focus.

III. By causing the lens to form the parallel rays, and then sending them back through the lens with the help of a plane mirror.

IV. By measuring u and v. and applying the formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$.

I **Measurement of Focal Length with an Optician's Lens-tester or Dioptremeter.** The strength of spectacle lenses is measured in terms of the **dioptre.** *The power of a lens whose focal length* is *100 cm. is one* *dioptre* (1D). A lens of 2 D has a focal length of $\frac{100}{2}$ or 50 cm., and one of 5 D a focal length of $\frac{100}{5}=20$ cm. The *larger* the dioptric strength (called "the number") of a spectacle lens, the *less* the focal length. As opticians regard the power of a convex lens as +, and that of a concave lens as −, a suitable formula for focal length is

$$\text{Focal length} = \frac{-100}{\text{Dioptric strength}}.$$

The lens-tester is a watch-like instrument (Fig. 108), with two fixed steel points and in the middle a pin sliding up and down in a hole. On pressing a convex lens on the three points, the middle pin will be depressed by an amount depending on the curvature of the surface. The depression of the pin (and therefore the curvature) is recorded on a dial by a hand rotated by the pin by means of a rack and pinion.

As spectacle lenses are generally made of *crown glass*, the instrument is graduated so as to give the dioptric strength directly for lenses of this material. The readings will apply to common laboratory lenses, but *not to flint-glass lenses.* The student will see that the instrument is based on the spherometer principle.

The instrument can be used also for determining the radius of curvature of spherical mirrors. The reading on such a mirror will be the same as on a spectacle lens of the same radius, the other side of the lens being supposed plane. The focal length of such a lens can be calculated from the lens formula $\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$. As one side is plane the formula becomes $\frac{1}{f} = (\mu - 1)\frac{1}{r}$. For crown glass $\mu = 1{\cdot}52$ Hence, $r = {\cdot}52\,f$. But $f = \frac{-100}{\text{D}}$. Hence

$$r = \frac{-52}{D}, \text{ or}$$

$$\textbf{Radius} = \frac{-52}{\textbf{Reading in dioptres}}$$

EXPERIMENT 74.—**To measure the focal length of a convex or concave lens, and the radius of curvature of a mirror, with a lens tester.**

Apparatus: Dioptremeter, lenses and mirrors, plate glass.

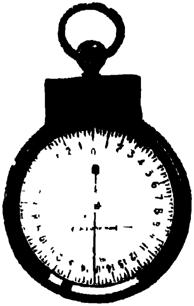


Fig. 108.—Optician's lens-gauge graduated to read dioptres.

Press the instrument on plane plate glass. It should read zero. If necessary twist the middle leg by means of the key supplied with the instrument till the reading on plate glass is 0. Next take the reading on both surfaces of the lens, and add algebraically. Suppose the reading on one surface is + 2 D, and on the other + 3 D. The power is + 5 D. The lens is convex, and the

focal length $\frac{-100}{+5} = -20$ cm.

Press the instrument on a spherical mirror. Suppose the reading is $-1{\cdot}75$ D. The mirror is concave and

the *radius* $= \frac{-52}{-1{\cdot}75} = +29{\cdot}7$ cm.

This method is useful for checking values obtained by other methods. By itself it is only an approximate method (accuracy 5%).

II. III. Determination of the Focal Length of a Convex Lens by Parallel Rays.

EXPERIMENT 75.—**To determine the focal length of a convex lens by means of (1) distant objects, (2) plane mirror.**

Apparatus: Lens, optical bench, plane mirror.

(1) Place the lens in a suitable holder on the bench, arrange a screen behind the lens, and direct the bench towards distant objects in direct sunlight. Move the screen about till images of *distant* (beyond 100 ft.) trees and houses are sharply focussed. Measure the distance between lens and screen. Repeat once or twice, upsetting and readjusting lens and screen positions every time. Calculate index correction, and the mean corrected focal length.

Observations :

Lens reading—screen reading = …… (1)
……… — ……… = …… (2)
……… — ……… = …… (3)

Mean : 30·2 cm.

Index correction rod = 20·00 cm.
Index correction = 20·00 − (difference between screen and lens upright readings).
= − ·4 cm.
Half thickness of lens = 1·2 mm. = ·12 cm.
∴ Corrected distance = 30·2 − ·4 + ·12 cm.
= 29·92 cm.
∴ f = $\underline{-29{\cdot}9\text{ cm.}}$

[Apply rule of signs *after* applying index correction].

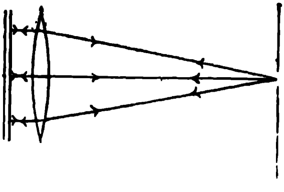


Fig. 109.—The plane mirror method of determining the focal length of a convex lens.

(2) If rays diverge from a bright object at the principal focus of a lens they will form a parallel beam after passing through it. This beam if reflected back along its own path by a plane mirror behind the lens, will, on passing through it, form an image coincident with the object. To apply this method, arrange a plane mirror behind the convex lens and place in front a screen with a central hole and cross-wires (Fig. 109) illuminated by candle or electric lamp. Adjust till the image of the cross-wires is sharply

focussed on the screen near the cross-wires themselves. Repeat once or twice, measuring the distance between the lens and screen each time. Add half the thickness of the lens and apply index correction as above.

Instead of the cross-wires, a needle may be adjusted till there is no parallax between it and its inverted image (Fig. 110). Repeat once or twice. Calculate mean reading. Keeping lens fixed, place the mirror and the *same* needle on the other side of the lens and readjust. Calculate mean reading. Half the difference between the two mean readings is the focal length *automatically corrected for index error and thickness of lens.*

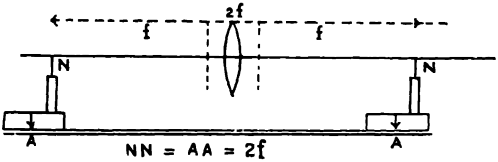


Fig. 110. – Index correction and thickness of lens automatically allowed for in the plane mirror method, by two readings on opposite sides of the lens.

Imagine N and N (Fig. 110) as the two positions of the needle. NN$=2f$. As an extreme case imagine the needle to be at one end of the upright, while the index is in the middle. The distance AA between the two positions of the index is also $2f$. Hence the difference between the two readings of the needle upright is twice the distance of the principal focus from the *centre* of the lens.

Record f, and calculate the power of the lens in dioptres. $D=\frac{-100}{f}$.

Methods II and III, involving the use of parallel rays, require a bench whose length is only a little more than f. They are, therefore, peculiarly suitable for lenses of *long* focus, because as the measurement of u and v requires a bench length of $5f$ to $8f$, even two metres may be insufficient, if f is large. With a bench less than $4f$ in length, no image can be obtained on the screen.

IV. Determination of the Focal Length of a Convex Lens by measurement of u and v. The method consists in measuring the distance from the lens of screen and wire-gauze when a sharp image of the latter is focussed. u and v can also be measured by means of two needles, each adjusted to be the image of the other by the method of parallax.

EXPERIMENT 76.—**To measure the focal length and dioptric strength of a lens by applying the formula** $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$.

Apparatus : Convex lens, optical bench with wire-gauze, screen, and two needles.

Get a rough idea of the focal length by obtaining on a sheet of paper images of distant objects. u or v cannot be *less* than this distance. Start with the illuminated wire-gauze at one end of bench, screen at the other end, and the lens close to the screen between the two. Move lens away from screen. At a distance about equal to or a little more than the roughly estimated value of f, a sharp image of the gauze will be obtained. Read the three uprights. Next move the lens about 10 cm. towards the gauze and adjust screen till image is sharp. Read the three uprights again. Again reduce the distance from lens to gauze, and repeat the adjustment. Obtain at least 4 or 5 sets of readings in this way, both *magnified* and *minified*, images being tried.

Exactly the same measurements can be made by means of two needles. When the needles are adjusted properly, parallax between any one needle and the image of the other will be absent. *It is better, however, to carry out the adjustment by removing parallax between the reduced image of the further needle* (*at the greater distance from the lens*), *and the nearer needle itself*. The other arrangement is less sensitive.

Measure index correction (1) between the gauze (or needle) and lens uprights, (2) between screen (or needle) and lens uprights.

Before proceeding to calculate, answer the following :

Q 1. Can f *numerically* exceed u or v irrespective of sign ?

Q. 2. Can $\frac{1}{f}$ be *numerically less* than $\frac{1}{u}$ or $\frac{1}{v}$ (irrespective of sign) ?

Q. 3. Can $\frac{1}{f}$ be equal *numerically* to $\frac{1}{v} - \frac{1}{u}$ or the *numerical difference* of the reciprocals of u and v ?

Good work is frequently spoiled by students wrongly applying the rule of signs. A safe rule (for use with *convex* lenses when producing *real* images only) is to **add** the reciprocals (both regarded +) of u and v and calculate f. Finally place a negative sign before the value so found.

Observations :

Approximate focal length by distant objects = ...cm.
Index correction, gauze to lens = 20·00 − (28·6 − 7·9)
= 20·00 − 20·7
= − ·7 cm.
" " screen to lens = 20·00 − (47·8 − 28·6)
= 20·00 − 19·2
= + ·8 cm.
Half thickness of lens = .../2 = ...cm.

u observed (cm.)	v observed (cm)	u corrected (cm.)	v corrected (cm.)	$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$ *arithmetically.*	f
125·23 —100·10 = 25·13	100·10 —83·75 =16·35(—)	25·13—·7 24·43 *add* also half thickness of lens *numerically*	16·35+·8 =17·15(—) *add* also half thickness of lens *numerically*	·04093 +·05832 = ·09925 $= \frac{1}{10·08}$ (from tables of reciprocals)	—10·08cm say —10·1 cm. f cannot be found correct to ·1 mm.

CAUTION. If you find f to be *very large or larger than* u or v, you have made a mistake in applying the rule of signs.

IVa. Determination of Focal Length by measurement of u+v and u—v or the Displacement Method. u and v can also be obtained from the sum and difference of u and v, that is from $u + v$ and $u - v$. These quantities can be obtained on the optical bench from a few readings of the uprights only, correction for thickness of lens and for indices being automatically applied (except in one case).

Consider Fig. 111. Needles N_1 and N_2 are conjugate for *both* positions of the lens. The distances in the 1st position are u and v. With needles fixed, move the lens to the 2nd position so that the points N_1 and N_2 are still images of one another. In the 2nd position what was formerly u has become v, and v has been reduced to u. The lens centre has moved through $v - u$ cm. This is exactly the distance through which the index of the lens upright has moved. Distance $N_1 N_2 = v + u$, and can be obtained from index readings of N_1 and N_2, *provided index correction is applied.*

From $v - u$ and $v + u$, v and u and hence f can be easily obtained. The method corrects automatically for the thickness of even unusually thick lenses.

EXPERIMENT 76*a*.—**To measure the focal length of a convex lens by the displacement method.**

Apparatus: Convex lens, optical bench, two needles on uprights.

Determine the focal length roughly by distant objects. Place one needle at about 3 times the roughly determined focal length from the lens and by means of a second needle on the opposite side locate the image of the first, so that the needles are conjugate foci. Read the two needle uprights, as also the lens upright. Keeping needles fixed, move the lens towards the needle farther from it, and adjust its position till needles (kept fixed) are again conjugate. Parallax (now judged from the other side) should be absent between the nearer needle and the reduced image of the farther needle. Read lens upright again. *The needles should not have moved.* Remove lens, and obtain index correction for the two needle uprights.

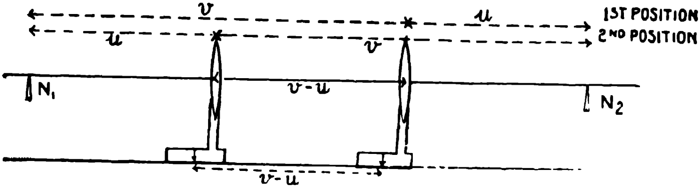


Fig. 111.—Determination of f for a convex lens by the displacement method.

Repeat with needles at different distances from one another, the index correction to be determined only once.

Corrected distance between needles= $u+v=D$ (*say*)
Difference between 2 readings lens upright $= u - v=d$ (*say*)

Arithmetically (regarding u, v, f as $+$)

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v} \text{ or } f=\frac{uv}{u+v}=\frac{\frac{1}{4}[(u+v)^2-(u-v)^2]}{u+v}=\frac{D^2-d^2}{4D}$$

Observations :

Approximate focal length = ...
Index correction between two needle uprights = 20·00—(... — ...)
= ....

Observed $u+v$ (difference two needle uprights)	Corrected $u+v$ D	$u-v$ (difference two lens readings) d	$\frac{D^2-d^2}{4D}$ or f
1 ... - ... = ...	... + ... = ...	... - ... = ...	
2			
3			
4			

Mean f = ...cm.

EXPERIMENT 76*b*.—**To show, for real images formed by a convex lens, that $\frac{\text{Size of image}}{\text{Size of object}}=\frac{v}{u}$.**

Use for object, illuminated glass scale or wire-gauze. Project real image on screen on which a paper millimetre scale is pasted. Read size of as much of the image as is clearly seen. With wire-gauze, transfer by dividers, distance covered by 8 or 10 spaces in the image, to a mm. scale. Similarly measure actual distance on gauze. Measure u and v as usual. Tabulate

$$u,\ v,\ \frac{v}{u},\ i,\ o,\ \frac{i}{o}$$

The experiment is similar to experiment 67*a* for a concave mirror. *Small* index corrections may be ignored.

EXPERIMENT 76*c*.—**Locating the virtual image formed by a convex or concave lens.** This can be done by means of tall needles seen over the lens as with a convex mirror (Expt. 67*b*). For convex lens, use as object a sewing needle, and for locating the image a knitting needle. For concave lens, use knitting needles. In both cases, the two needles are on one side of the lens and the eye on the other. Measure u and v as usual, and calculate f. u and v are both positive, and the formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ can be applied in the ordinary arithmetical sense.

NOTE.—For tables of reciprocals see Appendix.

CHAPTER XXVII

OPTICAL INSTRUMENTS

The simple Convex Lens as Magnifier or Eye-piece. A convex lens will form a magnified, erect, and virtual image of an object held between the lens and its principal focus. The image is formed on the same side as the object, and its position changes considerably if the object is even slightly moved.

EXPERIMENT 77.—**To measure approximately the magnification produced by a magnifying lens.**

Apparatus : Convex lens (focus 2 inches), metre rod, retort stand, etc.

Support the lens horizontally 10 inches (Fig. 112) above the metre rod placed flat on the table. Cut out of card a hole 1 cm. square. Support the card with the hole nearly 2 inches below the lens. Place the eye close to the lens and adjust the card up and down so as to see the virtual image of the hole *easily and distincty.* This will be the case when the image is formed at the *least distance of distinct vision, i.e.*, 10 inches away, where the scale is. If you apply one eye to the lens, and see the scale directly with the other, you can estimate in cm. the distance on the stick, covered by the virtual image of the hole. This is, numerically, the magnification. The magnification equals *numerically:*

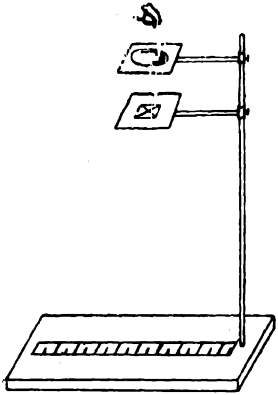


Fig. 112.—Determination of the magnifying power of a reading glass.

$$\frac{\text{Least distance of distinct vision}}{\text{Focal length of lens}} + 1 = \frac{10}{2} + 1 = 6.$$

The Telescope and the Microscope. The single convex lens or eye-piece used in the above experiment as magnifier, can be used to magnify real images formed close to it by another convex lens If the image so magnified is that of a distant object, the combination forms a telescope. If the already magnified real image of a near object is further magnified by the eye-piece, the combination is a compound microscope.

EXPERIMENT 78.—**To make (i) a simple astronomical or Galilean telescope, (ii) a compound microscope.**

(*i*) TELESCOPE.

Apparatus : Object glass (convex lens *f* : 25 to 30 cm.) convex and concave eye pieces (*f* : 2 inches or 5 cm.) candle, white card, etc.

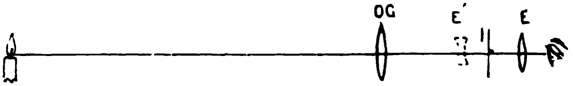


Fig. 113.—The construction of a telescope.

Adjust the object glass OG (Fig. 113) so as to throw a small sharp image I of the candle held 10 ft. away (distant object) on a piece of card.

Place convex eye-piece E *behind* card, and adjust so as to see a dot on the back of the card. The dot, and hence the candle image on the card, are in focus here. Remove the card. The candle will be clearly seen (inverted). Substitute for the candle a metre-rod. The divisions will appear clear and magnified. The combination is an **astronomical telescope.**

Repeat above experiment, placing *in front* of the card with the candle image (between object glass and card) the *concave* eye piece E' at about 2 inches (*f*) from the card. Remove card and look in. The candle will be clearly seen (erect). The combination is a **Galilean telescope (an opera-glass** or "**binoculars**" is a *pair* of such telescopes).

(*ii*) MICROSCOPE :

Apparatus: Object glass (convex lens, *f* : 1 inch), eye-piece (convex lens, *f* : 2 inches), candle, wire-gauze (fine) or glass scale.

Place well lighted *fine* gauze or glass scale O near the object glass OG (Fig. 114) and obtain, at a distance of 6 or 7 inches, a real magnified image I on a card (use a partially darkened room). Focus eye lens EL on a dot on the back of this card Remove card. A highly magnified image of the gauze or glass scale will be seen. The combination is a **compound microscope.**

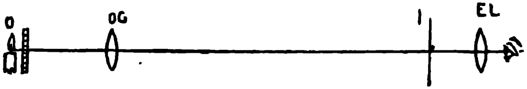


Fig 114.- -The construction of a microscope.

The Optical Lantern. Every physics student should be able to operate a lantern and project slides on a screen. The class should operate a lantern under the supervision of the instructor, and project a slide in the ordinary way. Next remove the condenser and project as before. The image will be poorly lighted, only the middle portions being at all bright. Explain.

EXEPRIMENT 79.—**To construct a simple magic lantern.**

Apparatus : Objective (ordinary convex lens *f* : 10 cm. or 4 inches), condenser (convex lens of large diameter : 2½ to 3 inches, *f* : 10 cm. or 4 inches), glass scale, fish-tail burner or 6 to 8 volt battery lamp as illuminant, white screen.

A small-sized light source is necessary in a lantern. A small battery lamp (used in motor car lights) is ideal. A fish-tail gas burner, used *edge on* so as to present its narrower side to the lenses, is nearly as good.

Place the lamp or burner behind the glass scale (Fig. 115) held horizontally, and project with the object glass (lens of smaller diameter) an image magnified two or three times. It will be noticed that one or two centimeters only of the scale are projected properly. This is because light rays passing through the small portion *a b* alone reach the object glass.

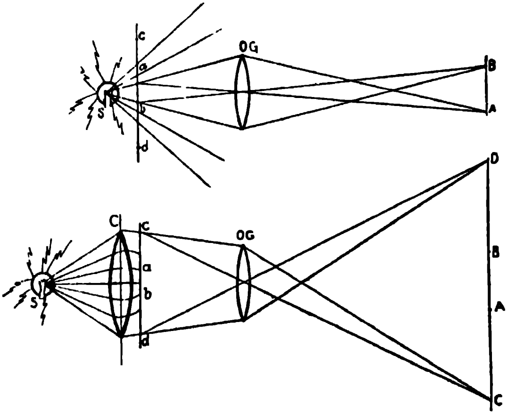


Fig. 115.—The optical or "magic" lantern.

Rays through the other portions *c*, *d*, pass above or below it. Now place immediately *behind* the scale (between it and the lamp), the lens of larger diameter. This serves as a condenser. Move back the light a little, so that the pencil through the condenser may become convergent. A bright and clear image of *c a b d* (as much of the scale as is covered by the condenser, 8 to 10 cm.) will be projected by the object glass. Explain. Consult the diagram.

Record answers to the following :

(1) What is the function of the condenser ?

(2) Has the condenser anything to do with the projection proper, or the magnification, of the image ?

Dispersion. Projection of a Spectrum. To project a **pure spectrum** on a screen, the light from an **illuminated slit** should be made convergent by a **convex lens,** and then passed through a **prism in the position of minimum deviation.**

EXPERIMENT 79*a*.—**To project a pure spectrum on a screen.**

Apparatus : Metal screen with narrow slit 1 cm. long and 1 mm. broad, convex lens 10 cm. focus. Ordinary prism. White screen.

Use a partially darkened room.

Illuminate the slit S (Fig. 116), placed vertically, by a fish-tail gas burner. Project a slightly magnified real image of the slit on the screen by means of the convex lens.

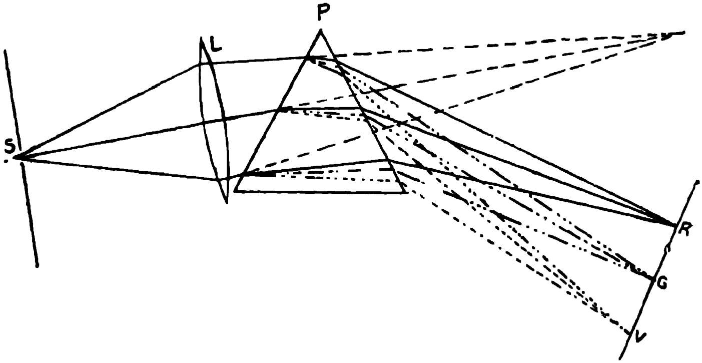


Fig. 116.—The projection of a pure spectrum.

Place close to the lens, in the path of the convergent beam from it, the prism P, and receive on a screen the spectrum RGV. Turn the prism till the deviation is a minimum. Adjust distance of screen from prism to be the same as that of the image formed by the lens without the prism. The single pencil converging to the image is converted by the prism into a number of differently coloured pencils each converging to a separate point image (R,G,V) whose deviation from the colourless image depends on the colour and there fractive index for that colour. The red, green and violet rays from every portion of the prism converge, each to their own focus (R,G,V), along the continuous lines (red), dash lines (green), and dot lines (violet).

[An ordinary glass prism gives only a narrow spectrum on account of the low dispersive power of crown glass. A flint-glass prism (very expensive) gives a much better spectrum].

Record answers to the following:

Q. For which colour is the deviation and the refractive index greatest and for which the least ?

Q. What is the lens for, and how does it improve the spectrum ?

PART VI

ELECTROSTATICS

CHAPTER XXVIII

REPULSION AND ATTRACTION. POTENTIAL

Apparatus. For the experiments described in this section, the following apparatus is necessary.

(1) Brass sheet oven (with one shelf) heated by burner, for keeping apparatus dry during use.

(2) Glass and ebonite rods two each, one rod half-brass half-ebonite, brass ball on insulating handle, silk and flannel rubbers. Stirrup with silk thread support.

(3) Gold-leaf electroscope with metal case and binding screws on disc and case. Small wooden table with three 3″ ebonite legs, or block of ebonite or paraffin, as insulating support. Pieces of chain.

(4) Two proof-planes: brass balls or discs with ebonite handles.

(5) One large conductor on ebonite support.

(6) Two copper calorimeters of different diameters.

(7) Two sheets of metal 15 cm. square, each supported vertically on ebonite sticks fixed to wooden blocks, as condenser. Thick sheet of ebonite or glass.

Ebonite, sealing wax and paraffin should always be preferred to glass where insulation is important. Ebonite and paraffin often get electrified by mere handling. The *only* way to discharge them, as also other insulators, is to pass them through a flame. This should always be done when ***unelectrified*** rods or slabs are required, as in condenser experiments.

Electrostatic Attraction and Repulsion.

EXPERIMENT 80.—**To study electrostatic attraction and repulsion.**

Apparatus: Two glass rods, two ebonite rods or sealing-wax sticks, silk rnd flannel rubbers, stirrup suspended by silk thread.

Suspend a negatively excited ebonite or sealing-wax stick in the stirrup. Have ready another similarly excited sealing-wax stick, as also a glass rod rubbed with silk. Make sure that the glass rod *is* excited by trying it on small bits of paper. Present the two rods in turn to the suspended sealing-wax stick. There will be attraction in one case, and repulsion in the other. As the suspended body is the same in the two cases, note down your conclusion as to the number of kinds of electrification that exist, and the kinds on the sealing-wax and the glass rods.

Present to the suspended rod, a metal plate held in the hand (uncharged body). Also an ***uncharged*** glass or ebonite rod. What do you observe ?

Repeat the above (*i*) with a suspended glass rod (excited), (*ii*) with an uncharged rod (glass or ebonite) in the stirrup.

Tabulate results thus:

Suspended body	Body presented	Result	Conclusion
Excited sealing-wax or ebonite (–)	Excited sealing wax or ebonite (–)	Repulsion	Similar charges repel
	Excited glass (+)	Attraction	Dissimilar charges attract
	Uncharged body	Attraction	A charged body and an uncharged body attract
Excited glass rod (+)	Excited ebonite (–)		
	Excited glass (+)		
	Uncharged body		
Uncharged rod (ebonite, glass or metal)			

Answer the following questions in your book :

(1) What would you conclude if you observed between a charged glass rod and an unknown body

(*a*) repulsion

(*b*) attraction ?

(2) What is the *sure* test of electrification on an unknown body ?

The Gold-leaf Electroscope.

EXPERIMENT 81*a*.—**To study the action of a gold-leaf electroscope.**

Apparatus: Electroscope (gold-leaf). ebonite and glass rods, rod half brass half ebonite.

(1) *To know the meaning of increased and decreased divergence.* Scrape with a knife or sandpaper the top of the electroscope disc so as to expose the bare metal. Flap the brass disc *gently* with flannel, so as to produce a moderate divergence of the leaves (angle 45°) Also excite the brass rod (holding ebonite end only) with flannel. The brass disc of the electroscope and the brass rod must have similar charges. Present the excited rod to the electroscope. What do you notice? How does a similar charge alter the divergence of the leaves. Present to the charged electroscope excited glass and ebonite rods. What do you observe in each case? Which is the *surer test*? What is the sign of the charge on the electroscope? Present uncharged body (metal plate in hand). Can you draw any *definite* conclusion from decreased divergence? Has increased divergence a *definite* meaning?

(2) *To charge an electroscope by conduction.* Excite *very feebly* the brass rod. The charge can be reduced by repeatedly touching the brass rod with a proof plane, discharging the latter after every contact. When the brass rod produces only a *small* divergence of the leaves when held *quite close* to the electroscope. it is charged to the right amount. Touch momentarily the electroscope disc with the brass rod. The electroscope has been charged by *conduction* and the leaves will diverge. Present the brass rod (moderately excited) to the electroscope. Increased divergence will be noticed. What is your conclusion as to the

sign of charge acquired by an electroscope when charged by conduction.

Excited glass or ebonite rods can be used for charging an electroscope by conduction. but the process is unsatisfactory, as the rods (insulators) do not easily part with their charge.

(3) *To charge an electroscope by induction.* Hold an excited rod (ebonite) above the electroscope. The leaves diverge Touch disc with finger. keeping rod unmoved. Leaves collapse Remove finger. *Last of all* remove the rod. The electroscope is charged by *induction*, and the leaves will diverge. Test the sign of the charge by means of glass and ebonite rods, drawing your conclusion only from *increased* divergence. What kind of rod is necessary for charging an electroscope negatively by induction?

Potential. Potential difference is necessary for a flow of electricity from one point to another. Metallic bodies connected by a conductor, ordinarily. soon come to the same potential. A body *touched* or otherwise *connected to the earth* is regarded as at "zero" potential.

EXPERIMENT 81.—**To show that the divergence of the leaves of an electroscope indicates a difference of potential and nothing else.**

Apparatus: Electroscope with a metal case. Ebonite and glass rods. Insulating tripod, or block of paraffin or ebonite.

(1) *Connect case to gas pipe* (Fig. 117, *right*). Give to the leaves, by means of a charged proof plane, a small (*i*) + charge, (*ii*) − charge. Notice divergence in both cases. The potential of the case is zero, and that

of the leaves (*i*) +, (*ii*) −. Keeping your finger on the disc, present excited rods of various kinds. There is no divergence.

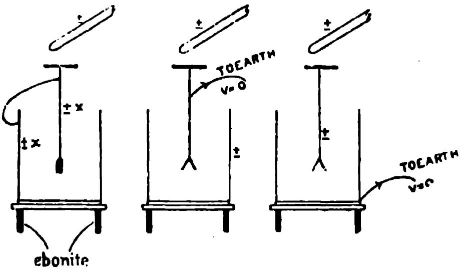


Fig 117.—The electroscope can indicate a difference of potential between the leaves and the case but nothing else.

(2) *Connect leaves to gas pipe, the case being on the insulating tripod or block of paraffin or ebonite* (Fig 117, *middle*). Rapidly pass the blocks through a flame to be sure they are *uncharged*. Charge case by proof-plane, positively or negatively. The leaves diverge. Keeping your finger on case, administer a charge or present excited rods. No divergence.

(3) *Connect the case and the disc by springy wire or easily removable brass chain* (Fig. 117, *left*).) Place the electroscope on the insulating tripod. Impart a charge or present excited rods. No divergence. After a charge is given, remove chain or wire by an ebonite rod, taking care not to let the chain connect the electroscope to the ground. Touch the *disc*. The leaves diverge. Reconnect, charge and disconnect. This time touch the *case*. The leaves diverge.

***Tabulate results thus*:**

	Experiment	Potential of case	Potential of leaves	Result
Case connected to gas pipe.	(1) + charge on leaves.	0	+	divergence
	(2) − charge on leaves.	0	−	divergence
	(3) Finger on leaves, excited rods presented.	0	0	no divergence
Case insulated and leaves connected to gas pipe.	(1) + charge on case	+	0	divergence
	(2) − charge on case	−	0	divergence
	(3) Finger on case, excited rods presented.	0	0	no divergence
Electroscope on insulating tripod or block.	(1) Leaves and case connected, + charge on both.	$+x$	$+x$	no divergence
	(2) Leaves and case connected, − charge on both.	$-y$	$-y$	no divergence
	(3) Case and leaves charged ±, disconnected, *leaves* earthed.	±	0	divergence
	(4) Case and leaves charged ±, disconnected, *case* earthed	0	±	divergence

Study the above table and *record* answers to the following questions :

(1) Do you *necessarily* observe divergence when there is a charge on the leaves ?

(2) Do you *necessarily* observe divergence when there is a charge on the case ?

(3) Do you *necessarily* observe collapse when the leaves are *not* given a charge ?

(4) What is in your opinion *the only necessary condition* for the divergence of the leaves ?

CHAPTER XXIX

INDUCTION

SEAT OF CHARGE. SIMULTANEOUS GENERATION OF + AND − CHARGES. ELECTROPHORUS

Seat of Charge.

EXPERIMENT 82.—**To show that the charge resides on the outside of a conductor.**

Apparatus: Electroscope, calorimeter, brass ball. insulating tripod, etc.

Place the copper or brass calorimeter on the insulating support, and charge it by an electrophorus or by flapping it with flannel. Lower into the calorimeter a brass ball, attached to a silk thread till it touches the sides or bottom, and withdraw without touching the rim of the calorimeter. Take the ball to an electroscope Repeat, testing this time the outside of the calorimeter. ·Where does the charge reside ?

A Positive Charge is always accompanied by an Equal Negative Charge. This can be easily verified experimentally.

EXPERIMENT 83*a*.—**To prove that friction generates simultaneously equal amounts of + and − electricity.**

Apparatus: Flannel cap with silk thread. Ebonite rod. Electroscope.

Prepare a cylindrical flannel cap about 5 to 8 cm. long to fit loosely the end of an ebonite rod. Attach a long silk thread to the top of the cap. Wrap the thread round the cap fitted on to the rod. Pull thread. The cap will rotate on the rod. Present rod and cap in position to an electroscope. No divergence.

Pull apart cap and rod; *each* one will cause divergence. Test the cap and rod for sign of charge. They will be found *oppositely* charged.

As the cap and rod *together* produce no divergence, what is your opinion as to the *amounts* of each kind produced?

Equal Amounts of + and — Charges by Induction.

EXPERIMENT 83*b*.—**To prove that induction generates simultaneously equal + and - charges.**

Apparatus: Large conductor on insulating stand. Two small balls or proof-planes with insulating handles. Electroscope.

Charge strongly, by flapping or by means of an electrophorus, the large conductor A on insulating support.

Determine the sign of its charge. Hold near A two proof-planes in contact as shown (Fig. 118) For the time being. the two proof planes become a single conductor. Separate the proof-planes and *afterwards* remove them from the neighbourhood of A. Test the charges on the two proof-planes for sign by means of an electroscope. Repeat the above experiment, but after showing the existence of opposite charges on the two planes, let them touch one another, and test both with an uncharged electroscope. The proof planes will be found discharged.

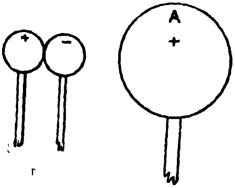


Fig 118. Simultaneous generation (by induction) of equal positive and negative charges.

Record answers to the following:

(1) What is the sign of the charge induced on (*i*) the *nearer* portion of a conductor, (*ii*) the *farther portion*?

(2) Are the quantities equal or unequal?

Ice-pail Experiments.

EXPERIMENT 83.—**To show that the maximum induced charge equals the inducing charge.**

Apparatus: Calorimeter on insulating tripod. Ball and silk thread. Electroscope.

Place on the insulating stool a large calorimeter or tin can connected to the disc of the electroscope. Slowly lower into the can, without touching, a charged metal ball held by a *dry* silk thread. The leaves will diverge, and the divergence come to a maximum when the ball is well within the can, though not touching. Lower the ball further. The divergence will be unchanged. Finally let the ball touch the can, and observe if there is any increase in the divergence. Withdraw the ball, and test it for any residual charge, after discharging the electroscope. The ball will be found to have lost *all* of its charge.

Record answers to the following:

(1) Suppose the ball had +10 units to start with. *Which* induced charge, do you think, has neutralized these + 10 units? What must be the *amount* of this charge, and *where* does it reside?

(2) What must be the *amount* and *sign* of the other *simultaneously induced charge*, and *where* does it reside?

(3) How has your experiment shown that these amounts are the *maximum possible?*

(4) Comparing the *initial* charge on the ball with the *final* charge on the can and electroscope, can you not say that the ball has merely given up the *whole* of its charge to the can, etc.?

(5) What conclusion do you draw from (4) regarding the *seat* of charge?

Electrophorus.

EXPERIMENT 83*c*.—**To study the action of the electrophorus.**

Apparatus: Electrophorus. Catskin. Electroscope.

Excite the vulcanite cake by flapping it with catskin or flannel. Test the sign of the charge. Place the cover (disc with insulating handle) on the cake. Touch. The repelled charge will pass to the earth. Remove finger. Lift the cover. Test sign of charge carefully, keeping at a distance from electroscope. The cover will be found charged oppositely to the cake. Discharge cover and repeat. If the weather is favourable, the experiment can be repeated dozens of times, a spark being obtained each time.

Record answers to the following :

(1) If the weather were perfect, do you think you could get an unlimited amount of electricity from a single charge ?

(2) As each charge means energy (the spark produces heat, light and sound) can you get an unlimited amount of energy from the first charge ?

(3) Where does so much energy come from ?

CHAPTER XXIX—*contd.*

INDUCTION

CONDENSERS. DIELECTRIC CAPACITY

Condensers. Capacity. The capacity of a conductor or a condenser is $\mathbf{C} = \frac{\mathbf{Q}}{\mathbf{V}}$ where **Q** is the charge and **V** the potential difference caused. **Capacity is, therefore, the charge necessary for unit rise of potential.**

EXPERIMENT 84*a*.—**To study the effect of an earthed conductor on the potential and capacity of neighbouring conductors.**

Apparatus: Electroscope, insulating tripod, two different-sized calorimeters.

Support a calorimeter (Fig. 119) on the insulating tripod as shown. Connect to electroscope and charge by means of electrophorus or otherwise.

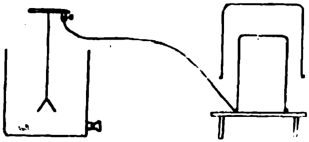


Fig. 119.—The Leyden jar principle.

Notice the *amount* of divergence. This is a measure of the potential of the calorimeter. Bring round the calorimeter, without touching it, a larger calorimeter held in the hand. The divergence (hence potential **V**) will fall. As **Q** is unchanged and **V** decreases, how does **C** change?

The two calorimeters illustrate the Leyden jar principle. The presence of an earthed conductor near an insulated conductor *increases* the capacity of the latter.

Condenser. Capacity (*contd.*). Dielectric Capacity.

EXPERIMENT 84.—**To study the action of a plate condenser.**

Apparatus: Two brass sheets 15 cm. × 15 cm. with rounded edges and corners, supported vertically on ebonite or sealing-wax sticks (Fig. 120) on wooden bases. Ebonite rod, etc.

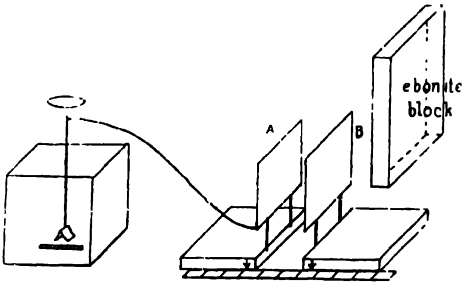


Fig. 120.—Studying the action of a plate condenser.

Connect plate A to electroscope. Remove B to a distance. Charge A. Notice the divergence. Bring B (fingered, or earthed by connection to gas tap) near A. The potential of A falls. When B is quite close, the potential may fall considerably. The presence of B increases the capacity of A. The change in capacity can be roughly measured.

Remove B to a distance and discharge A. Charge a proof-plane from the electrophorus disc or electric machine, and allow the charged proof-plane to touch A. Let the charge Q so given to A be called a *unit*. Arrange the size of the proof-plane or the speed of the machine, so that a *single* unit produces a moderate divergence. Bring B quite close to A, without touching. A sheet of mica or paraffined paper may be interposed. The divergence will decrease Give to A unit charges by means of the proof-plane till the divergence is at its original value. If 3 more units are required, the total charge is $4Q$ and the increased capacity $= \frac{4Q}{V} = 4 \times \frac{Q}{V} = 4$ *times the capacity of* A *without* B.

Set A and B so that the leaves are considerably diverged.

Interpose between A and B a dry, but *uncharged* (pass through a flame) *thick* sheet of glass or ebonite about 20 cm. × 20 cm. The divergence will decrease and the capacity increase This is because glass and ebonite have greater **dielectric capacity** than air.

Measurement of Dielectric Capacity. An apporoximate measurement of the dielectric capacity of an insulator available in the form of a *thick* slab can be easily carried out. Liquids can be used in large flat battery jars of glass.

EXPERIMENT 84*b*.—**To measure the dielectric capacity of kerosene.**

Apparatus: Large rectangular battery jar, kerosene, condenser, electroscope.

Fill the glass jar with kerosene. Smear the outside as well, to improve the insulation of the glass surface. Interpose this jar between the plates A and B which should be close to (but, preferably, not touching) the sides of the jar Charge A (B being earthed) so as to produce a small divergence (less than 30°) of the leaves Measure this divergence on the scale under the leaves, or estimate it

Read the position of B on the scale. Slide out the battery jar carefully from between the plates The divergence will increase. Now push B towards A till divergence is the same as at first. Measure the distance x through which B has moved

If d is the initial distance between the plates (need not be measured) and t the thickness of the insulator, we can say that the initial

t cm. dielectric + $d - t$ cm. of air = the final $d - x$ cm. of air

$\therefore$ t cm. dielectric equals $(d - x) - (d - t)$ or $t - x$ cm. of air.

$$\therefore \text{Dielectric capacity} = \frac{t}{t - x}$$

A value of 6 to 8 will be obtained. This will be an overall **value** for the kerosene and the glass sides of the jar. A *thick* **block** of paraffin-wax (pass through a flame previous to interposing) **can be used.**

> Q. - A thick disc of ebonite, if introduced between the plates A and B in the above experiment, reduces the divergence of the leaves, but if introduced between an electroscope and an excited rod above it, increases the divergence Explain.

The Action of Points. Lightning Conductor.

The *density* of the induced charge **on a point** held near a charged conductor is so great, that air particles near the point get strongly charged, and are then repelled. This stream of charged particles can completely neutralize the opposite inducing charge. The ordinary **lightning conductor** is an application of this principle. It protects the building on which it is installed, by *continuously* neutralizing the charge on clouds overhead, **so that the difference of potential never becomes high enough for a direct flash**

EXPERIMENT 85—**To study the action of the ordinary lightning conductor.**

Apparatus: Disc with insulating handle, ball on stand, electric machine. Sewing needles, etc.

Support the disc (electrophorus disc is excellent) horizontally to represent the *cloud* (Fig. 121). Underneath, with a gap of 1″ to ½″, arrange a metal ball on stand to represent the *building*. Earth this by connection with the gas pipe. Connect one pole of a Wimshurst machine to the disc. Earth the other pole. On turning the machine, the potential of

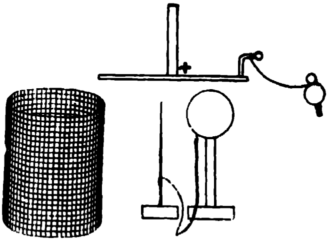


Fig. 121.—The principle of lightning conductors.

the "*cloud*" (assumed +) will steadily rise and a spark finally pass between it and the "*building*" This will be repeated at intervals if the machine is kept in action.

Now erect near the ball a sharply pointed clean new (not rusty) sewing-needle, tied to a knitting-needle on a cork base. Earth the knitting-needle by connection to gaspipe. Turn the machine continuously; no sparks will pass. Remove the earth connection between the "*lightning conductor*" and the gas pipe, placing the cork on paraffin or ebonite to make the earth connection thoroughly "faulty." Turn the machine. .The "lightning" will "strike" again, the faulty earth connection rendering the lightning conductor useless.

Improved Lightning Conductor. A more dependable protection against lightning is secured by encasing the building completely in a net-work of wires forming a cage round it.

EXPERIMENT 85*a*.—**Study of improved lightning conductor.**

Apparatus: Same as in Expt. 85. Wire-gauze cylinder. Electroscope.

Substitute for the metal sphere in experiment 85 an electroscope placed inside a cylinder of wire-gauze with a wire-gauze cover. The leaves will be unaffected even when the "lightning" actually strikes the cage The cage in this experiment need not be earthed. In the case of an actual building on the ground, the cage has also to be grounded, to avoid mutual differences of potential. This form of conductor will protect a building even when, due to special atmospheric conditions, the ordinary lightning conductor with a good earth connection is of no service

PART VII

MAGNETISM

CHAPTER XXX

MAGNETIC MOMENT. MAGNETIC FIELDS UNIFORM AND NON-UNIFORM

Unit Magnetic Pole. A magnetic point charge that repels an equal magnetic point charge held 1 cm. away from it in air with a force of 1 dyne is called a **unit magnetic charge** or **unit pole**.

Pole Strength. The magnetic charge on a pole measured in terms of the "unit magnetic pole" is the **pole strength** of that pole. A magnetic charge of m magnetic units on a pole is the equivalent of pole strength m.

Magnetic Moment. The product (mL) of m and L, where L is the distance between the two poles of a bar magnet and m the pole strength, is a very commonly occurring quantity, and is called the "magnetic moment" (**M**) of the bar magnet.

The Fundamental Formula for Magnetic Attraction and Repulsion. The fundamental law governing magnetic attractions and repulsions can be put in the form of a formula: $f = \frac{m_1 m_2}{d^2}$. that is. **the force between two poles (in air) equals the product of their strengths divided by the square of the distance between them.**

Magnetic Field. Strength of Field. Round a magnet there is a region of space in which magnetic attractions and repulsions can be observed. This region is called a "**magnetic field.**" The **strength of the field at any point** is the force (in dynes) exerted on a *unit* pole placed at that point. The strength is h, if a unit pole placed at the point is repelled or attracted with a force of *h* dynes, or a pole of *m* units with a force of *mh* dynes.

Uniform and Non-uniform Fields. If the force on a unit pole placed at any point within a certain region is the same in magnitude and direction, the field within that region is called a **uniform field.** **The field due to a magnet** is evidently **non-uniform**, as it is much stronger near the poles than at other points. *In regions far away from the magnet, or for small regions anywhere, the field may be considered practically uniform.*

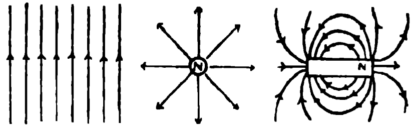


Fig. 122 —*Left*, straight and uniform magnetic field *Middle*, non-uniform radial field due to single pole. *Right*, non-uniform curved field due to bar magnet.

Lines of Force. In a magnetic field, lines can be drawn so that they coincide in direction at every point with the direction of the field at that point. Such lines are called **lines of force**. A small magnetic pole, if free to move in a field, will travel along a line of force, and a small freely suspended magnetic needle will place itself along the line of force through the point where the needle lies. **The lines of force for a uniform field are straight and parallel** (Fig. 122 *left*), because straight and parallel lines alone can represent a field of constant direction. The lines of force for *non-uniform* fields are either *radial straight lines* (Fig. 122 *middle*), or *curves* (Fig. 122 *right*) arranged in various ways.

The Compass Needle and Iron Filings as Field Indicators. Imagine a small magnetic needle, pivoted at the centre, lying at any inclination θ (Fig. 123 *dotted lines*) to the lines of force of a field, whose strength in the neighbourhood of the needle is h. If the needle is of pole strength m, the forces on the two poles will be equal and opposite in direction (mh and $-mh$), but not coincident in line of actoin. They will, therefore, form a couple, and the needle will rotate till it occupies a position parallel to the field. In this position (indicated by full lines) the forces

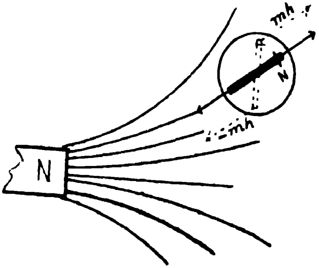


Fig. 123.—Compass needle as field indicator.

mh and $-mh$ neutralize one another and the needle will be at rest. The direction of the needle indicates the direction of the field. Similarly, fine iron filings sprinkled in the field, become small magnets by induction, and lie with their lengths along the lines of force. A single compass needle indicates the field only where it is, but a gramme of iron filings will picture a whole field at once, by forming everywhere along the lines of force continuous chains which are easily seen. Filings are, however, unaffected by *weak* fields; consequently, the compass needle method alone is of service in mapping fields of magnets at a considerable distance from the poles.

CHAPTER XXXI

THE FIELD OF THE EARTH. THE FIELD OF A BAR MAGNET. MAPPING OF FIELDS.

The Gravitational Field of the Earth. The student is familiar with the gravitational field of the earth. It is in virtue of this field that **a pendulum remains vertical** or **oscillates round the vertical line.** The **strength** of this field for a whole city may be the same, and is denoted by the value of 'g' for the place. The field is, therefore, a uniform field of strength 'g', the force on the unit of mass (1 gramme) being 'g' dynes and the force on m grammes mg dynes. The field may be represented geometrically (Fig. 124) by parallel vertical lines. *A pendulum, or a suitably pivoted bar, will either remain vertical, or if displaced, vibrate round the vertical, which position it will ultimately assume.*

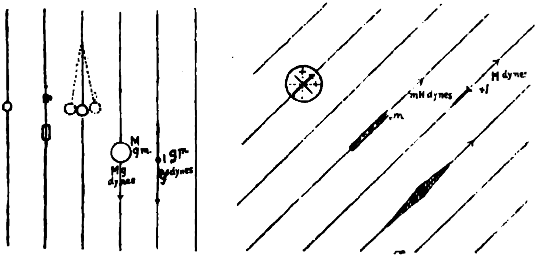


Fig. 124 —The Gravitational and Magnetic fields of the earth. While differing in nature and direction, their geometry is the same.

The Magnetic Field of the Earth. Very similar to the **Gravitational Field** is the **Magnetic Field** of the earth. Whether we like it or not, they are both always present. All our experiments are affected by them. The magnetic field of the earth may be sensibly uniform for a whole city, if much of iron or iron bearing rocks is not present. It is in virtue of this field that a **magnetic needle points** (if free) **approximately north and south,** or whatever direction the **magnetic meridian** at the place may have. If displaced from this position, it will **oscillate** about it **like a pendulum,** and ultimately come to rest in the meridian. If the strength of the magnetic field is H, the force on a unit pole is H dynes, and that on a pole of m units mH dynes. The formulæ for the times of vibration are also exactly analogous (see chap. 33). The only important difference (besides nature) between the gravitational and magnetic fields is in their direction The former is always vertical, while the latter may have any direction. Experiments are generally so arranged that the **horizontal component (H) of the earth's magnetic field** alone operates, the vertical component being neutralized by the reactions of suitably arranged pivots or axes.

An Important Practical Application of the Earth's Magnetic Field. The magnetic field of the earth has been used since the earliest times for determining directions, steering ships, etc. The following simple experiment illustrates the principle of a ship's compass.

EXPERIMENT 86*a*.—**To prepare a simple ship's compass, and to study the principle of steering.**

Apparatus : 2″ or 3″ compass needle on pivot, white card, pins.

Cut out of stiff paper or cardboard, a boat-shaped piece about 10″ long and 4″ wide. Draw a longitudinal line (Fig. 125) through the centre, from end to end, to represent the keel.

Next cut out of white card a circular piece of diameter equal to that of the needle. Draw four diameters at intervals of 45°, and mark the ends as the principal points N, S, E, W, NW, SW, SE, NE. Cut out of the centre of the card a hole for the cap of the needle, and stick the card with gum or seccotine to the needle, so that the North Pole is exactly under N. Now place the compass so prepared centrally on the boat-shaped piece of cardboard. Stick a tall *brass* (not iron) pin P on the forward half of the keel, close to the compass card. Rotate the "ship" till the pin is opposite N. In this position "the ship" is steaming northward. To make for the north-west, turn ship till pin is opposite NW, and so on.

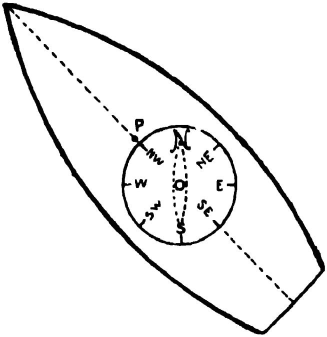


Fig. 125- Ship's compass illustrated. In the position shown the vessel is making for North-West.

Mapping of Magnetic Fields. Magnetic fields are best mapped by means of a *small* compass needle pivoted in a small brass box ($\frac{1}{2}$ inch diameter) with glass sides. The black or grey steel bar is the "needle"; the brass cross-bar (often provided) is used for mapping equipotential lines.

EXPERIMENT 86.—**To map with a compass needle (i) the earth's horizontal magnetic field, (ii) the compound field due to a bar magnet and the earth.**

Apparatus: Small compass in brass box with glass sides, drawing board, 3″ bar magnet, large sheet of paper.

The drawing board should be free from iron nails. Nails or screws, if used, should be of brass. Use stout brass pins (ordinary) for fixing the paper to the board instead of steel-pointed drawing pins. A hidden nail or screw (iron) often causes great confusion by distorting the lines of force near it.

(i) *To map the earth's field :* Fix the paper on the board. Draw a pencil line midway. Use one side for (*i*) reserving the other for (*ii*).

Remove the magnet as also any iron keys, knives or chains you may have to a distance.

Place the compass near the bottom corner, and turn the board round till the pencil line already drawn is roughly parallel to the needle. This is for convenience only. Now mark the position of the ends of the needle by small dots. Slide the compass upward (Fig. 126 *left*) till its lower end, as seen through the glass, lies exactly on the dot where the upper end at first was. Make a third dot opposite the upper end. Slide upward again, adjust as before, and make a 4th dot. Repeat till you have reached the upper end of the paper.

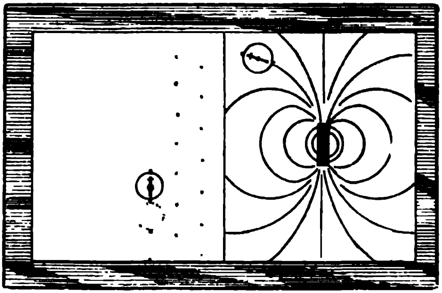


Fig. 126.—Mapping fields. *Left*, straight uniform field due to the earth. *Right*, compound field due to earth and bar magnet.

Connect all these dots. They will be found to be on a straight line. Indicate by an arrow the direction in which a N-pole placed on the '*line of force*' will tend to move. Start again from the bottom, but an inch or two to one side of the first starting position. Obtain a second '*line of force*.' Four or five such lines are enough. They should be all straight and parallel.

(*ii*) *To map the compound field due to the earth and a bar magnet.*

Place the 3″ magnet in the centre of the other half of the paper with its axis *parallel* to the earth's field as obtained in (*i*). Place the compass near one pole, and trace out a line of force up to the edge of the paper, or till you reach the other pole of the magnet. Next start from another point half a cm. away from the first and map out another line. Obtain in this way a number of lines till the map resembles Fig. 126, *right*. Put on arrows as before.

The portion of this field quite near the magnet is very similar to that given by the magnet alone in the absence of the earth's field. The portion at 10 to 20 cm. from the magnet will be considerably different from that given by the magnet or the earth alone. Portions still further removed (where the effect of the magnet is inappreciable) will be similar to the earth's field obtained in (*i*).

The above field has been obtained with the bar magnet in the meridian. In any other position an asymmetrical map will result. Fig. 133 shows the resultant field for a magnet placed approximately East and West or ⊥ to the meridian.

EXPERIMENT 86*b*.—**To map fields by iron filings.**

Apparatus: Two bar magnets, horse-shoe magnet, sheet of glass about 8″×10″. Iron filings in a muslin bag.

Place the horse-shoe magnet on the table or board. Place the glass sheet with a sheet of paper on it on the magnet. Sprinkle iron filings from the bag uniformly on the paper. Tap the glass sheet gently. The iron filings are temporary magnets due to the inductive action of the field, and each one serves as a compass, lying with its axis along the line of force through it. The filing chains along the lines of force are easily seen (Fig. 127).

If paraffined paper is employed, the chains can be fixed by melting the wax by means of a Bunsen flame passed momentarily along the surface. The filings sink into the liquid wax, and are permanently embedded on cooling.

Obtain chains for (1) single bar magnet, (2) two bar magnets side by sides 3 cm. apart, with like poles together, (3) same as (2) but unlike poles together, (4) two magnets in the same line North pole of one and South pole of the other 4 or 5 cm. apart, (5) same as (4) but with like poles 4 or 5 cm. apart.

Permanent records can also be obtained by exposing to sunlight, filing maps arranged on photographer's P.O.P. or engineer's ferrotype paper, shaking off the filings and "fixing" the image. The ferrotype method is specially convenient, as a simple washing in plain water is sufficient to fix the image.

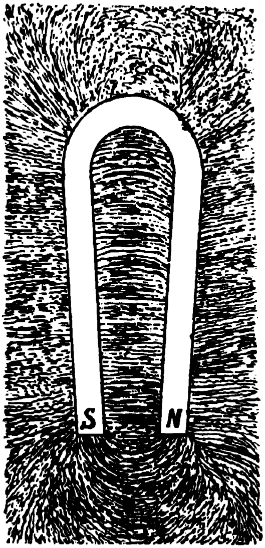


Fig. 127.–Magnetic field of a horseshoe magnet, mapped by iron filings.

NEUTRAL POINTS OF MAGNETIC FIELDS, MEASUREMENT OF POLE STRENGTH AND MAGNETIC MOMENT BY NEUTRAL POINTS

Neutral Points, The field round a magnet is compounded of a *uniform* field H due to the earth, and a *non-uniform* field due to the magnet. At certain points, the field due to the magnet alone may equal $-H$, *i.e.* be exactly equal and opposite to the earth's field. *At such points the resultant field is zero,* ***and*** **the compass needle does not oscillate, but lies indifferently in any meridian.** These points are called **Neutral Points.** No line of force can pass through a neutral point. Lines in the neighbourhood curve round so as to leave an empty space round it. This **neutral region** (Figs. 129—133) can be made fairly small and its centre (when small) can be taken as a neutral point.

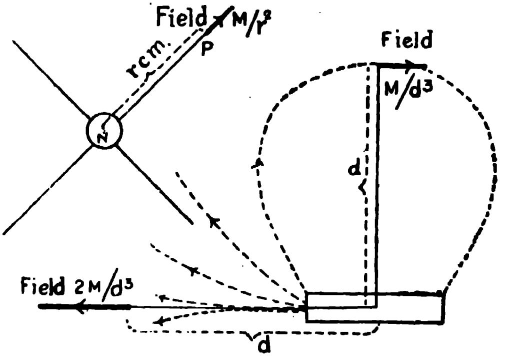


Fig 128.—Field due to single pole, and bar magnet at points on the equatorial line, and the axis produced

Field due to a Bar Magnet at Points on the Equator or the Axis produced. Before carrying out measurements with neutral points, it is convenient to know a few facts about fields due to poles and magnets.

Single pole: The field due to a single pole of strength m at a distance r cm. from the pole is (Fig. 128) $\frac{m}{r^2}$.

Bar magnet, points on axis produced: The field due to a bar magnet (of length $L=2l$, and pole strength m) is evidently the resultant of the fields due to the two poles taken individually. For a point on the axis produced at a distance d from the centre of the magnet (Fig 128), the forces due to the two poles are, respectively, $\frac{m}{(d-l)^2}$ and $-\frac{m}{(d+l)^2}$.

The resultant $\frac{m}{(d-l)^2}-\frac{m}{(d+l)^2}$ equals $\frac{4mld}{(d^2-l^2)^2}$ or $\frac{2Md}{(d^2-l^2)^2}$ which, for a small magnet (l negligible compared with d), becomes $\frac{2M}{d^3}$ where **M** is the magnetic moment of the magnet.

Bar magnet, points on the equator: For a point on the equator at a distance d from the centre the field can be taken to be $\frac{M}{(d^2+l^2)^{\frac{3}{2}}}$ or for a small magnet $\frac{M}{d^3}$.

Measurement of Pole Strength or Magnetic Moment by Neutral Points. At a neutral point, the earth's field H=field due to pole or magnet. The field due to a pole is easily calculated. In the case of a bar magnet, if the neutral points lie on the equator or the axis produced, the field and hence the magnetic moment can be easily calculated. The triangle of forces can be applied in other cases, no matter what the position of the neutral points.

EXPERIMENT 87.—**To determine the pole strength or magnetic moment of a magnet by means of (*i*) the neutral point due to a single pole, (*ii*) neutral points on the equator or axis produced of a bar magnet.**

Apparatus :—Magnetised knitting needle or ball-ended magnet, ordinary bar magnets, small compass, drawing board (without iron nails, etc.)

(*i*) *By neutral point due to single pole*: This method can be satisfactorily applied if the magnet is *long* or *weak* (knitting needle or ball-ended magnet) that is, when the distance of the neutral point from the pole is small as compared with the length of the magnet.

Remove all iron and extra magnets to a distance. Along the middle of a sheet of paper fixed on the board, trace with the compass, and mark with an arrow, the direction of the earth's field. Erect the *weak* magnet (preferably cylindrical) vertically with one pole on the centre of the line so traced. Use a *wooden* clamp for supporting the magnet. *Think out* the side on which the neutral point will be. To do so, mark small arrows all round the pole to indicate the direction of its lines of force. The arrow exactly opposite in direction to the arrow on the line marking the earth's field denotes the side of the pole on which the neutral point will lie.

Start from different sides of the pole, and trace, with the compass, lines of force up to a short distance only, except on the side on which the neutral point is expected. To get the lines beyond the neutral point, start from suitable points near the edge of the field, *not from the pole.*

Make the neutral region as small as you can by filling up with portion of suitable lines of force. The

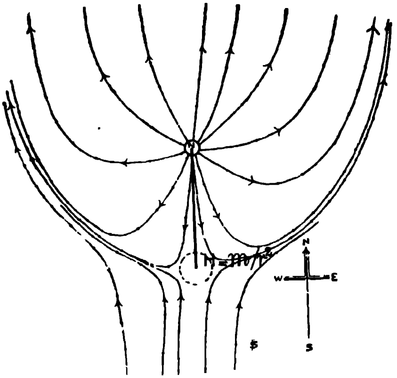


Fig. 129.—Neutral point and field for a single North-pole. $H = m/r^2$ *or* $m = r^2 H$. *(Reproduced from actual map by a student.)*

centre of this region is the neutral point. Measure the distance of this point from the centre of the pole. (marked with pencil passed all round). If this distance is r.

$$H = \frac{m}{r^2} \text{ or } m = r^2 H$$
$$= {\cdot}32r^2 \; (H = {\cdot}32.)$$

Repeat experiment with the other pole. Calculate mean m. Measure length of magnet L. $M = mL$.

[If the distance of the neutral point from the pole is not large the force due to the upper pole is nearly vertical, and hence inoperative.]

(*ii*) *By neutral points on the equator or the axis produced* Trace the direction (one line only) of the earth's field carefully with the compass as before. Place the magnet with its edge along this direction. The position of the neutral points depends on *which* pole is pointing north. Indicate by arrows the direction of the field due to the magnet at points on the equatorial and axial lines. *Think*

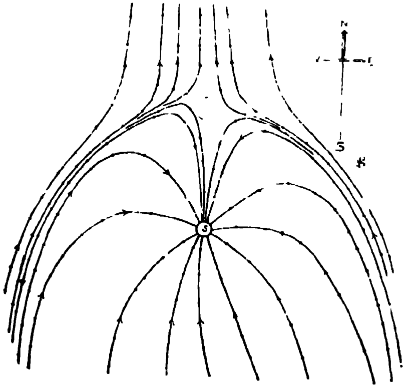


Fig. 130.—Neutral point and field due to a single South-pole. *(Reproduced from actual map by a student.)*

out, as in (*i*), where the neutral points can be expected. Trace out the lines of force as shown in Figs. 131 and 132.

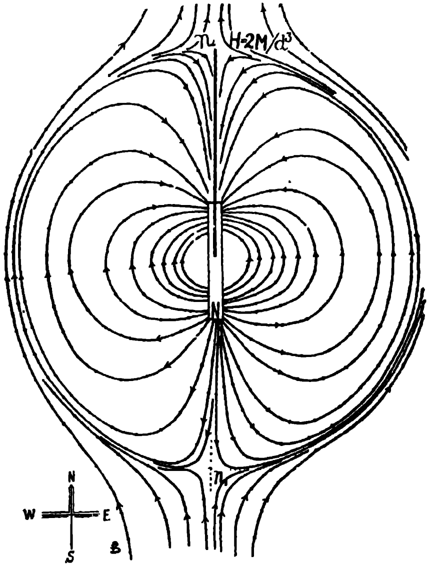


Fig. 131.—Neutral points and field for a magnet in the meridian, North-pole pointing south. (*Reproduced from actual map by a student.*)

As this requires considerable time, the two neutral regions and the field round only one of them may be carefully mapped, a few lines sufficing for the rest of the field

For a neutral point on the axis produced, measure D_1 and D_2, distances from the two poles.

$$H = \cdot 32 = \frac{m}{D_1^2} - \frac{m}{D_2^2} \text{ (see page 331)}$$

$$\therefore m = \frac{\cdot 32}{\frac{1}{D_1^2} - \frac{1}{D_2^2}}$$

Determine m from the second neutral point as well. Calculate the mean. Measure L, length of magnet. $M=mL$.

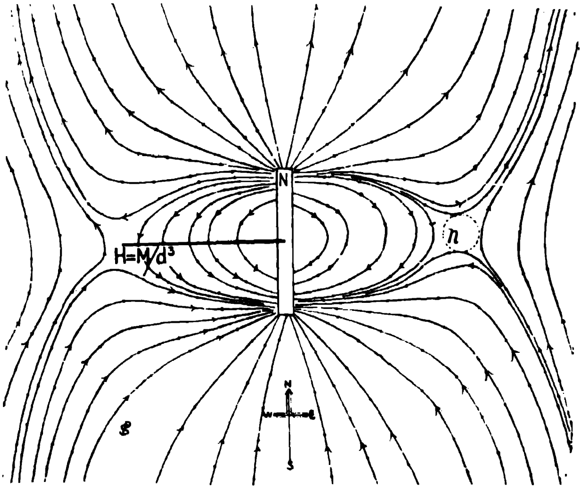


Fig. 132.—Neutral points and field for a bar magnet in the meridian, North pole pointing north. *(Reproduced from actual map by a student.)*

For the two neutral points the equatorial line, measure D_1, D_2, D_3, D_4, distances from the two poles. Let the mean of these be D. This equals $(d^2+l^2)^{\frac{1}{2}}$ and as for this case

$$H = \frac{M}{(d^2+l^2)^{\frac{3}{2}}} = \frac{M}{D^3} = \cdot 32 \text{ (see page 331)}$$

we have $M = \cdot 32 D^3$.

Enter observations thus :

Length of magnet $L = \ldots\ldots$cm.

POSITION OF MAGNET	NUMBER AND POSITION OF NEUTRAL POINTS	DISTANCES OF NEUTRAL POINTS FROM POLES OF MAGNET		POLE STRENGTH m		MAGNETIC MOMENT $M = mL$
Vertical, N pole on paper Vertical, S pole on paper	one south of pole one north of pole	$d_1\ldots\ldots$ $d_2\ldots\ldots$	Mean : $\ldots\ldots(d)$	$\cdot 32d^2$ $= \ldots\ldots\ldots$	$\log d = \ldots\ldots$ $\log d = \ldots\ldots$ $\log d^2 = \ldots\ldots$ $\log \cdot 32 = \ldots\ldots$	$\ldots\ldots$
Parallel to meridian. S pole pointing north	Two on axis produced	$D_1 = \ldots\ldots$ $D_2 = \ldots\ldots$ $D_1 = \ldots\ldots$ $D_2 = \ldots\ldots$	For neutral point N_1 For neutral point N_2	$\dfrac{\cdot 32}{\dfrac{1}{D_1} - \dfrac{1}{D_2^2}} = \ldots\ldots$ $\dfrac{\cdot 32}{\dfrac{1}{D_1^2} - \dfrac{1}{D_2^2}} = \ldots\ldots$	Mean m : $\ldots\ldots$	$\ldots\ldots$
Parallel to meridian, N pole pointing north	Two on equatorial line	$D_1 = \ldots\ldots$ $D_2 = \ldots\ldots$ $D_3 = \ldots\ldots$ $D_4 = \ldots\ldots$	Mean $= D =$			$\cdot 32\ D^3$ $= \ldots\ldots$ $3 \log D$ $= 3 \times \ldots$ $= \ldots\ldots$ etc.

EXPERIMENT 87*a*.—To determine m or M for a bar magnet by measurements on neutral points in any position.

Apparatus: As for expt. 87.

Trace carefully down the middle of the sheet one or two lines of force due to the earth alone. Place the bar magnet so as to cross these lines at any angle whatever. 70° to 80° is suitable. *Think out* as before the regions of the field where the neutral points are expected. These can also be found roughly, by watching the oscillations of the compass. *Near the neutral points, the oscillations are very slow or altogether absent.* Carefully map out the two neutral regions, and mark the centres as the neutral points N_1, N_2 (Fig. 133)

Join N_1, one of the neutral points, to the two poles Draw any line AB parallel to the meridian, so as to complete the triangle N_1 AB Mark on AN_1, N_1B and BA, the directions of the three forces acting at N_1, *two* due to the two poles, and the *third* due to the earth. These are in equilibrium.

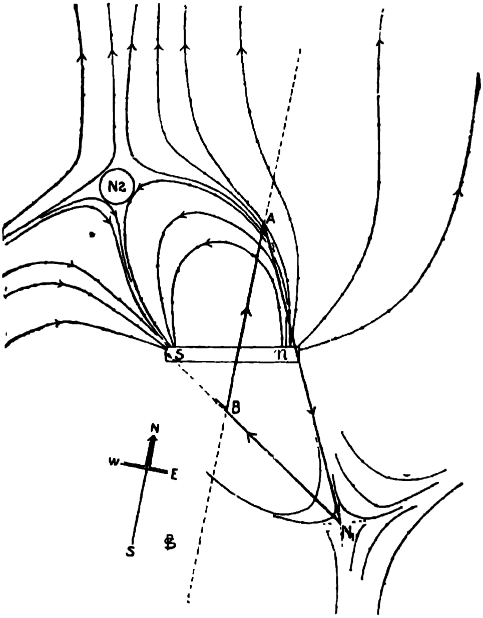


Fig 133.—Neutral points and field for a bar magnet in any position. *(Reproduced from actual map by a student)*

Measure the three sides of the triangle N_1AB.

Measure also distances of N_1 from the poles of the magnet. If these are d_1 and d_2, the forces due to the poles are $\frac{m}{d_1^2}$ and $\frac{m}{d_2^2}$. The third force is $H = \cdot 32$. By the triangle of forces.

$$\cdot 32 : \frac{m}{d_1^2} : \frac{m}{d_2^2} = BA : AN_1 : N_1B.$$

$$\text{or} \quad \frac{\cdot 32}{BA} = \frac{m/d_1^2}{AN_1} = \frac{m/d_2^2}{N_1B}$$

$$\therefore \quad m = \cdot 32 d_1^2 \times \frac{AN_1}{BA} = \ldots\ldots$$

$$\text{also} \qquad m = \cdot 32 d_2^2 \times \frac{N_1B}{BA} = \ldots\ldots$$

The two values should be nearly equal. If time permits, obtain another pair of values from the second neutral point. Calculate mean m, and determine M as usual.

Observations :

Length of magnet = ...: $H = \cdot 32$.

Distance of neutral points from poles d_1, d_2	Forces at the neutral point f_1, f_2, f_3	Corresponding sides of △ a, b, c.	Pole strength m	Magnetic moment M
N_1 ......(d_1)	$\frac{m}{d_1^2}$	......(a)	$\cdot 32 d_1^2 \times \frac{a}{c} = \ldots\ldots$	
......(d_2)	$\frac{m}{d_2^2}$	......(b)	$\cdot 32 d_2^2 \times \frac{b}{c} = \ldots\ldots$	$M = mL$
	$H = \cdot 32$	... ..(c)		$= \ldots \times \ldots$
N_2 ... ..(d_1)	$\frac{m}{d_1^2}$	......(a)	$\cdot 32 d_1^2 \times \frac{a}{c} = \ldots\ldots$	$= \underline{\ldots\ldots\ldots}$
... ..(d_2)	$\frac{m}{d_2^2}$	......(b)	$\cdot 32 d_1^2 \times \frac{b}{c} = \ldots\ldots$	
	$H = \cdot 32$	......(c)	Mean $m = \underline{\ldots\ldots}$	

CHAPTER XXXII

MAGNETIC MEASUREMENTS BY COMPOSITION OF PERPENDICULAR FIELDS, OR THE METHOD OF DEFLECTIONS

Composition of Fields. In neutral points we have a special case of **composition of fields**, *i.e.*, the neutralization of field H due to the earth, by an equal and opposite field due to a pole or a magnet. The resultant is zero, and from the equations $H=\frac{m}{r_2}$ or $H=\frac{M}{D_3}$ (see page 331) we can obtain $\frac{m}{H}$ $(=r^2)$ or $\frac{M}{H}$ $(=D^3)$, *i. e.* the ratio of m or M to H, so that if either is known the other can be found. Instead of reducing the resultant field to 0 in order to obtain $\frac{M}{H}$, the two fields could have been suitably compounded so as to give a finite resultant. From the direction of this resultant (as indicated by a compass needle) the ratio of the two fields, and hence $\frac{M}{H}$, can be easily obtained. The most convenient arrangement is to place the fields to be compared at right angles to one another, because in that case, the ratio of the fields can be easily obtained by means of a simple " tangent law."

Forces at Right Angles. The Tangent Law. In figure 134 a horizontal force F combined with a vertical force H gives a resultant (parallelogram of forces) R inclined to H at an angle θ, so that

$$\mathbf{Tan}\ \theta = \frac{\mathbf{F}}{\mathbf{H}}.$$

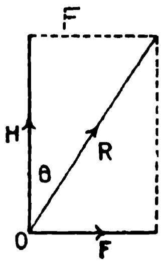


Fig. 134.—The tangent law for magnetic fields.

Imagine *F* and *H* to be magnetic fields. The diagram can be used to represent forces on a unit pole. These forces are numerically *F* and *H*, and the pole will move along *R*, which therefore is the resultant field. A compass needle will lie along *R*, and can thus indicate the direction of the resultant field.

Imagine *H* to be the earth's field, and *F* the field due to a pole or magnet.

On account of the field *F*, the needle is **deflected** through an angle θ from its usual psiotion along the field *H*, which ordinarily **controls** it, so that we have

$$\frac{\mathbf{F}}{\mathbf{H}} = \frac{\textbf{Deflecting field}}{\textbf{Controlling field}} = \tan\ ,$$

θ being the deflection from the original or *controlling* field.

This is the well known **"Tangent law for magnetic fields"** and is the the basis of measurements with the deflection magnetometer and the tangent galvanometer. The significance of the following simple experiment should be carefully understood, as it illustrates the conditions necessary for the application of the tangent law to magnetic and electrical measurements.

EXPERIMENT 88.—**To study the conditions necessary for the application of "the tangent law for magnetic fields."**

Apparatus: Vertical board (as for parallelogram of forces) with a single adjustable pulley, weights and scale pans, tin or cardboard strip cut out to represent a magnetic needle, squared paper, plumb line.

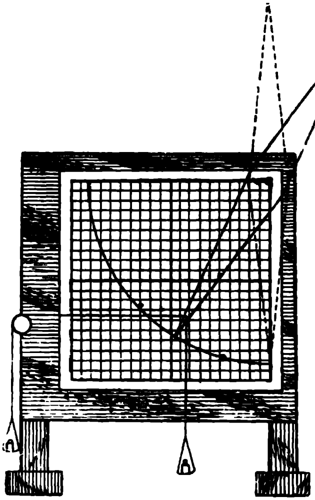


Fig. 135.—Experimental verification of the tangent law for magnetic fields.

The two fields *F* and *H* should essentially be *perpendicular* to one another.

Cut out of stiff cardboard or light tin-plate, a lozenge-shaped piece 12″ long, to represent a magnetic needle. Bore one fine hole through the centre and another near one end Fix a sheet of squared paper, 7″×8″ or so, near the top right hand corner of the board (Fig 135), adjusting one set of lines to be exactly vertical by means of a plumb line. Select the two axes, and through the origin drive a fine smooth tack or a stout pin to serve as an axis of rotation for the needle. Place the "needle" in position as shown, with the axis through the centre hole If the hole is well centred, the needle will remain indifferently in any position. Provide the fixed "*controlling field*" *H* by attaching to the lower hole a thread with a 200 or 300 gramme weight at the other end. The needle will now be vertical (*in the "meridian"*). Deflect the "needle" by a 100 gramme weight attached to a second thread adjusted *horizontally* or parallel to the other set of lines, and passing over the pulley as shown. Adjust the pulley till the lower edge of a rectangular wood block held in contact with the thread becomes parallel to the horizontal lines on the squared paper. Make a dot opposite the end of the needle. Repeat with "*deflecting forces*" of 200 300, 400 and 500 grammes, the *controlling field H* being kept constant. The needle in each case lies in the direction of the resultant field.

Remove the squared paper, and read the ordinates (vertical) and the abscissæ (horizontal) for the dots obtained, as if they were plotted points. $\frac{Abscissa}{Ordinate}$ for any dot equals $\tan\theta$ (θ being the angle between the deflected needle and H or the *controlling* field). $Tan\,\theta$ will, in every case, equal $\frac{F}{H}$.

Enter observations thus :

Controlling force H	Deflecting force F	$\frac{F}{H}$	$\tan\theta = \frac{abscissa}{ordinate}$
250 gms.	100 gms.		...
	200		...
	300		...
	400	...	...
	500	...	...

The values in the last two columns will be equal in each case.

The chief thing to be learnt from this experiment is that *the tangent law is applicable only when the two fields to be compared are* **at right angles.**

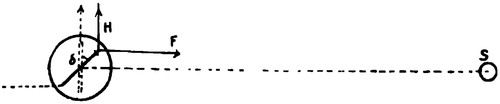


Fig. 136. The principle of the deflection magnetometer. The needle is deflected through δ. **Note that H and F are at right angles.** Provided with a scale of degrees and two millimetre scales, the compass needle becomes a magnetometer.

The Deflection Magnetometer. The small compass needle used for mapping fields can be used for indicating the direction of the resultant of two magnetic fields. In Fig. 136 the field due to a single pole (deflecting field F) is being compared with the earth's controlling field H. The needle is deflected from the normal position by an amount δ. Note that the pole is placed to the *east* of the needle, **so that its field may be at right angles to H.** It could have been placed to the *west* as well, but **not** to the north or south We can therefore, apply **the tangent law for magnetic fields** (page 340). The compass needles gives us the direction of the resultant, δ standing for t in the **Tangent law**. Consequently, if the pole is of strength m, and is at a distance d from the compass needle, we have

$$\frac{F}{H} = \frac{m/d^2}{H} = \tan \delta.$$

The simple compass needle is not provided with arrangements for actually measuring δ or d. With a scale of degrees round a large circle concentric with it, and mm. scales on two opposite sides, it becomes a very useful instrument called the **deflection magnetometer.**

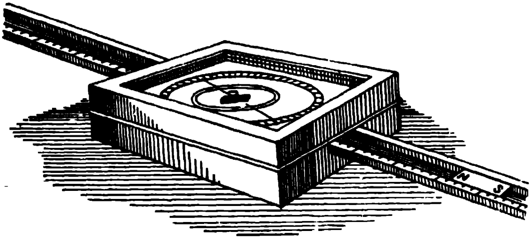


Fig. 137 —The deflection magnetometer.

Fig. 137 shows a regular deflection magnetometer. The small compass needle is provided with a long but light aluminium pointer which moves over the scale of degrees below it. A silvered mirror forms the bottom of the magnetometer box. and is of use in reading deflections without the error of parallax. Two millimetre scales graduated so as to read distances from the centre of the needle are fixed on opposite sides of the box. As long as the needle is small, its size and magnetic moment are theoretically immaterial. Weak or strong. its indications should be the same ; but in actual practice, *it is best to see that magnetometer needles are periodically magnetised to saturation to render the magnetic couples as large as possible.* In this way, the effect of errors due to the friction of the pivot becomes a minimum.

The "End on" and the "Broadside on" Positions.

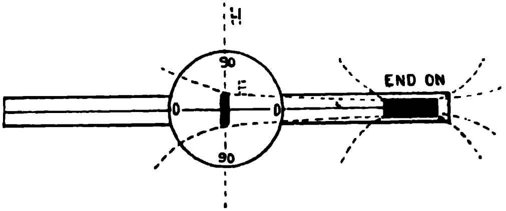


Fig. 138.—The **end on** position. H and F are **at right angles** F being equal to $2M/d^3$. *The needle is shown undeflected.*

The commonest use of the magnetometer is to determine $\frac{M}{H}$ by compounding with H (the *controlling field*), the field due to the magnet at some point on its equator or its axis produced (*deflecting field*). The two positions are illustrated in figures 138 and 139.

In Fig. 138, representing the **end on** position, the field on the axis produced of the bar magnet (placed with its axis east and west) is $\frac{2M}{d^3}$ towards the east or west, and is therefore at right angles to H.

In Fig. 139, showing the **broadside on** position, the magnet has its axis still east and west, but now it is the field on the equator $\frac{M}{d^3}$ which is used. This field is to the east or west, and hence at right angles to H. Note that *in both of these positions whether* "**end on**" *or* "**broadside on,**" *the axis of the deflecting magnet, as also its deflecting field, are* **east** and **west.**

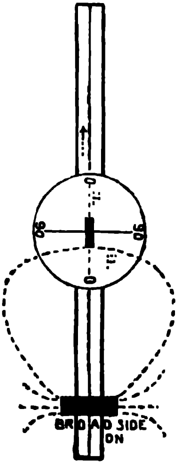


Fig. 139.—The **broadside on** position. H and F are, as always, **at right angles.** $F = M/a^3$ *The needle is shown undeflected.*

Fig. 140 shows a magnetometer with two pairs of scales. The east and west pair can be used for "*end on*" work, and the north and south pair for "*broadside on*" work. The instrument need not be rotated at all.

Innumerable failures are due to the fact that the ordinary magnetometer, with a single pair of scales, has to be *turned round* to place the scales east and west or north and south as necessary, and the student often does not know where to stop for the end on or the broadside on position. Failure can be avoided if the following one single rule is not forgotten; *The* **axis of the deflecting magnet** *should always* (*for " end on" or "broadside on"*) *be* **east** and **west** or **perpendicular** *to the* **needle** and **never parallel** *to it.*

In this position alone is $F \perp$ to H, and the tangent law, consequently, applicable.

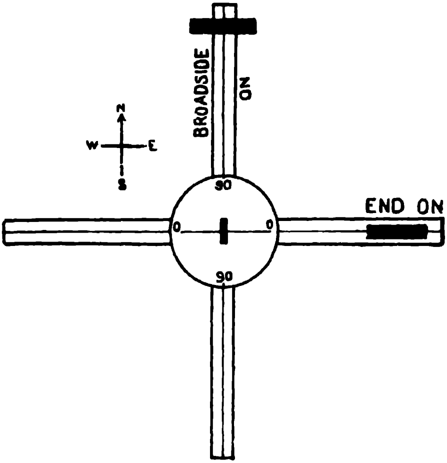


Fig. 140.—A magnetometer with two perpendicular scales, one for **end on** and the other for **broadside on** work. Notice that in *both* positions, the magnet as well as the field due to it near the needle is **east** and **west**.

The Essentials and Non-essentials of Magnetometer Adjustment.

The **essentials of magnetometer adjustment** are :

(1) Axis of deflecting magnet *always* East and West.

(2) Millimetre scales accurately

East and West ("End on " position);

or North and South ("Broadside on" position).

A *convenience*, generally provided, is

(3) That when (2) is satisfied, the pointer should read 0° or 90° on the scale of degrees.

This last (3) is a **non-essential** and should claim attention only *after* (1) and (2) are secured. If after adjustment (2) the pointer reads 3° or 4° or even more, deflections can be obtained by subtraction.

EXERCISE.—**To distinguish betwen correct and incorrect positions of the magnet and scale in a deflection magnetometer.**

Study the accompanying diagrams and enter in your book the impossible positions. State the objection in each case. Re-draw in your book the correct positions supplying the scale where necessary, and state whether the position is an "*end on*" or a "*broadside on*" position. The needle is shown undeflected throughout (imagine magnet replaced by brass bar). The dotted line is the meridian for each place.

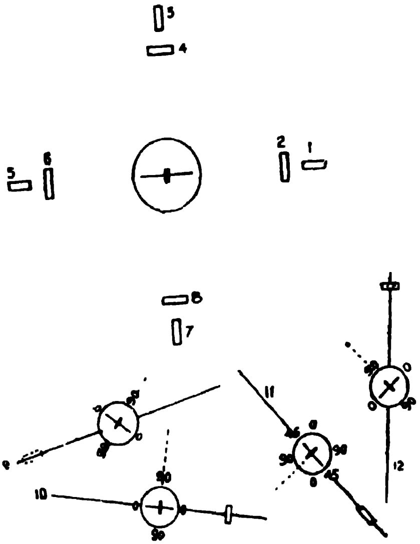


Fig. 141.—Wrong and right positions for magnetometer and magnet.

Enter thus:

Position No.	Correct or incorrect	Objection if incorrect.	"End on" or "Broadside on" if correct
......	Incorrect	Axis of magnet North & South	
......	Correct	......	"Broadside." *Redraw with scale*
......	......	......	

EXPERIMENT 89.—**Given H=·32, to determine m or M for a small bar magnet with a deflection magnetometer.**

Apparatus: Deflection magnetometer, 3″ or 4″ bar magnet.

Everything else remaining the same, the "*end on*" position gives bigger deflections than the *broadside on*" position, and is therefore always to be preferred. Scales of degrees are generally so arranged that the pointer reads 0° in this position.

Examine the needle to see if the pointer is at right angles (or parallel) to the needle. If it departs appreciably from these positions, twist it carefully till the adjustment is correct (use a set-square). Level the instrument till the needle moves freely. Place the scales *east and west*, *i e.* perpendicular to the needle. Use a set-square, if necessary. Draw chalk lines round the magnetometer, to recover the position if displaced. The pointer should read in this position nearly 0° or 90°.

In any case, whether this is so or not, DO NOT CALCULATE A ZERO CORRECTION. The zero correction will be applied automatically, as explained below.

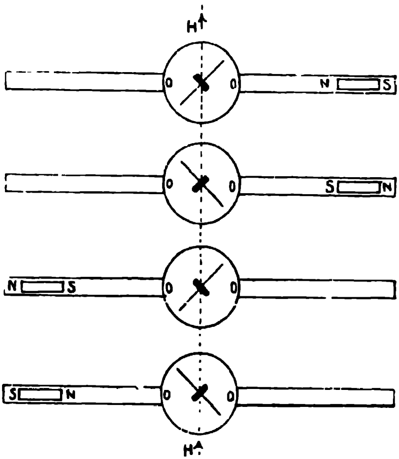


Fig. 142.—Four positions for determining the deflection produced by a magnet.

Mark an equatorial line all round the centre of the magnet.

Place the magnet respectively in the four positions shown in Fig. 142, keeping the distance of the centre always the same (20 or other round number of centimetres is convenient). The distance chosen should be such that the deflection is between 40° and 50°. Read both ends of the pointer for each of the four positions, tapping the instrument *gently* with the finger each time, to be sure that the needle is not sticking. To avoid parallax error, place the eye when reading, so that the pointer covers its own image in the mirror. The mean of the eight readings is automatically corrected for errors due to zero reading, eccentricity of needle, etc.

As F in this case is $\frac{2M}{d^3}$, we have

$$\frac{F}{H}=\frac{2M}{d^3H}=\tan\theta$$

$$\therefore M=\tfrac{1}{2}\,d^3H\tan\theta$$

Enter observations thus:

Length of magnet L = ...

Distance between centre of magnet and needle = ...

Magnet	Readings a		Deflection: θ	M or m
	N to east	N to west		
East of needle	(1) .	(3) .		$M = \frac{1}{2}d^3\ (\cdot 32)\ \tan\theta$
	(2) ...	(4)	Mean a = ... $\theta = a$ or $90^\circ - a$	$m = \frac{M}{L}$
West of needle	(5) ...	(7) ...		
	(6) ...	(8) ...		Use tables of logarithmic tangents.

NOTE 1: –If, on adjusting the scales east and west, the pointer seems to be near neither 0° nor 90°, and the compass box cannot be rotated relatively to the millimetre scale, the deflection can still be obtained without reference to the initial reading. *The deflection is always one-half of the angle between readings of the same end of the pointer when deflected in opposite directions, irrespective of the zero reading.* Suppose one end initially reads 35°·7 and the readings of this end with opposite deflections are 59°·0 and 12°·0 on the same side of the zero The deflection is $\frac{59^\circ - 12^\circ}{2} = 23^\circ{\cdot}5$. For readings 81°·0 and 12°·0 on opposite sides of the zero, the deflection is $\frac{81^\circ + 12^\circ}{2} = 46^\circ 5$. The same applies to the other end of the pointer.

NOTE 2: –The relation $F = \frac{2M}{d^3}$ is correct within 5 to 6% for distances three times the length of the magnet, and within 3% for distances four times this length (*e.g.* $L = 2l = 10$ cm, $d = 30$ and 40 cm., respectively). Unfortunately, suitable deflections are often not obtainable with such distances. A more correct relation should in that case be employed. To do so, it is convenient to read the distances of the poles D_1. D_2.

$$F = m\left(\frac{1}{D_1^2} - \frac{1}{D_2^2}\right) = H \tan\theta, \text{ which gives } m = \frac{H \tan\theta}{\left(\frac{1}{D_1^2} - \frac{1}{D_2^4}\right)}.$$

For "broadside on" use
$$F = \frac{M}{(d^2 + l^2)^{\frac{3}{2}}}.$$

Errors in Magnetometer Work. In experiment 89, eight readings of the deflection were taken. These are necessary for corrections necessitated by the following defects and imperfections, which are practically always present:

(1) *Needle pivot not in the centre of the scale of degrees* : corrected by reading both ends of the pointer.

(2) *Initial reading of the pointer not 0°*: corrected by reading deflections on both sides of the zero.

(3) *Adjustment of scales to E and W not quite correct* : corrected by reading deflections on both sides of the zero.

(4) *Magnets irregularly magnetized* : corrected more or less by using deflection produced with the N-poles and the S-poles facing the needle in turn.

EXPERIMENT 89*a*.—**To compare the magnetic moments of two small magnets by the method of deflections.**

(1) Obtain, for each magnet in turn, the deflection in the end on position, using in each case the mean of eight readings as in Expt. 89. The two magnets may conveniently be at the same distance from the needle. If this does not give suitable deflections (deflections less than 30° and more than 60° are not desirable), use different distances, adjusting the deflection in both cases to nearly 45°, say between 40 and 50°. If M_1, M_2, d_1, d_2, and θ_1, θ_2 refer to the two magnets respectively,

$$\frac{2M_1/d_1^3}{2M_2/d_2^3}=\frac{F_1}{F_2}=\frac{H\tan\theta_1}{H\tan\theta_2}=\frac{\tan\theta_1}{\tan\theta_2}.$$

$$\text{so that } \frac{M_1}{M_2}=\frac{d_1^3.\tan\theta_1}{d_2^3.\tan\theta_2}$$

$$\text{For equal distances } \frac{M_1}{M_2}=\frac{\tan\theta_1}{\tan\theta_2},$$

Enter observations thus :

$d_1 =$ ....

$d_2 =$ ....

Magnet	Readings a		Deflection: θ	$\frac{M_1}{M} = \frac{\tan\theta_1 \,.\, d_1^3}{\tan\theta_2 \; d_2^3}$
	N to east	N to west		
A				
East of needle	(1) ...	(3) ...		$\frac{M_1}{M_2} =$ ...
	(2) ...	(4) ...	...	
West of needle	(5) ...	(7) ...		
	(6) ...	(8) ...		
B East of needle	(1) ...	(3) ...		For calculation use tables of ordinary logarithms and logarithmic tangents.
	(2) ...	(4) ...	...	
West of needle	(5) ...	(7) ..		
	(6) ...	(8) ...		

(2) The above experiment can be varied by adjusting distances so that the deflection is the same for both the magnets. Using magnet A, obtain four values of d_1 (2 East, 2 West) which bring *one* particular end of the pointer always to *one* particular point of the circular scale (say 45° in *one* particular quadrant). Similarly obtain four values of d_2 for magnet B, adjusting the same end of the pointer to the same point of the scale as for A.

The magnets are evidently producing, when at d_1 and d_2 cm., respectively, from the needle, equal deflecting fields, and $\frac{M_1}{M_2} = \frac{d_1^3}{d_2^3}$, irrespective of zero errors, defective setting in the meridian, and any other defects that the magnetometer may have (except a sticking needle). Compare this method with "weighing with a false balance by the method of substitution."

EXPERIMENT 90.—**To compare magnetic moments by the null method.**

This is a rapid and accurate method of comparing the magnetic moments of two bar magnets.

Adjust magnetometer for "*end on*" position. Read any *one* end of the pointer.

Place one magnet A on the east with its centre at a convenient distance d_1, say 25 or 30 cm., so that deflection is about 40° to 50°. Place the second magnet B on the west, so as to bring the pointer to the initial position. Tap gently before deciding. Read d_2 the distance of the centre of B. Place magnet A in turn in the three other positions shown in Fig. 142, its distance d_1 remaining *unchanged.* Corresponding to these find three more values of d_2 by adjusting B against the magnet A as at first. Calculate mean d_2. $\frac{M_1}{M_2} = \frac{d_1^3}{d_2^3}$.

Record answers to the following questions:

Q. 1. Are the magnets producing equal fields at the needle when adjusted for equal deflections (used separately) or no deflection (used on opposite sides, as in the null method.)

Q. 2. If the magnetometer is 5° or 10° out of the east and west line, is the value of $\frac{M_1}{M_2}$ affected?

Q. 3. How will it be affected when using the method of deflections (Expt. 89) ?

Q. 4. Which is, in your opinion, the superior method ?

If time permits, the following instructive experiments may be performed.

EXERCISE.—**To prove experimentally the relation : Field on the axis produced of a bar magnet varies as $\frac{1}{d^3}$.**

Obtain $\theta_1, \theta_2, \theta_3,$...(mean of 8 readings in each case) for various distances d_1, d_2, d_3......The product $d^3 \tan \theta$ will be approximately constant.

EXERCISE.—To prove experimentally, that the field of a magnet at distance d on the axis produced, is twice the field at the same distance on the equatorial line.

Measure θ_1 (mean of 8 readings) for a suitable value of d (θ_1 should be nearly 60°) in the "*end on*" position. Measure θ_2 for the same value of d in the "*broadside on*" position. $\frac{\tan \theta_1}{\tan \theta_2}$ will be nearly 2.

EXERCISE.—To verify experimentally "the law of inverse squares" for a single pole.

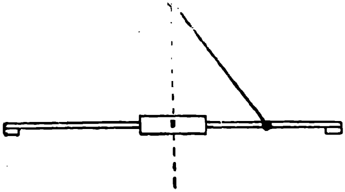


Fig. 143.—Verification of the law of inverse squares for magnetic poles.

Use a *long* magnet, placed vertically or, preferably, with one pole always exactly above the needle (Fig. 143), so that it is inoperative. Place second pole at various distances on the scale placed east and west (*end on* position). Measure $\theta_1, \theta_2, \theta_3,$......for d_1, d_2, d_3 respectively. $d^2 \tan \theta$ will be constant.

CHAPTER XXXIII

MAGNETIC MEASUREMENTS BY THE METHOD OF OSCILLATIONS

The Method of Oscillations. The value of $\frac{\mathbf{M}}{\mathbf{H}}$ can be found by the method of deflection, and if **H** is given, the absolute values of magnetic moments and pole strengths can be determined. If **H** is not known moments can be *compared*, relative strengths of fields at different points round a magnet determined, and various laws and relations verified. All this can also be done by another powerful method **the method of oscillations**. This method provides us with **MH**, so that if **H** is known, **M** can be determined. If **H** is not given, moments can be *compared*, because $\frac{M_1}{M_2}=\frac{M_1H}{M_2H}$. Fields can also be *compared* in a simple manner.

The Oscillations of a Magnet. The method is based upon the fact, that a magnet of magnetic moment M, suspended or freely supported in a field H, executes when disturbed, isochronous vibrations like a pendulum, the period being

$$T=2\pi\sqrt{\frac{K}{MH}}\,^{*},$$

where K is the "**Moment of Inertia**" of the magnet, a constant depending upon its mass, shape and size.

---

[*] This is exactly similar to the relation for a compound pendulum $T=2\pi\sqrt{\frac{K}{mhg}}$. The similarity is greater if the relation for a magnet is put in the form $T=2\pi\sqrt{\frac{K}{mLH}}$. The theory is identical for both cases. (See page 325).

On squaring, we have

$$T^2 = \frac{4\pi^2 K}{MH} \text{ or } n^2 = \frac{MH}{4\pi^2 K}$$

where n is the frequency $(= \frac{1}{T})$.

These equations form the basis of the method of oscillations. By swinging the *same* magnet in different fields, the fields can be compared. By swinging different magnets in the *same* field, the magnetic moments can be compared.

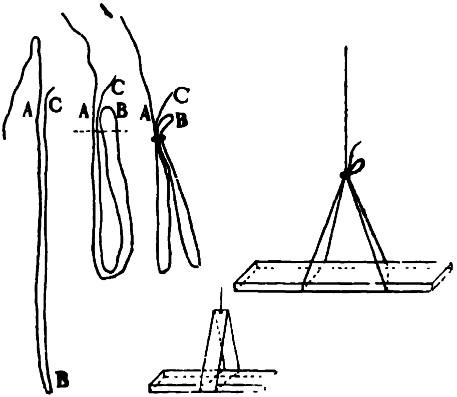


Fig. 143*a*.—Silk and paper suspensions for magnets.

EXPERIMENT 91.—**To compare the magnetic moments of two magnets by the method of oscillations without determining the moments of inertia.**

Apparatus : Vibration magnetometer, untwisted silk, the magnets to be compared, watch.

The Vibration Magnetometer. The essentials of this instrument (Fig. 144) are two :

(1) A magnet, freely suspended or pivoted so as to execute vibrations.

(2) A box with glass sides or a bell jar or beaker round the magnet, to protect it from draughts.

Suspensions. Avoid all "threads," silk or cotton. They all untwist under tension, with confusing results. The magnet will turn round and round, and, if weak, may settle in a direction quite different from the meridian. For light magnets, use single cocoon fibre. For heavier magnets, use just as much as may be necessary of the fluffy and untwisted silk known in the vernacular as '*putt*'.

The magnet may rest in a suitable stirrup, made out of paper, fine brass wire, or the thread itself (Fig.134*a*).

Vibrations. These should be started by allowing the magnet to come to rest, and then presenting to it carefully and at a distance, an iron knife, key, or another magnet, so as to obtain a swing of *small* amplitude. As in a pendulum, the period is appreciably *greater* for large amplitudes. Lateral or to and fro motion should be absent.

(1) With a pair of **identical magnets** (same mass and same dimensions), determine the period of each when swinging in the field of the earth, far away from all extraneous iron and magnets. Suspend one of the magnets *with axis accurately horizontal,* inside a vibration magnetometer. Make a suitable mark on the glass to denote the mean position, and keeping your eye properly adjusted, carefully present a key to set the magnet swinging, and determine the time of 10 to 15 complete vibrations. *The amplitude should be small.* Repeat with the second magnet. If T_1 and T_2 are the periods, we have

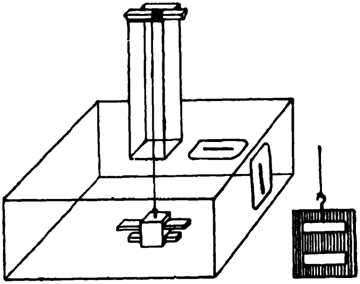


Fig 144—The vibration magnetometer.

$$\frac{T_1^2}{T_2^2}=\frac{4\pi^2\frac{K_1}{M_1H}}{4\pi^2\frac{K_2}{M_2H}}=\frac{K_1}{K_2}\cdot\frac{M_2}{M_1}=\frac{M_2}{M_1},$$

as $K_1=K_2$ for the two equal magnets.

(2) If the magnets are of different sizes, use the following **"sum and difference" method**. Suspend the magnets, with their axes parallel and similar poles together, in a double stirrup of brass or copper wire, or by insertion into two holes in a small block of wood as shown (Fig. 144); or they may be merely bound together by thread with a strip of wood between them. Determine the period T_1. Reverse one of the magnets, so that dissimilar poles are together. Determine the period T_2. The moment of inertia of the suspended system is the same in both cases; the magnetic moments are, respectively, M_1+M_2 and M_1-M_2. Therefore

$$\frac{T_1^2}{T_2^2}=\frac{M_1-M_2}{M_1+M_2}, \text{ which gives}$$

$$\frac{T_2^2+T_1^2}{T_2^2-T_1^2}=\frac{M_1}{M_2}.$$

[Method (1) should be applied only if the magnets are *really* equal in mass, length, etc. Weigh, if necessary. Method (2) becomes inaccurate if the magnets are of nearly equal moment, as in that case the system is nearly astatic in the difference position, and weak torsional forces become important.]

Special Cases. The sum and difference method is not suitable for magnets of nearly equal magnetic moment. For such cases, use the relation

$$\frac{T_1^2}{T_2^2}=\frac{M_2}{M_1}\cdot\frac{K_1}{K_2},$$

swinging the magnets singly as in (1), and calculating K_1 and K_2.

$K=m\left(\frac{l^2+b^2}{12}\right)$, where m is the mass, l the length.

and b the horizontal breadth of a rectangular magnet. For a cylindrical magnet $K=m\left(\frac{l^2}{12}+\frac{d^2}{16}\right)$, d being the diameter. Use the vernier callipers for measureing d.

EXPERIMENT 91*a*.—**To prove by the method of oscillations that the field of a magnet at a distance *d* on the axis produced is twice the field at the same distance on the equatorial line.**

Apparatus: Deflection magnetometer, 4″ bar magnet, stopwatch.

Determine the period of the magnetometer needle under the action of the earth's field alone, all iron and magnets being at a distance.

Next place the scales north and south, as for the "broadside on" position, but place the magnet "*end on*" (Fig 145), with the south pole pointing south. [For measuring deflections, this is an impossible position]. There should be no appreciable deflection, as the field of the magnet coincides with the field of the earth in this case.

If the needle turns round, the magnet is presenting the wrong pole.

Determine the period T_1. Repeat with the magnet on the opposite side at the same distance. Next place the scales east and west, and put the magnet (d same as before) "*broadside on*" with the south pole pointing north, so that the field of the magnet coincides with the earth's field where the needle is. Determine the period T_2. Repeat with the magnet on the opposite side. If the fields due to the magnet in the two positions are F_1 and F_2, and n, n_1, n_2, are the frequencies for the three cases, we have

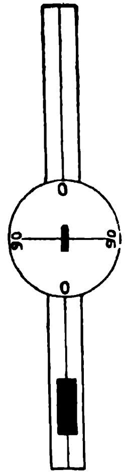


Fig. 145.—Deflection magnetometer used as vibration magnetometer.

$$n^2 = 1 = \frac{M}{4\pi^2 K} \times H \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad (1)$$

$$n_1^2 = \frac{1}{T_1^2} = \frac{M}{4\pi^2 K} \times (H + F_1) \quad \ldots\ldots \quad \ldots\ldots \quad (2)$$

$$n_2^2 = \frac{1}{T_2^2} = \frac{M}{4\pi^2 K} \times H + F_2 \quad \ldots\ldots \quad \ldots\ldots \quad (3)$$

Subtracting (1) from (2) and (3) respectively, and dividing, we have

$$\frac{n_1^2 - n^2}{n_2^2 - n^2} = \frac{\frac{1}{T_1^2} - \frac{1}{T^2}}{\frac{1}{T_2^2} - \frac{1}{T^2}} = \frac{F_1}{F_2} \left\{ \text{should equal 2 nearly.} \right.$$

T_1 and T_2 should each be determined in two positions at the same distance on opposite sides of the needle. If the magnetometer needle does not oscillate long enough, use a compass needle 2″ to 3″ long on stand, measuring the distances with a metre stick

Observations :

Distance d = ... cm. throughout.

Field.	Time of 10 or 20 vibrations.	Vibrations in 60 seconds n, n_1, n_2.	(Frequency)² n^2, n_1^2, n_2^2	$\frac{n_1^2 - n^2}{n_2^2 - n^2} = \frac{F_1}{F_2}$
H only	.... Mean :	n		
H + F_1	.... Mean :	n_1	$n_1^2 - n^2$	......
H + F_2	.... Mean :	n_2	$n_2^2 - n^2$	

If time permits, perform the following exercises.

EXERCISE.—**Prove by the method of oscillations the law of inverse squares for a single pole.**

Use a long magnet. Place one pole (N) to the south of a compass or magnetometer needle, the other pole being vertically above the needle (Fig. 143). Determine the frequency of vibration N with pole at different distances as also the frequency n under the action of the earth's field alone. Allow for earth's field as usual. $(N^2-n^2)/d^2$ will be constant.

EXERCISE.—**Prove by the method of oscillations that the force on the axis produced of a bar magnet varies as $\frac{1}{d^3}$.**

Place magnet to north or south of needle with axis north and south. The south pole should point south. Determine period (and frequency) at various distances. Allow for earth's field as usual. $(N^2-n^2)\times d^3$ will be constant.

Simultaneous Determination of Absolute Values of M and H. The deflection magnetometer gives us $\frac{M}{H}$, so that either of **M** or **H** can be found if the other is known. The same remark applies to the vibration magnetometer, which gives us, however, **MH**. It is evident that with the two instruments combined, **M** and **H** can be simultaneously determined.

EXPERIMENT 92.—**To determine the absolute values of H (horizontal component of the earth's magnetic field) and M.**

Apparatus: Deflection and vibration magnetometers, 4″ bar magnet, stopwatch, balance, callipers, etc.

(1) Determine exactly as in Expt. 89 the deflection θ (mean of eight readings) produced by the magnet in the *end on* position at any convenient distance d. We have

$$\frac{2M/d^3}{H}=\tan\theta\ ;\ \text{or}\qquad \frac{M}{H}=\frac{1}{2}d^3\tan\theta\ \ldots\ldots\ldots\ldots \text{I.}$$

(2) Determine exactly as in Expt. 91, the period T of the magnet used in (1), under the action of the earth's horizontal field H. We have

$$T=2\pi\sqrt{\frac{K}{MH}}\ ;\ \text{or}\qquad MH=4\pi^2\frac{K}{T^2}\qquad \ldots\ldots\ldots\ldots \text{II.}$$

Combining I and II we have by, (*i*) multiplying and (*ii*) dividing,

$$M^2 = 4\pi^2 \frac{K}{T^2} \times \tfrac{1}{2}d^3\, \mathit{taa}\ \theta; \text{ or } \mathbf{M} = \frac{2\pi}{T}\sqrt{K \times \tfrac{1}{2}d^3 \tan\theta}$$

$$H^2 = 4\pi^2 \frac{K}{T^2} \div \tfrac{1}{2}d^3\, \mathit{tnn}\ \vartheta\, ; \text{ or } H = \frac{2\pi}{T}\sqrt{K \div \tfrac{1}{2}d^3 \tan\theta}$$

To obtain K, weigh the magnet, and measure the length l and the horizontal breadth b. $K = m\left(\frac{l^2 + b^2}{12}\right)$ where m is the mass of the magnet.

Record an answer to the following question :

Q. Does the magnetic field of the earth affect the weight of the magnet ? Give reasons.

PART VIII

CURRENT ELECTRICITY

CHAPTER XXXIV

QUALITATIVE STUDY OF CURRENT AND CURRENT SOURCES

Sources of Current. A **current** of electricity can be obtained in a wire by connecting its ends to **conductors at different potentials.** For a *steady current*, it is essential that the **potential difference should be permanent,** and not such as to disappear or diminish rapidly, after the current has been flowing for a short time.

There are two well known methods of obtaining a steady potential difference, and both are extensively used for the production of current for experimental, domestic and industrial use.

(1) **The city electric supply system,** the two (+and−) **mains** of which are kept at a fixed difference of potential (usually 220 or 110 volts).

(2) **Cells, voltaic (primary), or secondary,** maintaining a potential difference of one to two volts between the two terminals. These cells are of various kinds. We shall study (1) the simple voltaic cell, (2) the Daniell cell, (3) the Grove or Bunsen cell, (4) the Leclanche, wet or dry, (5) the secondary cell or the accumulator.

How to Detect a Current. A currect can be detected by its **magnetic, thermal** or **chemical effects.** Quantitative measurements on currents are most conveniently carried out by use of the magnetic effects. The simplest appliance for this purpose is a small compass needle (with a graduated card underneath) like the one used for tracing fields, or a magnetic needle supported on a vertical pivot above a scale of degrees. A magnetic needle provided with a scale of degrees and a coil of 20 to 30 turns, with connecting terminals on a wooden base, becomes a very useful **galvanoscope**.

The heating of wires and lamp filaments, and the electrolysis of water are examples of the thermal and chemical effects. Measurements of current can be based on these effects, but the large majority of voltmeters, ammeters and galvanometers used for such measurements are *magnetic* instruments.

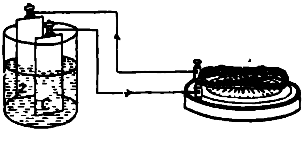


Fig 146—The simple voltaic cell, and a useful galvanoscope

EXPERIMENT 93*a*.—**To study the effects of a current, and the properties of (1) a simple voltaic cell, (2) a Grove or Bunsen cell.**

Apparatus : Plates of copper and amalgamated zinc, dilute sulphuric acid, Grove or Bunsen cell. magnetic needle on stand, galvanoscope, fine iron or German silver wire, small electric lamp on stand. Potassium iodide paper, connecting wires, etc.

Place the copper and amalgamated zinc plates in dilute sulphuric acid in a beaker. Prepare a plan in which to record the results of the following simple experiments as shown at the end.

(1). With the zinc and copper plates apart, what do you observe ?

(2). Touch the zinc plate *under the acid* with a bare brass, lead or copper wire in the hand. What do you see ? Is the copper wire *dissolving* ? What *is* being dissolved, to evolve the gas that you see ? What would happen if the zinc itself were to contain in it particles of copper, lead, etc., exposed to the acid ?

What is the use of amalgamating the zinc?

(3). Let the zinc and copper plates touch inside the acid. What do you observe?

(4). Attach copper wires to zinc and copper, and connect the two wires. What do you observe?

(5). Place the terminals ½″ or 1″ apart on filter paper moistened with potassium iodide solution. Which wire gives the brown spot?

(6). Place one of the wires above, and parallel to a compass needle. What happens?

(7). Connect the wires to the terminals of a simple galvanoscope, or the 50 turn coil of a tangent galvanometer. Read the deflection. Let the current continue for half an hour. In the mean while study the Bunsen or Grove cell.

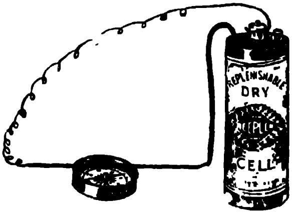


Fig 147.—The action of a current on a magnetic needle.

In the outer pot place the amalgamated zinc. Fill two-thirds or three-fourths with dilute H_2SO_4. In the inner pot place the carbon or platinum and fill with strong *concentrated* nitric acid. Attach copper wires. The following observations may be made with the Grove or Bunsen cell so set up, or if it is more convenient, with an accumulator.

(8). Connect the two poles and handle the bare wires to see if they are heated. Connect the two wire ends to one or two inches of *very fine* iron or German silver wire. Is this heated? Connect similarly to a small battery lamp. This will light up.

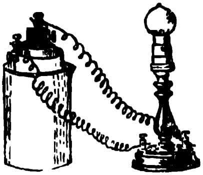


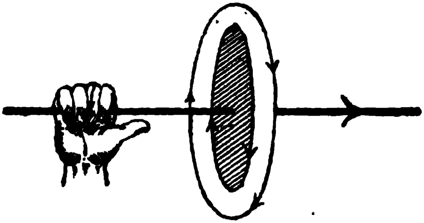
Fig. 148.—Heat and light produced by the electric current.

(9). Place the terminal wires ½″ or 1″ apart on paper moistened with potassium iodide solution. The positive wire (from the platinum, carbon or, in case of an accumulator, the red terminal connected to the red brown plates) will give a brown spot due to liberated iodine.

(10). Connect the wires, and place a portion (*i*) above, (*ii*) below, (*iii*) vertically near the ends of, the needle. Turn the wire round to use a reversed current, and repeat (*i*), (*ii*), (*iii*). Assuming the current to flow from the positive to the negative terminal, verify for every case the following rules :

Ampere's Rule.—*Imagine yourself swimming in the direction of the current* WITH YOUR FACE TOWARDS THE NEEDLE. *The* **north** *pole will turn to your* **left** *hand.*

Right hand Rule.—*Encircle the conductor completely with the fingers of the* **right** *hand, with the thumb pointing in the direction of the current. The field due to the current is (a north pole will turn) in the direction of the fingers (base to tips).*



Fig, 149.—The right hand rule for determining the direction of the magnetic field of a current.

Go back to the simple voltaic cell, and see the deflection. Is the current as strong as it was at first. Is the simple voltaic cell a *constant* cell ? Remove the copper plate, wash and dry over a flame, and replace in the cell. Does this improve the deflection ? What caused the decrease in the current ? Of the two cells studied, which gives the stronger current ? What function does the nitric acid in a Grove cell serve, or the $CuSO_4$ solution in a Daniell cell ?

It may be stated that Grove and Daniell cells will give fairly constant currents continuously for one or two hours.

Record as follows :

Experiment.	What is observed.	Conclusion.
(1) Copper and zinc plates not touching	No action on zinc or copper plate.	Contact of dissimilar metals necessary for voltaic action.
(2)		
(3)		
(4)		
(5) Wire ends placed on iodide paper.	Brown stain under wire from copper plate.	Wire from *copper* plate *positive.*
(6)		
(7)		
(8)		
(9)		
(10) Wire *above* needle current S to N.	North pole turns to west.	Ampere's rule verified.

EXPERIMENT 93.—**To examine Daniell, bichromate and Leclanche cells. To determine the positive pole in case of (1) any of the above cells, (2) the city electric supply mains (220 volts).**

Apparatus : Cells set up, or materials for setting up, magnetic needle on stand, wooden box, pole finding paper, 220 volts electric lamp (any candle power) on wooden base.

Cells: Examine the cells named above. *Record in your book the materials required for each in a table like the following* :

Cell.	Outer vessel.	Inner vessel.
Daniell ...	Copper vessel containing $CuSO_4$ solution + $CuSO_4$ crystals.	Zinc in dilute sulphuric acid.
Leclanche, wet or 'dry.'		
Bichromate ...	Only one vessel employed. Name materials for plates and solution.	

Place one of the cells in a closed box, leading the wires out through two holes. To determine which wire is + and which − perform the following experiments, *recording the results and conclusions briefly in tabular form as for previous experiment.*

POLE FINDING PAPER : Place the two wires ½″ or 1″ apart on paper moistened with potassium iodide solution. The wire which gives a brown stain is the *positive*. (Paper so prepared can be dried, and stocked. Moisten slightly when required for use).

AMPERE'S RULE : Applying Ampere's rule, we see that if a current passes *above a needle from south to north*, the north pole turns to the west. To make use of this rule in determining the direction of the current, connect the terminals to a spring key.

Support a portion of the wire ABOVE **the needle and parallel to it.** Press the key. **If the** NORTH **pole turns to the** WEST, **the current is from** SOUTH **to** NORTH **and** *vice versa.*

Electric supply wires : Proceed exactly on the same lines as for cells, with this difference, that as the voltage in this case is very high, the flow of excessive current must be prevented by interposing, at some point of the circuit, a lamp as a resistance.

Attach rubber insulated flexible wires to a wall "plug." Connect one of the free ends to one terminal of a lamp (any candle power) on a wooden base. The other wire from the plug, and a wire from the second terminal of the lamp may now be used for the test:

(*Do not handle the bare conductors.* It is safe to handle the rubber insulated wires.)

After (and *not* before) all arrangements are made, and a lamp correctly put in, put the plug into the wall socket.

Pole finding paper: Place on a piece of wood or glass the moistened iodide paper, and test exactly as for cells. This test is simpler and more convenient than the magnetic needle test. The test should not be carried out with the iodide paper *in the hand*. Place the paper on wood or glass.

Ampere's rule: With the lamp in circuit, proceed exactly as for cells, except that the key should be replaced by an ordinary lamp switch, or the plug itself may be used for making connection, *after* the wire has been properly arranged over the magnetic needle.

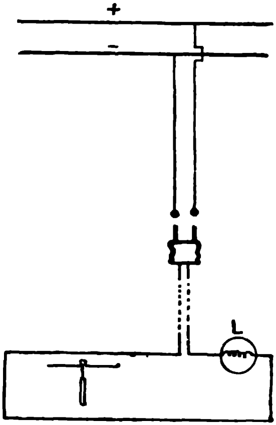


Fig. 150.—Determination of the polarity of lighting mains by means of a magnetic needle.

The Electromagnet.—A very important practical application of currents lies in the power they have of magnetising iron (temporarily, while the current lasts) and steel (permanently). The electric bell, the telegraph, the induction coil, dynamos, motors, and numerous other electrical appliances are based on this property.

EXPERIMENT 93*b*.—**To study the magnetising properties of a current.**

Apparatus : Spiral of insulated copper wire (50 turns) wound round a 7″ glass tube, thick iron wire, thick knitting needle, small horseshoe electromagnet, electric bell, push, two Leclanche cells (or 4 volts on table terminals from laboratory battery), magnetic needle.

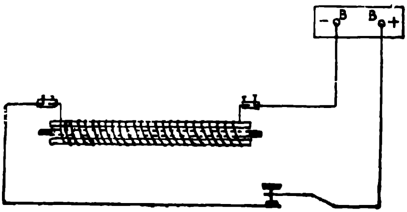


Fig. 151.—The magnetising power of a current.

(1) Connect the spiral to the current terminals through a spring key (Fig. 151). Place the iron wire inside the tube. Press the key. Present iron filings or a small iron tack. Attraction will be observed. Release key. The filings will drop off the wire.

Repeat with steel knitting needle. The magnetism remains even on stopping the current. Test the polarity, and verify the following rule after examining the winding of the spiral :

RULE :—**The end of the needle round which the current flows** ANTI-CLOCKWISE **becomes the** NORTH **pole and** *vice versa.*

(2) Replace spiral by electromagnet. This time use a plug key. Insert the plug and present the armature. It will be attracted. You can suspend a weight of one or two kilogrammes from the hook. A *flat* faced iron article of considerable weight may be suspended directly. With one hand under the attracted load, release the key. The weight falls.

(3) Next connect up the bell with push as shown (Fig. 152). A single Leclanche cell (reduce 4 volts to 2) is enough. Ring the bell. Examine the details of its construction. Trace the current from one terminal to the other through the various parts

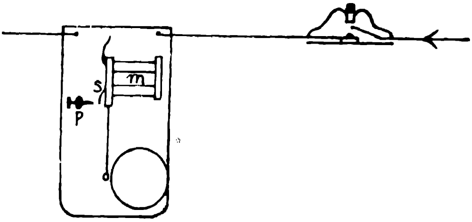


Fig. 152.—The electric bell and push.

(the electromagnet *m*, the contact spring *s* behind the armature, the contact screw on pillar *p*, as also the set screw for fixing the contact screw when correctly adjusted). What causes the vibratory action of the armature? Draw a diagram of connections, indicating by arrows the flow of the current through the various parts.

CHAPTER XXXV

CIRCUITS. WIRING DIAGRAMS. SWITCHES COMMUTATORS

Before taking up the quantitative study of currents, it is essential for the student to acquire considerable practice in connecting apparatus, and tracing a current to see if the connections are correct. Many students who know the complete theory of experiments with a slide-wire bridge or a tangent galvanometer, and take satisfactory observations *after the apparatus is connected for them*, often break down at the sight of an unfamiliar key or commutator, and are altogether unable to start. If the exercises in this chapter are performed, familiarity with the commoner instruments and types of connection will be gained, and the performance of the more elaborate quantitative experiments, later on, will not be hampered by difficulties connected with switches, commutators, and other indispensable accessories. If any particular exercise cannot be done, drawing a clear and neat diagram of connections is the next best thing to do.

Note.—As the experiments described relate to the mere building up of circuits, they do not take much time, and three or four of them are easily performed in one double period.

The City Electric Supply.—There seems to be no reason why students should not use, with proper guidance, current from the city electric supply for experiments on lamps and circuits. Very simple directions, if followed, render the work quite safe. It is but proper, that a physics student sholud be able to take an intelligent interest in the lights of his own house, if not actually to remedy small defects occasionally. The current is most conveniently taken from wall plugs (fitted to the tables, if possible). In the absence of an electric supply, battery current can be used with pocket electric (2 volt) or motor car (6—8 volt) lamps.

Special Equipment for Experiments on Circuits, etc.

Besides the apparatus usually available in practical laboratories, the following should be provided :

(1) Electric bells and pushes.

(2) Morse keys and sounders. In the absence of sounders, lamps, galvanoscopes or even magnetic needles will do. A spring key can be converted into a Morse key by means of an additional contact above the spring (see page 376).

(3) Lamps of proper voltage. Single lamps with terminals on wooden bases (Fig. 153) are convenient for experimental work by students

(4) Switches with terminals on wooden bases (Fig 153).

(5) Experimental cutouts consisting of two copper strips (2″ and 4″) with four terminals, two on each strip. With these, long rubber covered wires need not be spoiled by being bared for branching off leads to lamps, etc. Short pieces can be used (Fig. 153.)

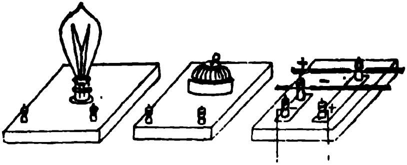


Fig. 153.—Lamp, switch and cutout with terminals on wooden bases, for experimental work by students.

(6) Rubber covered flexible wire, single and twin (the variety with the two members *differently* coloured is more convenient) for connections.

EXPERIMENT 94.—**To examine single-way, two-way, three-way...switches, and to arrange a bell, a lamp and a galvanoscope so that any one at a time can be worked by the same battery.**

Apparatus : As in the heading, wire for connections, two Leclanche cells or an accumulator.

In Fig. 154 (*bottom left*) we have a *single*-way plug key. It has *two* terminals Connection between wires connected to the two terminals can be made or broken by insertion or withdrawal of the plug. Used on the simple single voltaic circuit, for connection or disconnection.

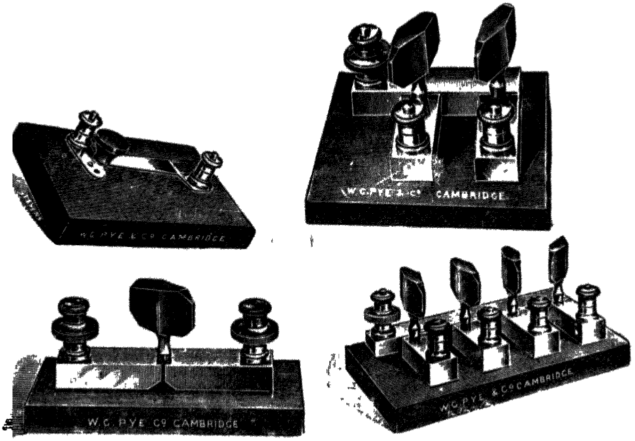


Fig. 154—*Top left*, plain press or spring key, *bottom left*, ordinary single-way plug key, *top right*, two-way key, *bottom right*, four-way key

Fig. 154 (*top right*) shows a *two*-way key (often wrongly called a three-way key on account of its three terminals). By inserting the plug in the proper hole, connection with any ***one*** of two circuits (lamp or bell) can be made at a time With two plugs in the two holes, both circuits can be simultaneously connected.

Fig. 154 (*bottom right*) shows a four-way key (*five* terminals) for controlling four instruments in four circuits. With one plug, only one at a time can be worked.

Arrange the lamp, bell and galvanoscope (Fig. 155) as shown (using a 3-way key, or a 4-way key, leaving one terminal unused), and operate one or the other by insertion of the plug in the proper hole

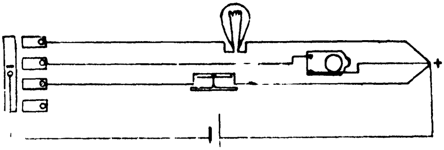


Fig 155 - Three circuits controlled by a plu and four-way key (one terminal unused).

EXPERIMENT 95.—**To study (i) a Morse key, (ii) a Morse sounder, and to arrange a simple telegraph (receiving and sending).**

Apparatus : Two Morse keys, two Morse sounders (or two galvanoscopes, lamps or magnetic needles). Two Leclanche cells, bare or cotton covered wire.

Morse key · Examine the Morse key and its three terminals. It is merely a two-way key of the "press" kind, arranged so that, normally, one circuit (or contact) is closed. On pressing the lever, the second circuit (or contact) is connected, the first one being unmade Normally, the lever touches the back stud. On pressing the knob, the lever touches the front stud, while connection with the back stud is broken.

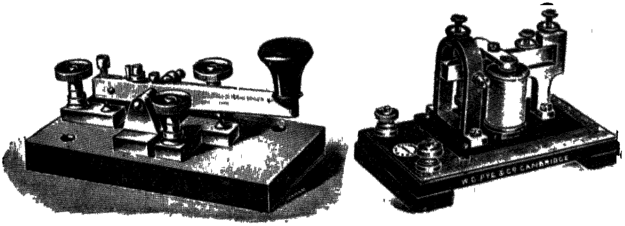


Fig 156 — *Left*, Morse key *Right*, Morse sounder

An ordinary spring key can be converted into a Morse key by arranging, just above the spring, a pointed piece of stiff sheet brass, the other end of which is connected to a third terminal screwed to the base. Bits of silver or platinum foil soldered to the brass point, and to the spring just under the point, provide a reliable contact.

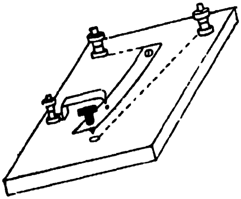


Fig 157.—Improvised Morse key.

The sounder: The sounder is merely an electromagnet, with a soft iron armature pivoted so as to move between two adjustable stops. The tick tack of the armature against these stops, under the action of the current, constitutes the message.

Arrange the simple telegraph as shown in Fig. 158. Connect the positive terminals to the blocks under the knobs, the negatives being connected together or what is better to the gas pipe. Thoroughly scrape the latter, using four or five turns of the bared wire so as to have a good "earth" connection.

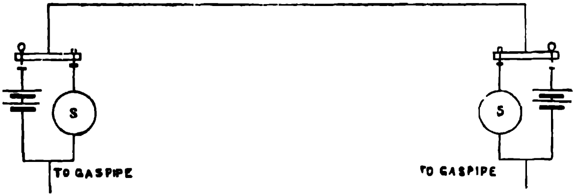


Fig. 158.—A simple telegraph, sending and receiving both ways.

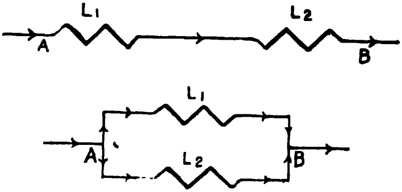


Fig. 159.—Lamps "in series" and "in parallel."

Send and receive, in turn, from each side, tracing the current through the various parts in each case. Draw a clear diagram.

Conductors Connected " in Series " and in "Parallel."—Fig. 159 (*top*) shows two lamps L_1, L_2 connected **"in series"**. The same current passes, through both lamps. Fig. 159 (*bottom*) shows the same two lamps, connected **"in parallel."** Here the current through one does *not* pass through the other. The currents through the two combine at B and return to the battery.

It is easily seen that in the SERIES arrangement, *a breakdown in* ANY ONE *lamp will extinguish both*. This is not the case in the PARALLEL arrangement. *The lamps are independent. Either one can be lighted or put off without affecting the other.*

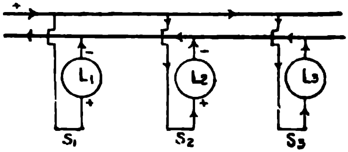


Fig. 160,—Three lamps connected "in parallel" through separate switches.

Fig. 160 shows three lamps " in parallel."

Rules in connection with the Use of the Lighting Circuit.

(1). Connect to mains through wall plugs (preferably fitted to the tables) only. Each plug should have, close to it, a porcelain covered pure tin fuse of a kind easily replaced by the laboratory bearer.

When replacing a fuse, turn off power (if practicable); or stand on a *wooden* stool, *using always only one hand. Do not touch the wall.* Pliers and screw drivers with *wood or ebonite handles* should alone be employed.

(2). Connect to power AFTER the apparatus is completely connected and passed by the instructor as correct.

(3). If anything is wrong, FIRST disconnect and THEN investigate.

(4). Conductors bared for connection, or otherwise, should be wrapped over with insulating tape for *temporary* connections during an experiment, *dry* cotton rags *securely* fastened are sufficient.

(5). If both + and − mains, running close together, have to be bared for connection, **choose two points (one on each main) two or three inches apart** so as to reduce the chances of a "**short circuit**" to a minimum.

(6). Do not allow water or chemicals on tables when experimenting with the supply current.

(7). A separate main switch for each room is desirable.

EXPERIMENT 96.—**To connect three lamps, independently of one another, to a battery or the electric supply, arranging the power wires along the top edge of the table, the lamps along the middle line, and the switches along the bottom edge.**

Apparatus: Three lamps, three switches of the usual household variety (with a battery, three one-way plug keys will do), power (battery or line).

Question: Will you connect the lamps in series or in parallel? Why?

Before proceeding to connect, read the "**rules**" *in connection with the use of power wires, given above on this and the preceedig page.*

Attach one end of each of two long rubber covered wires to a "plug." The wires should be 10 to 12 ft. long. If the "experimental cutouts" are used, expensive wires need not be bared at intermediate points; small pieces can be used repeatedly

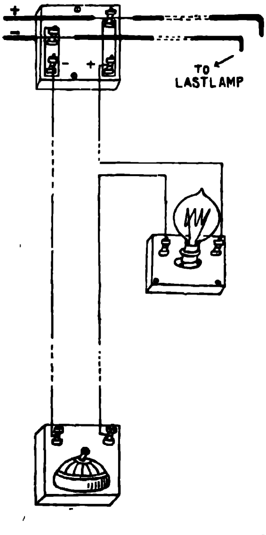


Fig. 161.—A laboratory study of the method of installing incandescent lamps.

without loss. Bare the wires at two points, or put in two experimental cutouts, one at the table corner nearest the plug, and the other in the middle of the table. From each cutout or pair of bared points, lead out two wires, placing the lamp and switch as shown. For connection, twist the branch wires round the bared mains, without cutting the latter. For the last lamp one main may end at one lamp terminal, and the other main reach the second terminal of the lamp through the switch. See that all bared portions are *securely* covered over with rubber tape or *dry* rags. With the switches in the "off" position, put the plug into the power socket. Light the lamps, and see if they are independent of one another. Draw a neat diagram, *all wires being shown straight, either horizontal or vertical. Avoid other angles.* When two wires *cross without contact*, put a *loop* on one near the point of intersection in the diagram.

Question : Do you think you could put a switch for a particular lamp at *any* required point ?

A regular installation (Fig. 162) will include, in addition to above, a main switch (a pair of ordinary switches, one on each main), a pair of " fuses " (one on each main) and a " meter." The connections are exactly as above, except that all terminals, as also the points where the mains are bared for leading off the branch wires, are placed under switch covers or inside porcelain cutouts, so as to be completely out of the way. For details of fittings see the description under Fig. 162.

Record answers to the following :

Q. 1. Suppose you remove a lamp, and connect the terminals of the holder with a piece of wire. What will happen (*i*) with the switch *off*, (*ii*) with the wistch *on* ?

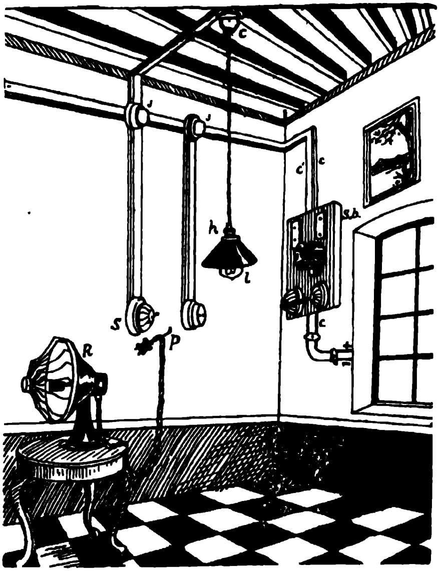


Fig. 162.—A portion of a typical electric installation, consisting of

±, Mains, copper cable No. 16.
cc, Cleats or insulating supports for cables.
sb, Switchboard with (*i*) 2 fusible cutouts or "fuses" mounted on porcelain. In case of a "short circuit," or excessive current due to leakage or misconnection, the tin wires under the porcelain covers melt, thus cutting off the current automatically and protecting the house from fires due to the burning of the rubber insulation. (*ii*) Double pole main switch, consisting of 2 switches one on each main, and (*iii*) a meter connected in series on one main.
c'c', Casing and capping, wood, for protecting the wires (copper No. 18).
jj, Cutouts or junction boxes, for leading out branches from the mains to lamps or fans, and protecting the necessary connections.
s, Switch
C, Ceiling rose, for connecting cable to flexible wires, and relieving the junctions from the weight of the lamp and shade.
h, Holder with bayonet catch for lamp.
l, Lamp.
p, Wall plug for desk fan, table light or
R, 660 watt electric heat radiator.

Q. 2. Suppose the conductors (accidentally bared) touch one another near the switch terminals. What will happen (*i*) with the switch *off*, (*ii*) with the switch *on* ?

Q. 3. Suppose dilute sulphuric acid (or dirty rain water from the ceiling) thoroughly wets (*i*) a cutout, (*ii*) a ceiling rose. What will happen ? To explain, trace the current in each case.

Q. 4. In case of accidents like the above, what protects a house from possible fires, etc ?

EXPERIMENT 96*a*.—**To wire up a lamp for a flight of stairs, so that a person using the stairs can always switch the light "on" at any one end and switch " off " at the other.**

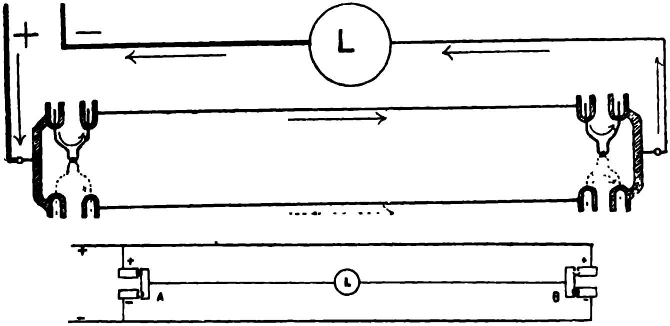


Fig. 163.—Two wiring diagrams for a staircase lamp, controlled from either end by 2-way switches.

Apparatus : Battery, battery lamp, two two-way keys. (For line current use the special two-way switches meant for the purpose. *With care*, plug keys can also serve).

Connect as shown (Fig. 163).

The upper figure shows regular *two-way* switches as used in actual practice. The lower figure shows ordinary two-way keys as used in the laboratory, No matter in which position the plug or the switch at one end is the lamp can be lighted or put out from the other end by placing the plug or the switch at that end in one or the other position. In the positions shown the lamp is lighted.

Evidently, this arrangement will enable the occupant of a room to control his lamp at the entrance or from the bed.

Perform, or draw diagrams for the following exercises.

EXERCISE.—**Put up three bells in three rooms so that the principal of a college, when at his office table, can ring up any one of his three clerks.**

Apparatus: One cell, three pushes or three-way key, three bells.

EXERCISE.—**Arrange a single bell to be rung from different rooms, differently coloured lamps or electromagnetic indicators (Fig. 164) near the bell indicating the room from which the call has come.**

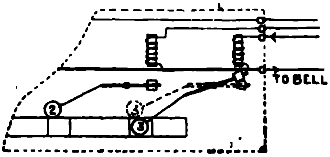


Fig. 164.—Magnetic indicator for bell circuits.

EXERCISE.—**Draw diagrams to show how you will lead off wires to a bed switch (1) from the ceiling rose, (2) from the switch, of a lamp installed in the ordinary way, the existing wires not to be cut or otherwise seriously altered.**

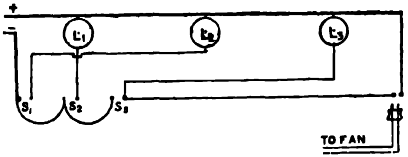


Fig. 165.—An example of misconnection in wiring. Detect the mistakes, and redraw.

EXERCISE.—**A gentleman writes to the wiring contractor to complain (1) that his switches S_1, S_2, S_3**

(meant for lamps L_1, L_2, L_3, respectively) light up the wrong lamps, (2) that the table fan, connected at the plug, works only at night. Detect the mistakes in the wiring and draw a correct diagram (Fig. 165).

EXERCISE (*special*).—*Draw diagrams for burglar alarms for a bank as follows: (i) One bell to ring as soon as the window is opened, (ii) three bells to ring (resident manager's bed-room, servants' quarters, and police station) as soon as the door of the strong room is opened.*

EXERCISE (*difficult.*)—*An old gentleman of moderate means wants the following lighting comforts: to (i) control a 32 C.P. lamp with a switch near the door, (ii) even if he has forgotten to switch off this light before getting into bed, to control this or a 2 C.P. night light from the bed, provided also (iii) that both lamps should never be simultaneously lit*

Apparatus: Two switches, one two-way, the other three-way.

Reversing Switches or Commutators.—It is often necessary to reverse the current in the whole or a part of the external circuit of a battery. This merely means that the end of the circuit connected to the positive pole of the battery should be connected to the negative pole and *vice versa*. The necessary interchange of connections can be rapidly and conveniently carried out, without unscrewing the connecting wires, by special switches called **reversing keys** or **commutators**. There are various designs, and it frequently happens that a student accustomed to one kind is completely upset by another. Four wires (two from the battery, and two from the circuit in which the current is to be reversed) have to be connected to four binding posts on every commutator, and as long as some principle is not followed, the arrangement selected by the beginner will, generally, result in failure.

The Battery Circuit and the Galvanometer Circuit.—It is a help to think of the whole current circuit as made up of two distinct portions :—

(1). **The battery circuit**, containing the battery and any other apparatus in which the current should not or need not be reversed (voltameters, lamps, resistance boxes, etc.). We will call the ends of this circuit + B and — B.

(2). **The galvanometer circuit,** containing the galvanometer, and any other apparatus in which it is desired to reverse the current. The ends of this circuit may be called G_1 and G_2.

Rules for connecting Commutators.—The four terminals of a commutator are to be divided into two pairs, one to be used for + B and — B, the other for G_1 and G_2. Whether *diagonally situated terminals* belong to the same pair or *the adjacent terminals* depends on the design, and the most important thing is to be able to decide this point, for the particular instrument used. The descriptions below are meant to enable the student to do this, for the usual types. Having divided the four terminals into two pairs, the following rules should be followed :

(1) **The two terminals of any one "pair" should be connected to the pair + B and - B, the two terminals of the other pair being used for G_1 and G_2**

(2) Although + B and - B can *as a pair* be connected where G_1 and G_2 *as a pair* are connected, and *vice versa*, **the interchage cannot be made between any** ONE **terminal of the battery circuit and** ONE **terminal of the galvanometer circuit.**

EXPERIMENT 97.—**To study various commutators, and to connect them up for reversing the current in a galvanoscope.**

Apparatus. Various commutators, Leclanche or dry cell. Galvanoscope, galvanometer or magnetic needle. Connecting wires, lamps, resistance coils, etc.

Connect up the various commutators in accordance with the diagrams supplied. Reverse the current. Draw diagrams for both positions of each commutator. When connecting up, divide the circuit into three parts as shown in every diagram: **battery circuit** (battery and lamps or resistance coils) on the *left*, **commutator** near the *middle*, and **the galvanometer circuit** on the *right*. **Plug keys are unnecessary with commutators** (excepting the disc type, Fig. 167) as the latter can *make* and *break*, as well as reverse, the current.

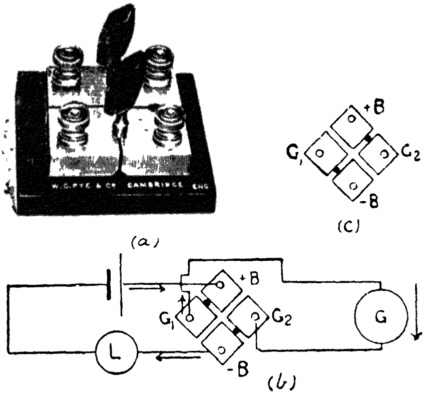


Fig 166.—The plug reversing switch or "commutator"

The Plug Commutator shown in Fig. 166,*a* is reliable and convenient. **Two diagonally situated screws form a "pair."** If +B and −B (or G_1 and G_2) are connected to *adjacent* terminals, failure will result. The two plugs are to be on *one diameter* or the other, but not on *adjacent* radii. Fig. 166,*b*, shows the current entering G_1 and leaving G_2. Draw a diagram and trace the current for the reversed position (Fig. 166,*c*).

The Disc Commutator shown in Fig. 167,*a*, is also very efficient. In principle it is the same as the plug commutator, with the difference that the gaps are bridged over, not by plugs, but by semicircular discs of brass mounted on a revolving ebonite base. **Diagonal screws form a pair.** Fig. 167,*b*, shows the current entering G_1 and leaving G_2. Draw a diagram and trace the current for the reversed position (Fig. 167,*c*). With the disc commutator a separate plug key at K is necessary for *breaking* the circuit.

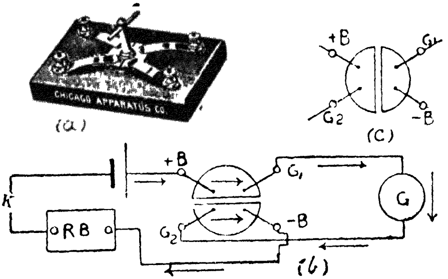


Fig 167 —The disc commutator

The Knife-edge Commutator.—This consists of a rectangular insulating base (Fig. 168,*a*) on which are mounted four clips of springy copper, the diagonal ones being connected permanently by conductors crossing one another without contact. A pair of hinged knife-edges, insulated from one another can make good sliding contact with the two clips on one

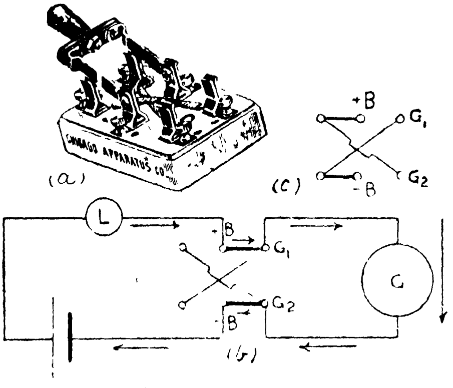


Fig 168 —The knife-edge commutator

side, or the two on the other. The two clip terminals on any *one* side form one pair, and may be connected to G_1 and G_2 The terminals on the knife-edge hinges should be connected to + B and - B. Of the *six* terminals shown, *four* alone are used, and it is best to remove two, as they cause unnecessary confusion. Fig. 168,*a*, shows the current entering G_1 and leaving G_2. Draw a diagram of connections for the reversed position (Fig. 168,*c*).

The Crossed Plug Commutator (Fig. 169,*a*) is efficient and easy to understand. The two terminals to which the plugs are permanently connected form one "pair," and should be used for + B and − B. Use the remaining two terminals, provided with holes for the plugs, for G_1, G_2. One position is shown. Draw a diagram and trace the current for the other position.

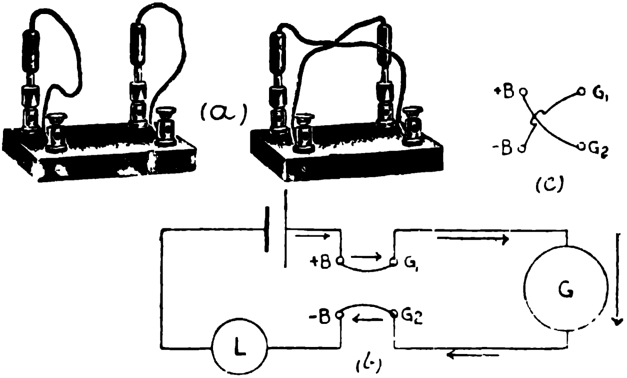


Fig. 169.—The crossed plug commutator.

The Revolving Brush Commutator (Fig. 170,*a*) consists of a wood or ebonite base with two metallic half-rings, on which press a pair of springy brass brushes mounted under a revolving arm. The rings can be connected to G_1, G_2, and the revolving brushes to +B and −B. One position is shown. Draw a diagram and trace the current for the alternative position (Fig. 170,*c*).

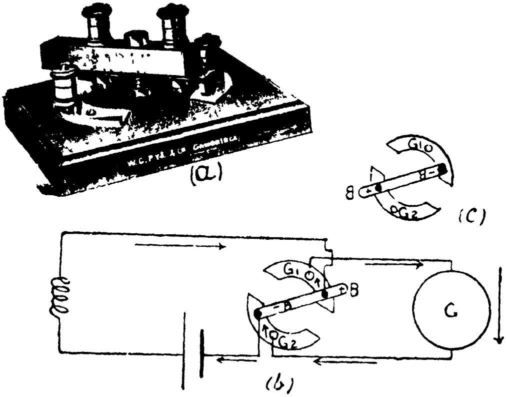


Fig. 170.—The revolving brush commutator.

Pohl's Commutator (Fig. 171,*a*), often met with, is messy and troublesome. It is exactly the same as

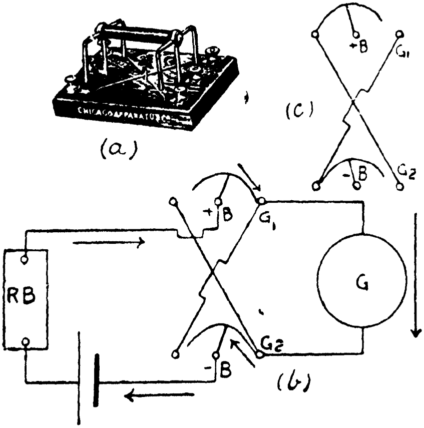


Fig. 171.—Pohl's commutator.

the knife-edge commutator, except that the interchange of contacts is carried out by a six-legged 'spider,' the connected set of three legs on one side corresponding to one knife-edge, and the other set of three to the other. The legs dip in mercury cups connected to six terminals. Connect +B and − B to the cups in which the central legs of the spider permanently dip. For connecting G_1 and G_2 select from the remaining cups any pair (preferably on the longer side of the base) not connected together by a diagonal conductor. The remaining two terminals should be removed. If this is done, the problem of connecting is much easier. One position is shown. Draw a diagram for the other position (Fig. 171. *c*.) obtained by rocking the spider so that the two side legs formerly *out* of the cups are *in* them, and *vice versa*.

The **Commutators of Bertin** and **Ruhmkorff are** rarely used. Explanatory diagrams are appended (Fig. 172, *a*, *b*, *c* and figure under Induction Coil).

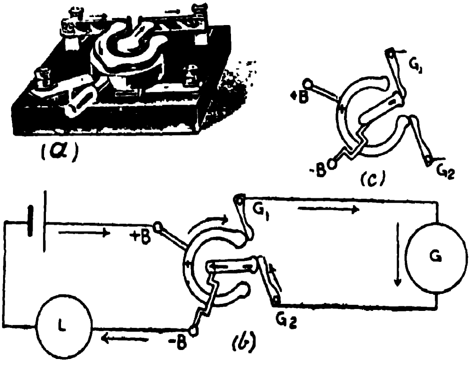


Fig. 172.—Bertin's commutator.

CHAPTER XXXVI

QUANTITATIVE STUDY OF CURRENTS

OHM'S LAW. ELECTROMOTIVE FORCE. RESISTANCE
GALVANOMETERS. VOLTMETERS. AMMETERS

Current.—The quantity of electricity passing any cross section of a conductor, per second, is defined as the **current** at that cross section.

Ohm's Law.—The student is perhaps already familiar with the law of Ohm, best stated thus: **The ratio of the difference of potential (E) applied to the two ends of a conductor, to the current (C) passing through the conductor, is constant for that conductor.** In other words, $\frac{\mathbf{E}}{\mathbf{C}} = \mathbf{R}$, where R is a constant. Evidently, on doubling E, C must double, if R is to be constant. E being constant, C is found to be less (and, consequently R more) for long thin conductors than for short thick ones of the same material. This constant R, whatever it may mean, is called the **resistance** of the conductor.

Hydrostatic Analogy. (Fig. 173 *left*), shows a vessel of water with a small bore tube a filled with coarse sand, from the end of which water trickles in drops at a constant rate w, when the hydrostatic pressure is P. On doubling the pressure, the number of drops per minute will be found to be doubled, so that $\frac{P}{w}$ is a constant (R) for the tube. For the wider and shorter tube A, $\frac{P}{W}$ is also constant, but as compared with a, this constant is less (r), because W (water current) in this case is greater than w. Similarly in Fig. 173 *right*, we have two conductors b and B, to both of which a potential difference or electromotive force E is applied, as shown. The current c in the long thin wire is less than the current C in the short thick wire, and, consequently, constant R ($= \frac{E}{c}$) for the thin wire more than the constant $r \left(= \frac{E}{C}\right)$ for the thick wire. R and r may be called the resistances of the conductors.

Resistance.—As stated above, the proper way of regarding a resistance is to look upon it as the ratio $\frac{\mathrm{E}}{\mathrm{C}}$, especially as, although we have electrometers for measuring E directly, as also galvanometers for so measuring current, we have no instrument for directly measuring resistance. It is nevertheless useful and easy to think of electrical resistance as a physical property of conductors (as in the case of the sand-filled tube), something which increases if the length increases, and decreases if the area of cross section increases

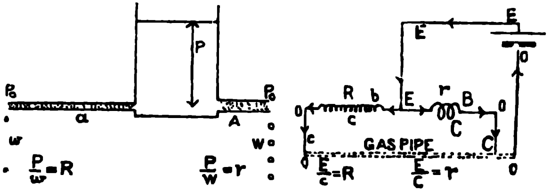


Fig. 173.—Hydrostatic analogy for current, electromotive force and resistance.

Ohm's Law (*continued*).—Assuming electrical resistance to be a definite physical property of conductors, Ohm's law $\left(\frac{\mathbf{E}}{\mathbf{C}} = \mathbf{R}\right)$ can also be stated in the following two ways:

(*i*). *The current in a conductor varies directly as the E.M.F. applied to it, and inversely as its resistance* $\left(\mathbf{C} = \frac{\mathbf{E}}{\mathbf{R}}\right)$.

(*ii*). *The potential difference at the ends of a conductor equals the product of its resistance and the current through it* (**E=CR**).

The Volt, the Ohm and the Ampere.—These are, respectively, the *practical* units of **electromotive force, resistance** and **current,** and the relation between them is thus stated:

$$\frac{\textbf{E. M. F. in Volts}}{\textbf{Current in Amperes}} = \textbf{Resistance in Ohms}$$

or $$\frac{\textbf{Volts}}{\textbf{Amperes}} = \textbf{Ohms} \text{ and } \frac{\textbf{Volts}}{\textbf{Ohms}} = \textbf{Amperes};$$

also, $$\textbf{Volts} = \textbf{Amperes} \times \textbf{Ohms}.$$

A *Volt* is roughly equal to the E. M. F. of a Daniell cell.

The E. M. F. of a Cell (*E*).—The difference of potential between the two plates of a cell, called its **electromotive force** or **E. M. F.**, is independent of the size of the plates or the quantity of liquids in the cell, and *depends only on the nature of these materials.* The E.M.F. of a Daniell cell (large or small) is about 1·08 volt. That of a Grove or Bunsen cell is about 1·8 volts; Leclanche 1·5 to 1·3 volts; lead accumulator 1·8 to 2·2 volts.

The Internal Resistance of a Cell (*b*).—The materials of a cell offer some resistance to the current. This is the internal resistance of the cell. Accumulators have very low internal resistance ($\frac{1}{50}$ to $\frac{1}{10}$ ohm). The internal resistance of a Grove cell may be as low as $\frac{1}{10}$ to $\frac{1}{5}$ ohm. Leclanche and Daniell cells may have any resistance from $\frac{1}{2}$ to 5 ohms.

The External Resistance of the Circuit (*R*).—Connecting wires, and apparatus through which the current passes, have their own (called external) resistance. The resistance of a 1 cm. cube of any material is called its **resistivity** or **specific resistance** (*s*). If the length of such a cube is increased to *l* cm., and its area of cross section becomes *a* sq. cm. (more or less than 1 cm.^{2} the cube becomes a wire of length *l* and cross section *a*, and its resistance $\mathbf{R} = s\frac{\mathbf{l}}{\mathbf{a}}$.

Cells in Series and in Parallel.—

Cells in series: If cells of E. M. F. $e_1, e_2, e_3, \ldots$ and of resistances $b_1, b_2, b_3, \ldots$ are joined in series (Fig. 174), the potentials of the plates of each cell will be as shown. Each cell must have its own peculiar *potential difference* between its plates irrespective of their *actual potentials*. Consequently, if the potentials of the zinc and copper plates of the first cell are 0 and e_1 the potential of the zinc of the second cell will also be e_1, and that of the copper plate of this cell $e_1 + e_2$. The final potential difference between the terminals of the battery of three cells will be $e_1+e_2+e_3$; the total resistance is, similarly, $b_1+b_2+b_3$. For a battery of three *similar* cells each of E. M. F. e and internal resistance b, the total E. M. F. and resistance are, respectively, $3e$ and $3b$.

Cells in parallel: In this arrangement, the positives of all the cells are joined together to form the positive pole of the battery, and all the negatives similarly joined to form the negative pole. The arrangement is used only with cells of equal E.M.F. The figure shows three cells each of E. M. F. e, joined in parallel. The E. M. F. of the battery equals that of a *single* cell, e, because this satisfies the condition that each positive must remain e volts above its own negative. The resistance of the battery is equal to

$$\frac{\textit{Resistance of one cell}}{\textit{Number of cells}} = \frac{b}{3} \text{ (see Fig. 174).}$$

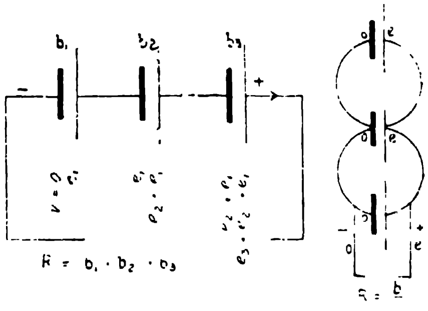


Fig. 174.—A study of E. M. F. and resistance for cells joined "in series" and "in parallel."

Galvanometers, Voltmeters, Ammeters.—No instrument will measure a resistance *directly as such.* E. M. Fs. are also rarely determined directly. As a rule, *what is actually measured is a current.* Measurements of E. M. F. and resistance are generally carried out **indirectly**, either by (1) measuring *currents* (deflection methods) or (2) reducing *currents* to zero (null methods). Of these, null methods are the more important and the more accurate.

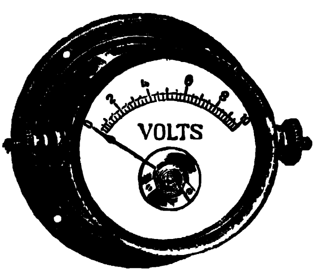


Fig. 175.—A *high resistance* galvanometer or "voltmeter," graduated to give the potential difference between its terminals in *volts*.

A *current* measurer or *galvanometer* is therefore *the* instrument for measurement of E. M. F., resistance and current. A **galvanometer** *is a current detector designed so that the value of the current can be easily obtained from its readings (either directly, or by a simple calculation).*

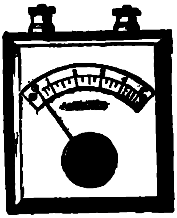


Fig. 176.—A *low resistance* galvanometer, or "ammeter" graduated to give the current through it in *amperes*.

The most useful way of classifying galvanometers is the following simple one:

(1). **High Resistance Galvanometers.**—In these the current carrying coil consists of many turns of *thin* copper wire, so that the resistance of the galvanometer is *high*. Such galvanometers, no matter of what variety, may be called *volt* measurers or **voltmeters** (we shall see why).

(2). **Low Resistance Galvanometers.**—In these the current carrying coil consists of a few turns of *thick* copper wire, so that the resistance of the galvanometer is *low*. Such galvanometers, no matter of what variety, may be called *ampere* measurers or **ammeters** (we shall see why.)

It must be remembered that *voltmeters and ammeters are merely galvanometers specially wound and graduated to suit the special purposes for which they are designed.* *Primarily* they measure only the *current* through their coils, the measurement of *volts* with a voltmeter being an *indirect* process. Figs. 175—177 show respectively, a voltmeter, an ammeter and a galvanometer. All three are moving coil instruments of identical construction, and differ from one another merely in resistance, sensitivity, and the mode of graduating the scale.

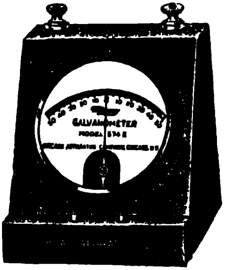


Fig. 177.—A sensitive galvanometer of medium resistance, with an *arbitrary* scale of equal divisions. The instrument is durable and easy to use Levelling is unnecessary and there are no broken suspensions to repair.

Ohm's Law for a complete Current Circuit: Almost all measurements of E.M.F., current and resistance are carried out by including a galvanometer in a complete current circuit, and measuring the current produced (or sometimes by reducing this current to zero). The formula

$$\textbf{Current} = \frac{\textbf{Electromotive Force}}{\textbf{Internal Resistance} + \textbf{External Resistance}}$$

$$\text{or } C = \frac{E}{R+b}$$

is the basis of all such measurements. By making R or b negligible according to the needs of the case, resistances and electromotive forces can be measured.

Verification of Ohm's Law.—To do this, it is necessary to apply different E. M. F.s to a complete circuit and show that in each case the ratio of the E. M. F. to the resulting current is the same.

It is not easy to verify Ohm's law with simple apparatus. Arrangements sometimes suggested merely *assume* the law. In the following experiment it is assumed that (*i*) the E. M. F. of two or three similar cells in series is two or three times the E. M. F. of a single cell. This has been proved (page 393) independently of Ohm's law; (*ii*) that the resistance of the cells (Daniell or Leclanche) used is of the order of 1 ohm or less.

EXPERIMENT 98.—**To verify Ohm's law.**

Apparatus :—Three similar Daniell, Leclanche or dry cells. Voltmeter, range 0-5 volts, or any high resistance galvanometer or any galvanometer with a high resistance in series with it.

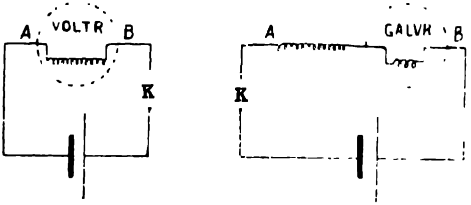


Fig 178—Verification of Ohm's law.

If a voltmeter is used connect it to the cell through a plug key K. The high resistance coil of a tangent galvanometer can be employed, but the use of the instrument is not desirable at this stage. A Weston galvanometer (Fig. 177) can be easily used with a high resistance (10 to 20 thousand ohms) in series as shown (Fig. 178).

Connect one cell to the voltmeter, or its substitute.

Read the deflection. Repeat with two cells in series. The E. M. F. applied to the circuit is now doubled. *The circuit itself is practically unchanged*, as the resistance of the additional cell (one ohm or less) is *negligible* as compared with the resistance of the voltmeter, etc. (some thousands of ohms). Connect three cells and read the deflection. Three times the E. M. F. has now been applied to what is practically the same circuit.

The current C varies as the deflection δ (in a tangent galvanometer it various as tan δ). $\frac{E}{\delta}$, therefore, stands for $\frac{E}{C}$. Calculate $\frac{E}{\delta}$ for the circuit. This will be found constant in every case. *Assume* the E.M.F. of a Leclanche cell to be 1·5 volts, and that of a Daniell cell as 1·08 volts.

Record observations thus:

E	Deflection. δ	varies as δ or *tan* δ	$\frac{E}{\delta}$ that is $\frac{E}{C}$
1·5 volts	δ_1	......	$\frac{1{\cdot}5}{\delta} =$ ....
3·0 volts	δ_2	......	$\frac{3{\cdot}0}{\delta_2} =$ ....
4·5 volts	δ_3	......	$\frac{4{\cdot}5}{\delta_3} =$ .. ..

$\frac{E}{\delta}$ will be found constant. For a voltmeter $\frac{E}{\delta}$ will be 1, because *a voltmeter is a galvanometer specially graduated to read volts direct.*

Important Conclusion from Experiment 98.—The result of experiment 98 can also be stated in this form :—

Different batteries or cells connected to a high resistance galvanometer give deflections proportional to their electromotive forces.

Such a high resistance galvanometer can be graduated so that a 2 volt battery gives a deflection of 2 divisions, and a 4 volt battery a deflection of 4 divisions. In this case the galvanometer becomes a voltmeter.

This will not be true if the circuit or galvanometer is of *low* resistance, because the effect of varying battery resistance will no longer be negligible.

Suppose that the Leclanche cells used in the last experiment were each of E. M F 1·5 volts and internal resistance 1 ohm. The resistance of the voltmeter or the total of galvanometer and additional resistance may have been 2000 ohms.

The current can be obtained in each case by substituting the proper values in the equation

$$C = \frac{E}{b+R}.$$

If the current is proportional to the deflection we can write $c = k\delta$ where k is a constant for the galvanometer used.

With *one* cell in series,

$$c_1 = k\delta_1 = \frac{1·5 \text{ volts}}{1+2000 \text{ ohms}} = \frac{1·5}{2000} \text{ amperes (neglecting 1 as compared with 2000).}$$

With *two* cells, we have

$$c_2 = k\delta_2 = \frac{3 \text{ volts}}{2+2000 \text{ ohms}} = \frac{3}{2000} \text{ amperes (neglecting 2 as compared with 2000).}$$

Dividing,

$$\frac{\delta_2}{\delta_1} = 2,$$

that is the deflection with two cells is double the deflection with one cell, that is, it is proportional to the E. M. F.

If the circuit resistance had been 1 ohm instead of 2000 ohms, the currents with one and two cells would have been, respectively, $\frac{1·5}{1+1}$ and $\frac{3}{2+1}$ or ·75 amperes and 1·0 amperes, respectively. The deflection in the second case is no longer *double* but is only 4/3 times the deflection of one cell. The deflection is *not* proportional to the E. M F. because the battery resistance is no longer *negligible*, as compared with the low resistance of the circuit

EXPERIMENT 99.—**(1) To distinguish between the E. M. F. and internal resistance of a cell. (2) To measure E. M. Fs. of cells with a voltmeter. (3) To study the E. M. F. of cells in series and in parallel.**

Apparatus : Two Daniell cells, one large, the other small (the smaller the better), one Leclanche or Grove cell, voltmeter (0-5 volts), ammeter (0-5 amperes (use suitable galvanometers, if voltmeters and ammeters are not available).

(1) Connect to the *voltmeter* (*i*) small Daniell, (*ii*) large Daniell. The *deflection* is the same, showing that the *current* through the voltmeter (a galvanometer of high resistance) is the same.

In the case of either of the cells pull the zinc rod partly out of the liquid, keeping it, however, steady. This alters b. Is the deflection affected ? Move the zinc laterally. Does this affect the deflection ?

The current equals in every case $\frac{E}{b+R}$. R is *very high*, perhaps 2000 ohms. b may be 1 or 2 ohms for any of the two cells, so that it is negligible compared with R. What is your conclusion about E in case of the two cells ? Connect now a Leclanche or Grove cell. The deflection is different. Do you think this difference due to a change in E or a change in b ? Do you understand why the *high* resistance galvanometer used is called a *voltmeter* ?

(2) Connect to the *ammeter* (*i*) small Daniell. (*ii*) large Daniell. The deflections are different. Again consider the expression $\frac{E}{b+R}$. E is the same in both cases. R is now very small (an ammeter is a *low* resist-

ance galvanometer. $R = \frac{1}{100}$ ohm, say). b is now important. What is your conclusion ? On connecting a Leclanche, is it necessary that the deflection should be more or less ? Try.

(3) Connect the two Daniells (*i*) in parallel, (*ii*) in series, connecting the battery terminals in each case to the voltmeter. Connect also (*iii*) all the three cells in series to the voltmeter. Record the deflections.

Calculate from the voltmeter and ammeter readings the internal resistances of the various cells. $b = \frac{volts}{amperes}$.

[The galvanometer shown in Fig. 177, with a high resistance in series, can replace the voltmeter, and when shunted by a copper or German silver wire of suitable size, the ammeter. Even the high resistance (500 turns) and low resistance (2 turns) coils of a tangent galvanometer will serve, the *tangents* of the deflections being added, etc. Neither of the above arrangements is really convenient. In the case of a tangent galvanometer especially, the details of setting, levelling and reversing of current are sure to divert the attention of the beginner from the fundamental facts of E.M.F., resistance, etc., that are studied in the above experiment. It is best for the beginner to handle this complicated and antiquated instrument at a later stage, when he will understand it better and more easily than here].

Record observations as follows:

Fundamental formula: deflection varies as $\frac{E}{b+R}$.

Cell or cells	Voltmeter or Ammeter	Reading	Which is negligible? b or R	Conclusion
Small Daniell	voltmeter	θ_1	b	θ_1, θ_2 nearly equal, $b+R$ nearly constant, $\therefore$ E for the two cells......
Large Daniell	,,	θ_2	b	
Leolanche	..	θ_3	b	
Small Daniell	ammeter		R	E constant, deflections different $\therefore$ b_1, b_2 .. .
Large Daniell	,,		R	
Leclanche	,,		R	
2 Daniells in parallel	voltmeter	θ_4	b	Deflection (E.M.F.) same as for one cell
2 Daniells in series	..	θ_5	b	$\theta_1 + \theta_2 = \dots$ nearly $\theta_5 = \dots$ equal hence, when cells are joined in series, E.M.F.s are *added.*
All the three cells in series.	,,	θ_6	b	$\theta_1 + \theta_2 + \theta_3 = \dots$ nearly $\theta_6 = \dots$ equal E.M.F.s are *added*

CHAPTER XXXVII

MEASUREMENT OF ELECTROMOTIVE FORCE, CURRENT AND RESISTANCE BY VOLTMETER AND AMMETER

Voltmeters and Ammeters.—Voltmeters and ammeters provide simple means of measuring E. M. F. and current. Resistances can also be easily calculated with the help of these instruments. As the readings are taken directly in volts and amperes, all relations become very easy to understand, and the significance of Ohm's law is quickly grasped by young students. For really accurate work, somewhat expensive moving coil instruments are necessary. They cost, however, no more than good tangent galvanometers. Weston's are the cheapest in the long run. A certain amount of valuable work, as far as educational value is concerned, can, however, be done even with moving iron instruments of reasonable size. These are cheap, and have the advantage of working equally well with currents in either direction so that the terminals can be connected irrespective of polarity. If moving coil instruments are used, the centre zero type are likely to give less trouble than the side zero type.

As it is not convenient to give more than one cell (2 volts) to each student, the following ranges for voltmeters and ammeters will be found suitable for the standard equipment :

Voltmeters :	0—2·5 volts by $\frac{1}{10}$ volt.
Ammeters :	0—1 ampere by $\frac{1}{20}$ ampere.

Occasionally, ranges of 6 volts (3 cells) and 4 to 6 amperes may be required. For experiments with fine wires, a range 0—·1 ampere will be convenient. Two or three ranges in one instrument can be obtained.

Battery.—A battery of accumulators, suitably installed and with wires leading out to tables, is an essential for efficient work by a class of 16 to 20 students. Leclanche and Daniell cells cannot supply strong currents, and can be used only for testing work with the bridge or the potentiometer. Grove and Bunsen cells, though quite strong, are, like all primary cells, expensive, untidy and unreliable, and the labour of putting up a supply for a whole class is a very serious consideration.

Wireboard.—Ordinary laboratory coils and resistance boxes are unsuitable for use with an ammeter, as they are designed for tests with weak currents or by null methods, a delicate galvanometer being employed. For the heavier currents required in many experiments, bare or insulated wires of German silver of suitable diameter can be used. As loose pieces of wire get lost, it is convenient and economical to stretch them on a wireboard, 110 cm. long, and 8 cm in breadth. If made as described, the wireboard can be used as a potentiometer or a slide-wire bridge, and a set of eight or ten (the apparatus can be made at small cost) forms a good substitute for expensive bridges and potentiometers.

The apparatus (Fig. 182) may be made as follows:

Base board. Non-warping hard wood (teak), 44" × 3½" × ¾".

Terminals. Four corner ones, large double grip for holding two wires simultaneously. Distance between terminals should be 100 cm. clear. Middle terminal for 50 cm wires should be a *small* ordinary binding screw. This terminal can also grip a 100 cm wire in the middle, and if *small*, it will not materially alter its effective length. All terminals should have the lowest contact surface nearly flush with or just a trifle *above* (if *below*, bad contacts may result) the wood, so that the straps when screwed down are on a level with the board, and yet preserve good electrical contact.

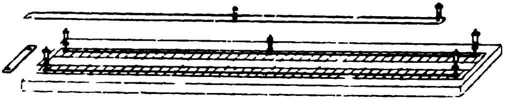


Fig. 179.- The wireboard for experiments on wires. Can be used as slide-wire bridge or potentiometer.

Copper straps. $\frac{1}{16}$" sheet copper or brass strips, ¾" wide. The longer strap, put on and screwed down with the three top terminals, converts the board into a slide-wire bridge. The shorter one put on a pair of terminals on the side, gives a potentiometer.

Knife-edge contact or "jockey" This can be purchased (Fig 183, *left*) Many forms are available A common binding screw filed to an edge, and fixed to a wooden handle (Fig. 183, *right*) by means of a metal strip soldered to the screw, is quite effective.

Cm scale. 100 cm. millimetre scale, graduated along both edges, to serve for both wires.

Wires. Any size required can be put on. Use the table given below. For potentiometer or bridge work. Eureka or platinoid No 28 is convenient

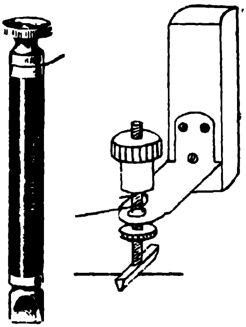


Fig. 180.—"Jockey" or contact maker.

The instructor will find the following table handy in selecting wires for the class. The numbers refer to the Standard Wire Gauge.

For resistance per metre (roughly)	USE WIRE					
	Iron		*German silver*		*Eureka, platinoid or manganin*	
	No.	Diameter mm.	No.	Diameter mm.	No.	Diameter mm.
1 Ohm	30	·31	24	56	22	·71
2	34	·23	28	·37	26	·45
3	36	19	30	·31	28	·37
4	38	·15	34	·23	30	·31
5			36	·19	34	·23
10			40	·12	38	·15
20					40	12
30						

The Current through any Galvanometer proportional to the Potential Difference between its Terminals.—This follows from Ohm's law, because the current carrying coil of a galvanometer is an ordinary conductor, for which $C = \frac{E}{R}$. The *current* through a galvanometer (given by its deflection) is, therefore, an indicator of the *potential difference* between its own terminals. The statement is essentially correct for every galvanometer, irrespective of the value of its resistance. Every galvanometer cannot, however, be actually used in practice for determining potential differences.

The Special Function of the High Resistance Galvanometer called "Voltmeter."—We have already seen that the deflections produced in a voltmeter or other high resistance galvanometer are proportional to the E. M. F.s of the batteries or cells connected to the terminals. This, however, is not true of a *low* resistance galvanometer. A voltmeter can be used not only for determining in volts the E. M. F. of a battery, but also for determining in volts the potential difference between any two points A and B (Fig. 181) of a current carrying wire. This can be done by connecting the VOLTMETER to the points A and B IN PARALLEL with the wire AB, which should not be cut or injured.

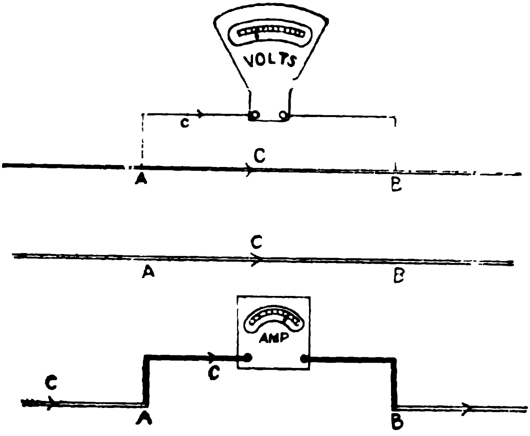


Fig. 181.—AB current carrying wire. *Top*, potential difference between A and B measured by high resistance galvanometer (voltmeter) connected at A and B in parallel with the conductor AB. *Bottom*, the current in AB indicated by low resistance galvanometer (ammeter) connected *by thick wires* in series, in a gap made by cutting AB at any one point.

The voltmeter of course indicates the *actual* potential difference existing between A and B, *after the voltmeter is connected.* This, however, is very nearly the same as the potential difference between A and B *before the voltmeter was connected,* because the voltmeter is taking *very little current* itself *on account of its very high resistance.* The current in the wire A B is practically undisturbed (in reality *very slightly less*) and the potential difference between A and B practically unchanged (in reality, *very slightly less*) by the connection of the voltmeter. Had a low resistance galvanometer been connected, a good deal of the current originally passing through the wire AB would have been diverted to the galvanometer and the potential difference between A and B completely disturbed. A voltmeter is so called because the scale is graduated to give the potential difference between the points connected directly in "**volts**."

[*Q.* What will happen if the voltmeter is connected *in series* between A and B ?]

The Low Resistance Galvanometer or Ammeter.—A *low* resistance galvanometer can be introduced *in series* at any point in a circuit, without materially altering the current in that circuit. When thus connected, it will indicate the current through it, which is the same as the current in the circuit before the galvanometer was connected. **If graduated so as to read amperes, the low resistance galvanometer is called an ammeter.** The current in the wire AB (Fig. 181) can be indicated, by *cutting the wire and connecting the ammeter in series with* **thick connecting wires** (why thick ?), as shown. The low resistance of the ammeter does not alter the current to be measured.

[*Q.* What will happen if the ammeter is connected to A and B *in parallel* with the conductor AB ?]

Fall of potential or "Drop of volts" along a Current carrying Wire.—Ohm's law expresses *quantitatively* what must be evident to the student *qualitatively* that as we travel along a current carrying wire ABCD (Fig. 182*a*), the potential must fall steadily, exactly as if we travel along a river current we must steadily come *down*. The potential at A must be *higher* than the potential at B.

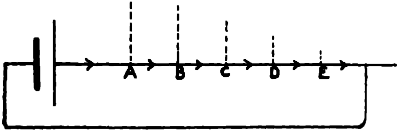


Fig. 182*a*.—Fall of potential or drop of volts along a uniform curtren carrying wire.

The fall of potential between two points on a current carrying wire of uniform cross section is proportional to the distance between the points.

EXPERIMENT 100.—**To study the fall of potential along a uniform current carrying wire.**

Apparatus : 2-4 volt battery, thin resistance wire

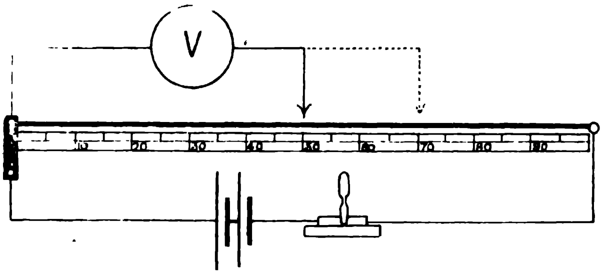


Fig. 182.—Fall of potential between two point on a uniform current carrying wire varies as the distance between the two points.

(100 cm.) stretched on wireboard (or the thin wire on a potentiometer), voltmeter (2-4 volts), jockey, plug key, wires, etc.

Connect, through a plug key, a 2-4 volt battery of constant cells (secondary or Daniell), to the two ends of the thin resistance wire. To the zero end of this connect also one terminal of the voltmeter. The other terminal should be connected to a jockey. Note the zero reading of the voltmeter, and calculate the zero correction. Start the current, wait a second or two, and placing the jockey on the wire opposite the 10 cm. mark read the voltmeter carefully. Take out the plug and record the reading. Repeat with the jockey at 20, 30,.........90 cm., taking care to pass the current only when necessary. Plot a graph with length as abscissæ and voltmeter readings as ordinates.

Record thus :

Zero reading of voltmeter = − ·04 volt.
Zero correction = + ·04 volt.

LENGTH OF WIRE l	VOLTMETER READING	CORRECTED VOLTS V	VOLTS PER CM. V/l
10	·14	·18	·0180
20	·31	·35	·0175
30	·49	·53	·0176
40	67	·71	·0177
50	·85	·89	·0178
60	1·03	1·07	·0178
70	1·21	1·25	·0178
80	1 38	1·42	·0177
90	1·55	1·59	0176

(Q. What sort of curve will you get for V and l. ?)

Mean fall of potential per cm. = ·0177 volt.

Hence fall of potential for any length L cm. = ·0177 × L volts.

Hence, fall of potential varies as length of wire.

Drop of Voltage along Electric Light Mains.—The P. D. between electric light mains becomes less and less as we travel from the power house to the place where the current is being used. The lamps at the far end of a line, therefore, burn somewhat dimmer than those at other points. This is illustrated by the following experiment.

EXPERIMENT 100*a*.—**To study the fall of potential along electric light mains.**

Apparatus : Two or three 220 volt lamps, fine resistance wire.

[With battery lamps the experiment can be more easily performed, as *short* pieces of German silver wire suffice (see below). 220 volt lamps illustrate an actual case].

Connect the lamps individually. They are brilliantly lighted.

Now connect them in parallel (Fig. 183) with ordinary flexible wire. They are still brilliantly lighted. The currents in the different lamps, and in different portions of the connecting wires are as shown.

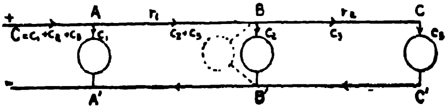


Fig. 183.—The "drop of volts" increases with (*i*) load, (*ii*) resistance of the leads.

The voltage drop (E = CR) from A to B is $(c_2 + c_3)\ r_1$, and from A to C, $(c_2 + c_3)\ r_1 + c_3\ r_2$. These are, ordinarily, negligible, because the resistances r_1 and r_2 of the connecting copper wires are negligible. The potential at all the three points, A. B. C is, therefore, practically 220 (assume A'B'C to be very thick and at potential 0). The potential at C can be decreased by (*i*) increasing the resistance of *any* portion between A and C, (*ii*) by increasing the current in *any* portion between A and C.

Increasing r_1 and r_2: Connect A and B, and B and C, with pieces of insulated German silver or manganin wire.

For battery lamps, short pieces of thick German silver wire will suffice. For 220 volt lamps, use one or 2 metres of very fine (No. 36) insulated Eureka or manganin wire Keep this wire well away from A'B'C' by coiling it up on a pencil or piece of glass tubing.

The voltage at B and C will decrease, the lamps B and C decreasing in brightness, while the lamp A remains fully bright.

Increasing c_1, c_2 *etc.*: Remove the resistance wire from between B and C, allowing that between A and B to remain. Add at B, as shown, a lamp of much higher candle power. The first lamp at B and the lamp at C will become *less* bright, because the increased current in AB has caused the potential of B (and hence of C), to fall another step.

On replacing the German silver wire by ordinary wire (decreasing the resistance of the feeding wires) all lamps become fully bright.

Q.—A house is lighted with four 16 C. P. lamps. On substituting in one room a 200 C. P. lamp, the lamps in two other rooms become dim while one is unaffected. What are the relative positions of the lamps on the mains ?

Q.—If you extend an installation for four lamps to twenty lamps, why is it necessary to use thicker wires ?

Determination of Resistance by Voltmeter and Ammeter.—Determination of resistance by voltmeter and ammeter is a simple matter. Resistance in *ohms* $= \frac{volts}{amperes}$. The volts at the ends of a conductor being given by the voltmeter, and the amperes through it by the ammeter, the resistance in ohms can be quickly calculated by simple division.

EXPERIMENT 101.—**By voltmeter and ammeter (i) to determine the resistance of a lamp, or coil of wire, (ii) to show that the resistance of a given uniform wire is proportional to its length.**

Apparatus : Voltmeter, ammeter. Battery lamp, or *thick* wire coil* (2 to 3 ohms), 100 cm. wire (No. 26 or No. 28 Eureka), wireboard, one or two constant cells (accumulators).

(*i*) Connect the lamp, wire or coil to the battery, interposing an ammeter in series (Fig. 184). Read the ammeter carefully. Connect the voltmeter to the ends of the resistance to be measured. The ammeter reading will remain practically unchanged (explain). Read the volts indicated. $\frac{\textbf{Volts}}{\textbf{Amperes}}$ gives the required resistance in **ohms**. Use a plug key in the circuit, keeping the current on no longer than is actually required for taking the readings (why ?). Try different currents. The resistance of a fully lighted lamp will be found considerably different from that when the lamp is a dim.

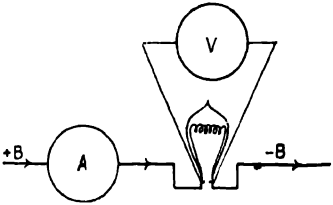


Fig. 184.—The resistance of a lamp determined by voltmeter and ammeter.

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* Ordinary laboratory coils are unsuitable.

(*ii*) Connect up the wire as shown (Fig. 185). For the same ammeter reading obtain the volts for 100 cm. as also for 75, 50, and 25 cm. of the wire as in experimet 100. Use the knife-edge contact for 25 and 75 cm. points. Watch the ammeter to be sure that its reading is unchanged. If necessary, wait between readings to give the wire time to cool. If the current varies, record ammeter readings corresponding to individual volt readings.

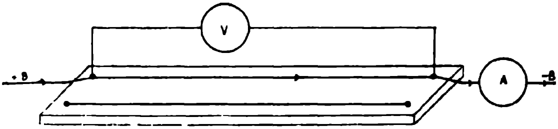


Fig. 185.—The resistance of a wire (stretched on the wireboard) determined by voltmeter and ammeter.

Enter observations thus:

Conductor		Volts (corrected) V	Amperes (corrected) A	Ohms $R = \frac{V}{A}$	Ohms per cm. R/l
Lamp:					
	White hot	...	...	...	
	Red hot	...	...	...	
Wire:					
	l = 100 cm.	...	...	...	...
	75 cm.	....	...	...	...
	50 cm.	...	...	...	...
	25 cm.	...	...	...	...

Mean resistance per cm. of wire = ... ohms.
Resistance of any length L = L × ...... ohms.
Hence, *resistance varies as length of wire*.

EXPERIMENT 101*a*.—(*i*) **To verify the formulae for the joint resistance of conductors "in series" and "in parallel." (*ii*) To determine how the resistance of a wire varies with its diameter.**

Apparatus : Voltmeter and ammeter, wires Eureka or platinoid No. 20 (two piece 100 cm. each), No. 26 and 28 (one 100 cm. piece each). Battery.

(1). Mount two wires, one No. 26 and the other No. 28 side by side on the wireboard. Determine the resistance of each. Next connect the two **in series** by a *short* piece of *thick* copper wire (1 mm diameter) between the terminals at one end (Fig. 185), and pass the current as shown. Read the ammeter. Read also the volts at the terminals on the battery side. Calculate the joint resistance. Do you find this **joint resistance** nearly **equal to the sum of the individual resistances ?** Next connect **in parallel** by short pieces at both ends, connecting the voltmeter as shown, and determine the joint resistance. Check to see if the **reciprocal of the joint resistance equals the sum of the reciprocals of the two resistances.**

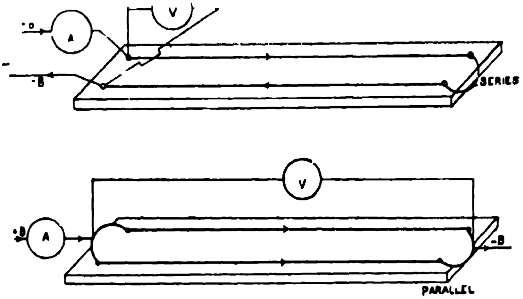


Fig. 186.—Resistances in series and parallel determined by voltmeter and ammeter.

(2) Next determine the resistance of 200 cm. (two 100 cm. pieces in series) of wire No. 20. The diameter of wire No. 20 is almost exactly double the diameter of No. 26 used in (1). Two metres are used, as the resistance of one metre is not enough. Calculate the resistance of one metre, and compare this with the resistance of wire No. 26. Is the ratio nearly 1 : 4 ? The diameters are as 2 : 1. How does the resistance vary with the diameter ?

Record observations thus :

No. and length of wire	P.D. in volts V	Current in amperes A	Resistance in ohms R = V/A	Compare with
(*a*) 1 metre No. 26	..	...	... R_1	
(*b*) 1 metre No 28	...	..	... R_2	
(*a*) and (*b*) in series	...	...	... R_3	$R_1 + R_2 = \ldots$
(*a*) and (*b*) in parallel	...	...	... R_4	$\frac{R_1R_2}{R_1+R_2} = \ldots$
(*c*) 2 metres No. 20	...	...	... R_5	
1 metre No. 20	Blank	Blank	$\frac{R_5}{2} = R_6$	$\frac{R_1}{4} = \ldots$
Diameter wire No 26 = ·45 mm...d_1			$\left(\frac{d_2}{d_1}\right)^2 =$	...... } equal.
„ „ No. 20 = ·91 mm...d_2			$\frac{R_1}{R_6} =$	.....

∴ Resistances of equal lengths vary inversely as the diameters squared or inversely as cross sectional areas.

For calculation of joint resistance in parallel, use tables of reciprocals thus :

$$\frac{1}{R_1} = \ldots\ldots\ldots$$

$$\frac{1}{R_2} = \ldots\ldots\ldots$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \ldots\ldots\ldots(x).$$

$$\text{Hence } R = \frac{1}{x} = \ldots\ldots\ldots\ldots \text{ ohms.}$$

Resistance of Cells in Series and in Parallel.—Resistance of single cells has already been measured in Expt. 99. Resistance of cells in series and in parallel can be similarly measured, readings being taken in each case (*i*) with a voltmeter to give E, (*ii*) with an ammeter (using *short thick* connecting wires) to give $C = \frac{E}{b}$, because R is negligible. The ammeter and voltmeter should be connected one at a time (*not* simultaneously) to the cell or cells. $b = \frac{E}{C} = \frac{\text{volts}}{\text{amperes}}$. The relations, $\dot{B} = b_1 + b_2$ for series, and $\frac{1}{B} = \frac{1}{b_1} + \frac{1}{b_2}$ for parallel connection can thus be verified.

CHAPTER XXXVIII

CHEMICAL EFFECTS OF CURRENTS

ELECTROLYSIS. FARADAY'S LAW. ACCUMULATORS

General.—The passage of a current through a liquid is accompanied (in case the liquid is a compound) by its decomposition into two parts called **radicles**, one of which is liberated at the point **(anode)** where the current enters the liquid, and the other at the point **(kathode)** where it leaves. The amount of a radicle liberated per second is proportional to the current. **The amount** *of a radicle* **liberated per second** *by* **a current of one ampere** *is called* **the electrochemical equivalent** *of that radicle.* This relation is represented by the equation (**Faraday's law**): $m = e\,C\,t$; where m is the mass liberated by current C in time t, and e is the **electrochemical equivalent**. The relation can be used for determining C if e is known, or *vice versa*, provided m and t are determined experimentally.

EXPERIMENT 102*a*.—**To study electrolysis with a water voltameter.**

Apparatus: Water voltameter, ammeter, voltmeter, plug key, 4-volt battery or electric supply mains and lamp resistance, watch.

Connect up the voltameter in series with an ammeter, plug key, and either a 4-volt battery or the electric supply wires with a lamp resistance (consisting of two or three lamps in parallel) in series on one main. For more current, use more lamps or lamps of higher candle power. Before making connection, fill up the voltameter and the tubes with dilute sulphuric acid. Tubes graduated in c. c. are necessary.

Preliminary experiment : Pass the current for about five minutes, so as to allow the electrodes to absorb as much gas as they will. If the hydrogen collected occupies much less than one-third the tube, use more current for the final experiment so that the time required for filling up the hydrogen tube does not exceed fifteen to twenty minutes.

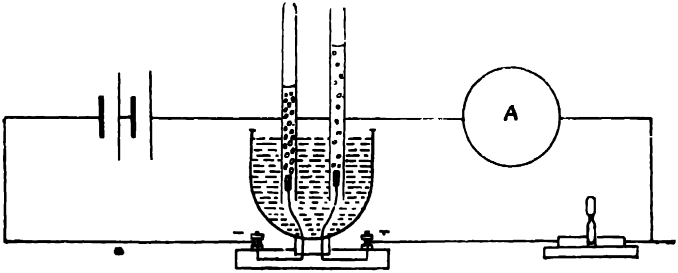


Fig 187.—Electrolysis of water.

Final experiment : Refill and invert the tubes, and put in the plug when the seconds-hand of your watch passes sixty. Read the ammeter. Withdraw the plug exactly after three minutes, and read the volume of hydrogen evolved. Insert the plug again when the seconds-hands passes sixty and read the volume and the current after three minutes. The ammeter reading should remain constant. Obtain five or six readings, or till the hydrogen reaches the level of the acid in the voltameter. Finally, disconnect the apparatus. Connect the voltameter to the voltmeter and observe what happens? Record the reading. Is the voltameter behaving like a cell? With pole paper, or otherwise, determine which electrode is acting as the positive pole of this cell.

Read the volume of oxygen obtained. Test the gases in the usual way for hydrogen and oxygen.

Calculate the volume of hydrogen evolved in each three minute interval. Also draw a graph with time as abscissæ and volume of gas evolved as ordinates.

Enter observations thus :

Volume of hydrogen (total) = ...

Volume of oxygen = ... Ratio = $\frac{\dots}{\dots}$ = ...

Time (minutes) *t*	Current (amperes) *c*	Total volume H evolved *v*	Volume evolved in three minutes	P. D. between electrodes after electrolysis	Positive pole of cell formed (anode or kathode)
0	...	...	...		
3	...	...	...		
6	...	...	...		
9	...	...	...		
12	...	...	...		
15	...	...			

Current = ... amperes.

Hydrogen evolved by one ampere in one second = ... c c.

Graph drawn for *t* and *v*.

What is the shape of the graph and what does it signify ?

How will you calculate *e*, the *electrochemical equivalent* for hydrogen ?

Q. The electrochemical equivalent of copper is ·000329 grammes per ampere second (coulomb). How will you test the accuracy of a given point of the scale of an ammeter by electrolysis of copper sulphate solution ?

Secondary Batteries or Accumulators.—In the above experiment we saw that, after electrolysis, the platinum electrodes of the water voltameter behaved like the two dissimilar plates of a voltaic cell. This was due to one platinum electrode absorbing oxygen, and the other hydrogen, so that for the time being, they became practically different metals immersed in dilute acid. The cell so formed registered a voltage of nearly 1·4 volts, and it was shown that the anode was the positive pole, that is, the current supplied left by the electrode where the **charging current** (current used for the original electrolysis) entered and *vice versa.* This results in a current, whose direction *inside the voltameter* is opposite to the direction of the charging current, so that oxygen is evolved on the electrode which has absorbed hydrogen, and *vice versa.* Consequently, in both cases the *occluded* gases are, after some time, converted into water, and the cell ceases to supply current or is **discharged.** It can be again **charged** by a current from the external battery as in the first case.

If **lead plates** *are used as electrodes in a water voltameter,* **the oxygen combines chemically with the anode to form lead peroxide,** and a much larger amount of the gas can thus be absorbed, than by a platinum electrode, which merely *occludes* it. The size of the electrodes does not affect the E. M. F. of the secondary cell formed, but as the *larger* plates can accumulate more lead peroxide, they have larger **capacity,** that is, they can supply a current of given strength (say one ampere) for a longer *time* than smaller plates.

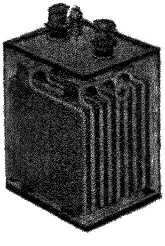


Fig 188 —Accumulator in celluloid case.

Secondary cell can supply strong currents for many hours and are therefore extensively used. When **discharged,** they can be **charged** at small cost with current from the electric supply and are, consequently, much cheaper to work with than primary batteries. Fig. 188 shows an accumulator. For larger **capacity,** three or more plates connected together form one pole, and a similar number the other pole. **The terminal connected to the red brown plates is the positive pole.**

EXPERIMENT 102.—**To study the principle of the lead accumulator.**

Apparatus : Pair of lead plates 6″ × 2″, dilute sulphuric acid, voltmeter, ammeter, 4-volt charging battery or current from electric supply wires through lamp resistance, pole paper.

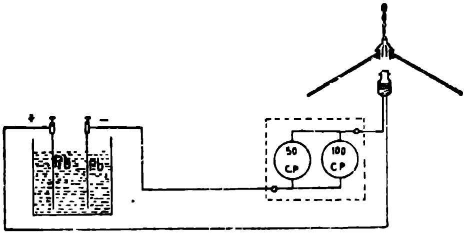


Fig. 189.—Charging an "accumulator" from the electric supply wires connected through a lamp resistance.

Connect the apparatus as shown (Fig. 189), or use a 4-volt battery for charging. Observe the plate connected to the positive wire or the *anode.* Is the *oxygen* coming up in bubbles? Next observe the *kathode.* Is the *hydrogen* coming up in bubbles? Do you notice more oxygen coming up as time elapses? What became of the oxygen liberated at first? Do you notice any change in the *colour* of the anode? Explain.

After fifteen minutes, or half an hour, disconnect the apparatus. Place the ends of the wires from the lead plates on moistened pole paper. Which is the positive pole? The brown or the blue plate?

Determine the E. M. F. of the cell formed, with the voltmeter. Finally, connect to an ammeter. The cell will be rapidly **discharged.** You can charge it again by current from the mains.

Record in your note book

(*i*) brief answers to all the above questions,
(*ii*) voltage of the secondary cell formed.

CHAPTER XXXIX

NULL METHODS OF MEASURING ELECTRO-MOTIVE FORCE AND RESISTANCE

The Use of the Straight Uniform Current Carrying Wire.—By combining cells *in series*, we can have E. M. F.s which are *exact multiples* of the E. M. F. of a single cell, *but not intermediate values*. A **straight uniform current carrying wire,** on the other hand, **provides** us with **points of gradually decreasing or increasing potential,** and is, therefore, extensively used in electrical measurements of every kind.

Null Methods.—These measurements are generally carried out by **null methods.**

The peculiarity of these methods lies in, that the measurement finally depends on reading, on a mm. scale, the position of a sliding contact on the uniform current carrying wire, the contact being so adjusted that the current in a sensitive galvanometer, suitably connected in the circuit, is zero. Such measurements are unaffected by imperfections in the galvanometer or in its setting, as long as the moving system is free, *because there are no galvanometer deflections to read.* They are, therefore, more accurate than when deflections have to be read and corrected for errors of eccentricity, etc.. as in **deflection methods**. In addition, more sensitive galvanometers can be employed than can be used with deflection methods.

In Expt. 90 we used a **null method** for comparing magnetic moments.

The Potentiometer.—Let ABCD (Fig. 190) be a current carrying wire. As the current flows from B to D, the potential will fall as we go from B towards D.

Connect the positive pole of a cell whose E.M.F. is required to the point B, and let the common potential after the connection be *E* volts. Let the potential of the negative pole of the cell, connected as above to be *e* (less than *E*) volts. The E. M. F. of the cell is evidently *E-e* volts. As the galvanometer is connected to the negative pole, both galvanometer terminals will also be at potential *e* volts. The potential of B is *E*, and falls towards D. Let *C* be the point whose potential is *e*. The potential of the free terminal of the galvanometer is already *e*. If this free terminal is pressed at C the galvanometer will show no current, as both its terminals remain at the same potential *e* volts. Contact at any other point of the wire (of potential *e′* volts), will result in current, as the two terminals of the galvonometer will be connected to points at different potentials. The E.M.F. of the cell (*E-e* volts) equals, therefore, the P. D. between the point B and the *balance point* (also *E-e* volts). This is the **potentiometer principle**, used for comparing the E. M. F. s of cells.

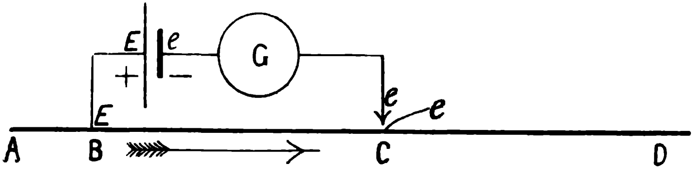


Fig. 190 —The potentiometer principle.

EXPERIEMENT 103.– **To compare the E M. Fs. of Daniell and Leclanche cells by means of the potentiometer.**

Apparatus : Wireboard, cells to be compared, two-way key, moving coil pointer galvanometer (the instrument shown in Fig. 177 or Weston model 375 is best). 4-volt auxiliary battery.

Convert the wireboard into a potentiometer, by means of the small side strap, using only one wire. Connect the two cells to be compared, as also the auxiliary battery as shown (Fig. 191), taking care to **see that all the three positives (or negatives) are at one end of the wire,** that is, on the strap. Insert a spring key at K. Use a pointer galvanometer of *medium* sensitivity. Connect up one of the cells (say, the Leclanche) by inserting the plug in the proper hole. Place the "*jockey*," or knife-edge contact on the wire near the strap; press key K and then the jockey. Next

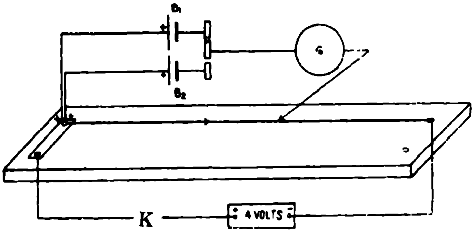


Fig. 191.—The wireboard converted into a potentiometer, and connected up for comparing the F. M. Fs. of two cells, B_1 and B_2.

repeat with the jocky near the other end of the wire. The deflections should be in *opposite* directions. If this is not the case, examine the connections (see Note 2 below). Obtain, for one of the cells, the point of balance, that is, the point where the knife-edge contact gives no current in the galvanometer. This is best done by determining two points on the wire, *1 mm. apart*, which give *small* deflections in *opposite* directions. The point midway between these two may be taken as the **balancing point** l. *Much time may be wasted if an attempt is made to actually locate the exact balancing point.* Obtain the balancing point, L_2, for the other cell, and then once again l' for the first cell. Let the mean of l and l' be L_1.

Record observations thus:

Fall of potential per cm. of wire : e (need not be known).

Strap reading : 0 cm.

Balancing point.		E. M. F. of cell tested.	Ratio $\frac{E_1}{E_2}$
Leclanche : l ... l' ...	Mean L_1	$e \times L_1$	
			$\frac{L_1}{L_2} = \ldots$
Daniell	L_2	$e \times L_2$	

E. M. F. of Leclanche $= \frac{E_1}{E_2} \times$ E. M. F. of Daniell $= \ldots \times 1{\cdot}08 = \ldots\ldots$

Note 1. Two readings for the first cell are necessary to allow for changes in e, due to a slight change in the auxiliary current which causes a steady shift in the point of balance. If the fall in this current is steady, the mean of l and l' will correspond to the same *time* and, consequently, to the same value of e, as that when L_2 was determined.

Note 2. Deflections on the *same* side on *both* ends of the wire are generally due to

(1) A bad connection somewhere.

(2) Auxiliary current being *too weak*, so that the total fall on the wire is less than the E. M. F. of the cell in series with the galvanometer. Two accumulators for the auxiliary battery are generally enough.

(3) Positives and negatives being promiscously connected to the strap. Check with pole paper, if necessary.

The Wheatstone Bridge Principle.—Fig. 192 shows two conductors *abd* and *aed*, connected in parallel at *a* and *d*. The total fall of potential along each branch is bound to be the same, as the ends of the two conductors are at the same potentials. The potential at an intermediate point b on one branch, will have the same value as the potential at some point e on the other branch, and a galvanometer whose terminals are connected to b and e will show no deflection. Let C_1 and C_2 be the currents in the two branches, and X, Q, S, R the resistances in the four **arms** of the **quadrilateral** *abdea*. The fall of potential along any conductor equals the product of the current and the resistance ($E=CR$).

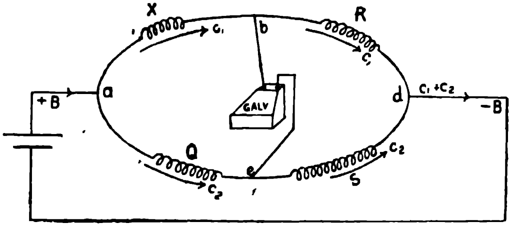


Fig. 192.—The principle of Wheatstone bridge.

Now, if b and e are at the same potential.

The fall from a to b = **The fall from** a to e

that is, $C_1 . X = C_2 . Q$ ... (1)

also, $C_1 . R = C_2 . S$ ... (2)

because, **The fall from** b to d = **The fall from** e to d.

Dividing (1) by (2), we have $\frac{X}{R} = \frac{Q}{S}$.

This is the principle of the Wheatstone bridge method of determining a resistance X, which can be found if R, Q and S are known.

The Slide-wire Bridge.—A common instrument for applying the above principle practically is, **the slide-wire bridge**, which is, essentially, like the potentiometer, a straight uniform wire, provided with suitable terminals for the necessary connections. The wireboard already used can be converted into a Wheatstone bridge by means of the long copper strap. In figure

193, we have the unknown resistance X connected up for comparison with the known resistance R from the resistance box. Figures 192 and 193 are similarly lettered, the whole copper strap, on account of its low resistance, representing the single point b of Fig. 192. Resistances Q and S of Fig. 192, are represented by the two segments of wire into which the balancing point e divides it. The equation

$$\frac{X}{R} = \frac{Q}{S} \text{ (see above)}$$

now becomes

$$\frac{X}{R} = \frac{\text{Resistance of wire } ae}{\text{Resistance of wire } ed}$$

$$= \frac{\text{Length } ae}{\text{Length } ed}$$

$$\text{or } X = R\frac{ae}{ed}.$$

EXPERIMENT 104.—**To determine the resistivity of German silver or platinoid by means of a slide-wire bridge.**

Apparatus: Wireboard with long copper strap, insulated German silver (100 cm., No. 36) or platinoid (100 cm., No. 34) wire, resistance box, moving coil pointer galvanometer, one Leclanche or dry cell, commutator. Screw gauge, metre stick.

For bridge wire use bare platinoid or Eureka No. 30 or 28. Connect up the apparatus as shown. Adjust box resistance to 10 or 15 ohms. All plugs should be *tight*. *Whenever any plug is taken out, the neighbouring plugs become slightly loose, and should be given a twist to tighten them.* Complete the battery circuit. Test with the "jockey" near one end of the wire and then near the other. *Opposite* deflections should result. If this is not so, examine the connections. All wire ends and contact surfaces on terminals should be rubbed clean and bright with sandpaper. Locate roughly the balance point, and adjust *R* so as to obtain a balance near the *middle* of the wire. Keep one commutator plug in, *the other being inserted only when actually testing with the jockey. If the current is passed continuously, the wire and the resistance box will be heated unduly.* Determine the balance point carefully. The jockey should be pressed on the wire *after* the battery circuit is completed. Do not test by first pressing the "jockey" at a certain point, and then completing the battery circuit to see if any of the current passes

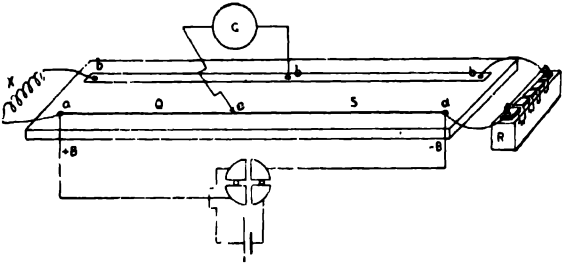


Fig. 193.—The wireboard converted into a slide-wire bridge and connected up for the determination of an unknown resistance *X*.

through the galvanometer. *If this is done, a momentary deflection due to induction may be obtained even if the point tested* **is** *actually the balancing point.*

Repeat with the current reversed. Obtain two other pairs of readings, one between 30 and 50 cm., and the other between 50 and 70 cm. To do so, adjust *R* as necessary. *Avoid readings outside the range 30 to 70 cm.*

With a sharp penknife, cut off the wire close to the terminals, so as to remove the portion whose resistance has been determined, and measure its length, *correct to half a millimetre*, with a metre stick. *The* *portion under the binding screws should not be included when the length is being measured.* Next remove the silk covering carefully, and **measure the diameter with the utmost care, at eight to ten points, with the screw gauge.** When removing the insulation, *do not scratch or pull the wire roughly*, as this will alter its diameter.

Q. Why is the *diameter* measured eight or ten times with special care, and the *length* only once with a metre stick ?

Q. What is the *weak spot* in this experiment ? (See pages 8 and 9).

***Observations*:**

Wire used : Platinoid No. 34 (1 metre nearly).

Resistance from box R	Distance of balance from X end or first segment l_1	Second segment l_2	$X = R\frac{l_1}{l_2}$
7 ohms	.... } Mean:	$100-l_1$	......
10 „	.... } Mean:	...	......
15 „	.... } Mean:	...	......

Mean value of resistance = ... ohms.
Length of wire = ... cm.

Diameter :

....
....
....
....

...... + ... ; ....../8 = ...cm.

$$X = \text{resistivity}\ \frac{\text{length}}{\text{cross section}} = s\ \frac{\text{length}}{\frac{\pi}{4}(\text{diameter})^2}.$$

$$\therefore \text{Resistivity } s = X.\ \frac{\pi}{4}\ (\text{diameter})^2/\text{length}$$

$= \ldots\ldots\ldots\ldots\ldots \times 10^{-6}$ ohms (express as *micro*-ohms).

(Determine $\log \frac{\pi}{4} = \log \cdot 7854$ from special entry in log table).

The Post Office Box.—In a slide-wire bridge, the three known resistances necessary for determining an unknown resistance, by applying the principle of the Wheatstone quadrilateral, are represented by the two wire segments and a known resistance from the auxiliary resistance box. The Post Office Box* (Fig. 194) is a compact form of the Wheatstone bridge in which all the three known resistances necessary for measuring an unknown resistance are represented by sets of coils. The scheme of connections in a Post Office Box is represented diagrammatically in Fig. 195, in which the battery, galvanometer and the unknown resistance X are shown connected up for the determination of X. Fig. 196 is lettered exactly as Fig. 195, and exhibits the same scheme of connections in a form specially designed to show that the three known resistances R_1, R_2 and S form with the unknown resistance X, a Wheatstone quadrilateral, so that when the adjustment for balance is made by unplugging the proper resistances,

$$\frac{X}{S} = \frac{R_2}{R_1}$$

$$\text{so that } \underline{\mathbf{X} = \mathbf{S} \times \frac{\mathbf{R}_2}{\mathbf{R}_1}}.$$

The resistances R_1 and R_2 each consist of a set of three coils (10, 100, 1000 ohms), and are called the

---

*So called on account of the post and telegraph departments using it for determining the resistance of telegraph wires. The letters L and E on two of the terminals stands for "line" and "earth," as by connecting one to a broken telegraph line and the other to the ground, and determining the resistance between the two, the distance of the place where the "line" has been grounded after the break can be estimated from the known resistance per mile of the wire used for the "line."

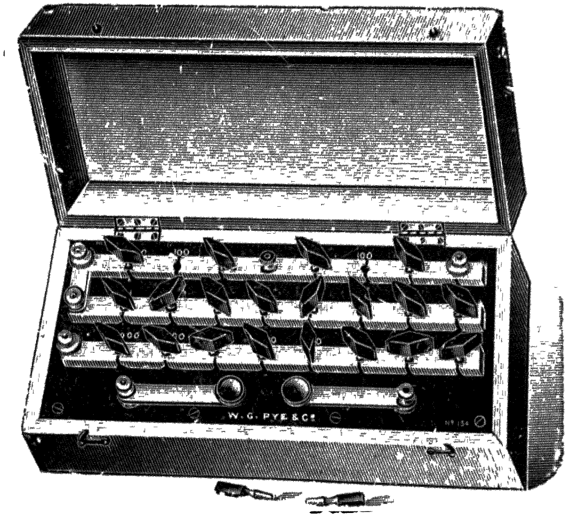


FIG 194 —The Post Office Box, a compact apparatus for determination of resistance by the Wheatstone bridge method

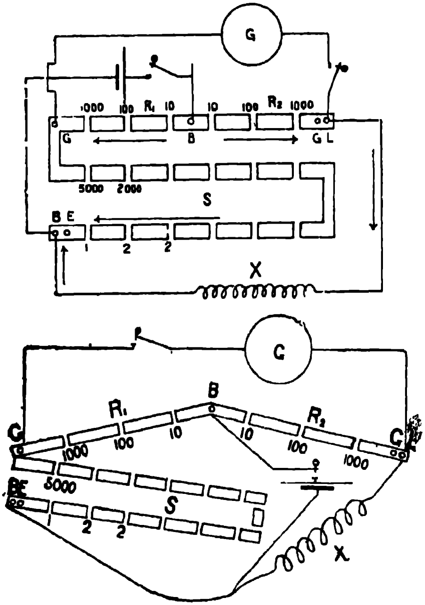
ratio arms. As R_2 and R_1 can each be adjusted to 1000, 100 or 10 ohms, the ratio between these $\left(\frac{R_2}{R_1}\right)$ can have any of the values $\frac{1000}{10}$, $\frac{100}{10}$, $\frac{10}{10}$, $\frac{10}{100}$, $\frac{10}{1000}$ that is, 100, 10, 1, $\frac{1}{10}$, $\frac{1}{100}$. As S ranges from 1 ohm to 10000 ohms or more, the Post Office Box can measure resistances ranging from ·001 ohm to one million ohms.

EXPERIMENT 195.—**To determine the resistance of a lamp or a coil of wire, by means of the Post Office Box.**

Apparatus : Post Office Box, Leclanche or dry cell, *sensitive* galvanometer, connecting wires. The unknown resistance (lamp or coil of wire of resistance 10 to 100 ohms).

[IMPORTANT NOTE. It is useless to expect success with the Post Office box as long as the instrument is not *scrupulously clean.* The plugs, *as also the holes into which they fit*, should be bright and clean. The plugs, should fit the holes tightly without touching the ebonite top. Plugs that press on the ebonite top should be shortened by filing till the fit is good. The plugs and holes can be cleaned by gentle grinding (in position) with a little rouge or fine emery powder between the two. Sand paper should never be employed, and the cleaning is best left to the laboratory *mistri.*

All dust should be removed from the ebonite top by means of a camel hair brush.



Figs. 195, 196.—The upper figure shows the scheme of connections in a P. O. box for measuring an unknown resistance X. The lower figure, lettered similarly to the upper, gives the same scheme in a shape designed to show that the four resistances R_1, R_2, S and X constitute a Wheatstone quadrilateral.

The form of P. O. box illustrated (and most commonly employed) is at best a somewhat unsatisfactory instrument on account of the numerous plug contacts causing trouble except when the instrument is new. Dial briges, on account of the fewer contacts involved, give better results.

Connect the unknown resistance X to the terminals marked L and E, the battery to the terminals marked B and B, and the galvanometer to the terminals marked G and G, interposing in each of the two latter circuits a spring key. The necessary keys for the battery and galvanometer are often placed on the ebonite top and marked B and G. To make use of these, transfer to the key terminal marked B that battery wire *which has been placed on the terminal marked B connected to the battery key by a white line on the ebonite top*. Similarly transfer the appropriate galvanometer wire to the key terminal G. All connections being firm and good, proceed to the measurement of X.

Press firmly, with a slight twisting motion, *every one of the plugs* into its proper hole, till the contact is firm and reliable. Adjust R_2 and R_1 to ten ohms each. and press firmly with a twisting motion, as before, the *neighbouring plugs. This should be done whenever any plug whatever is taken out*, as the removal loosens slightly the adjacent contacts. The plugs withdrawn should be placed in the lid of the box, *and nowhere else. Mixing up of plugs from different boxes should be regarded a very serious matter and carefully avoided.*

With all the plugs in S in position, press the battey key and *then* the galvanometer key. There will be a violent deflection (it is advisable to shunt the galvanometer suitably, the shunt being removed when balance is very nearly attained). Withdraw the infinity plug and test as before. An *opposite* deflection should result. If the deflection is in the *same* direction as before, the connections should be examined, and, if necessary, tightened or cleaned. Replace the infinity plug, and starting with 5000 ohms try the various resistances systematically in descending order, as also various combinations, *till two values of S, differing by one ohm (say 33 and 34 ohms), are found to give opposite deflections.* X lies between these two values. The process is similar to that used in trying the weights in a weight box when determining the mass of a body with a balance.

Next, adjust R_1 to 100 ohms instead of 10 ohms, the resistance R_2 remaining 10 ohms. Test as before but with values of S between 330 and 340 ohms (10 times the former resistances 33 and 34). Record the two resistances *differing by one ohm*, say 335 and 336 ohms, which give *opposite* deflections.

Repeat with 1000 ohms as R_1 and 10 ohms as R_2. Try values of S between 3350 and 3360 ohms (10 times the former resistances). Exact balance may be obtained this time. If two resistances differing by one ohm still give opposite deflections, the mean of the two may be supposed to be the necessary balancing resistance; and if this gives more than four figures in the result, the resistance giving the *lesser* deflection may be adopted.

Enter observations thus :

The unknown resistance X : Coil marked C.

Ratio arm R_1 (ohms)	Ratio arm R_2 (ohms)	$\frac{R_2}{R_1}$	S (ohms)	$X=S\frac{R_2}{R_1}$ (ohms)
10	10	1·0	between 33 and 34	between 33 and 34
100	10	·1	,, 335 and 336	,, 33·5 and 33·6
1000	10	·01	,, 3352 and 3353 3352 giving the *lesser* deflection.	,, 33·52 and 33·53 say 33·52 ohms.

If the unknown resistance X *exceeds* the total resistance in the arm S, no balance will be obtained with any values of $\frac{R_2}{R_1}$ used in the experiment described above. In this case $\frac{R_2}{R_1}$ should be made *greater* than one (10 or 100) by making R_2 100 or 1000 ohms, and R_1 10 ohms. Suppose balance is obtained with 10891 ohms. X will be, respectively, 108910 ohms or 1089100 ohms, according as $\frac{R_2}{R_1}$ is 10 or 100.

CHAPTER XL

MEASUREMENT OF CURRENT, ELECTROMOTIVE FORCE AND RESISTANCE WITH THE TANGENT GALVANOMETER

Relative merits of the Voltmeter and Ammeter, the Potentiometer and Wheatstone's Bridge, and the Tangent Galvanometer methods of measuring E. M. F. and Resistance, etc.—We have already used the **voltmeter and ammeter** for comparing E. M. F.s and resistances. On account of the ease and rapidity with which these measurements can be made, and the simplicity of the calculations involved, voltmeters and ammeters are extensively used by the electrical engineer for everyday electrical work. The accuracy of such methods, with *cheap* commercial instruments, is not high. Well made moving coil instruments, generally, give every satisfaction. The chief merit of the methods lies, however, in their convenience and simplicity.

Where extreme accuracy is necessary, **null methods** are always employed. These methods are fairly convenient, though they involve the use of costly bridges, potentiometers, P. O. boxes and sensitive galvanometers. The superiority of null methods lies in, that (*i*) the galvanometer serves only as a current *detector*, the actual *reading* of deflections being *unnecessary*, (*ii*) repeated readings for eliminating errors due to imperfections are unnecessary. *The moving system should, of course, be quite free, and sufficiently sensitive.* The resistance boxes used must, also, be accurate, and the bridge wires uniform, but these qualities are, fortunately, more easy to secure and to retain, than all-round accuracy in a galvanometer.

Measurements of medium accuracy can be carried out by **deflection methods** with any well made galvanometer, whose deflections are proportional, or otherwise simply related, to the current. The **tangent galvanometer** is an instrument of this kind. *Its proper function is to measure with accuracy a current* **in absolute units**, *or to calibrate ammeters, etc.* Its circular coil, with its simple geometry, easily lends itself to such calculations (see page 437), which cannot be carried out accurately with other forms of galvanometer. Frequently, however, the instrument is used merely for *comparison* of electromotive forces and resistances. For this work, it is much less suited than moving coil poniter galvanometers, ammeters and voltmeters, which are unaffected by the earth's field and require

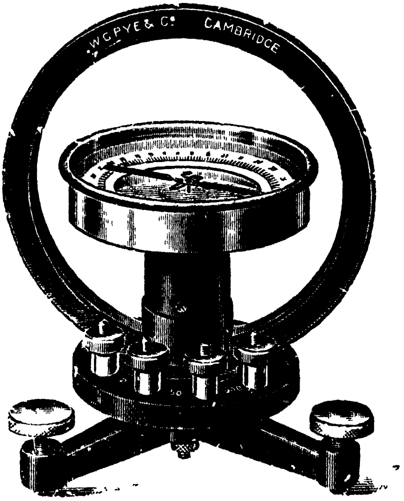


Fig. 197 —A popular and convenient from of tangent galvanometer, with a *very low* resistance or ammeter coil (2 turns very thick wire), a *high* resistance or voltmeter coil (500 turns fine wire), and a 50-turn coil for general use.

no levelling and adjusting. The tangent galvanometer is not easy to adjust. For eliminating errors due to eccentricity, etc , two readings have to be taken with the current flowing one way and an equal number taken after the current has been reversed with a commutator. All this takes time, and is worth while only when a galvanometer (otherwise more suitable for comparing E.M.F.s, etc.) has to be *tested for accuracy.* Outside schools and colleges the instrument is rarely employed.

The Tangent Galvanometer.—Fig. 197 shows a convenient and commonly used form of tangent galvanometer. The three principal parts are, (*i*) the base with its three levelling screws, (*ii*) the circular ring carrying three coils of wire and the necessary terminals for connection, mounted so as to rotate upon the lower fixed base round a vertical axis, (*iii*) the compass box with its agate-capped magnetic needle on pivot, graduated circle and anti-parallax mirror, fixed horizontally in the middle of the circular ring.

The Theory of the Tangent Galvanometer.—The tangent galvanometer measures currents by comparing the magnetic field of a circular current carrying coil with the magnetic field of the earth. The two fields are compared by being placed **at right angles** as in the case of the deflection magnetometer. For this purpose, the coil is turned round so as to be in the magnetic meridian (in the same plane as the mag-

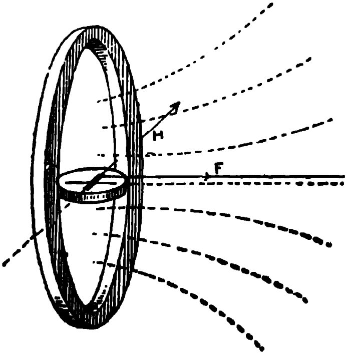


Fig. 198.—The application of "the tangent law of magnetic fields" to the tangent galvanometer. Note that H and F are **at right angles**.

netic needle at its centre). In this position (Fig. 198) the magnetic field of the coil (**F** or **deflecting field**) which is always at right angles to the coil, is **perpendicular** to the field of the earth (**H** the **controlling field**). We will, therefore, have (see "Tangent law of Magnetic Fields" page 340) as in the case of the deflection magnetometer,

$$\frac{\textbf{Deflecting Field}}{\textbf{Controlling Field}} = \frac{\mathbf{F}}{\mathbf{H}} = \tan\theta, \quad \ldots\ldots \qquad \textbf{I.}$$

where θ is the deflection of the needle from the meridian. But as F equals $\frac{2\pi n C}{r}$, where n is the number of turns in the coil, r the radius of the coil, and C the current in absolute units, (I) becomes,

$$\frac{2\pi nC/r}{H} = \tan\theta$$

$$\text{or} \qquad C = \frac{rH}{2\pi n} \tan\theta$$

$$\text{or} \qquad \mathbf{C = K\tan\theta} \qquad \textbf{II.}$$

where **K** is a constant for the coil used and the place of observation. **K** is called the **reduction factor** of the coil.

The Tangent Galvanometer as Ammeter, Voltmeter and Galvanometer.—The tangent galvanometer shown in Fig. 197 has three windings and can be used as

(1) **Low Resistance Galvanometer** or **Ammeter** by using the 2-turn coil whose resistance is negligible (about ·01 ohm), and whose reduction factor is about 1·75 amperes.

(2) **High Resistance Galvanometer** or **Voltmeter** by using the 500-turn coil, whose resistance varies from 150 to 250 ohms.

(3) **Galvanometer** for general use by using the 50-turn coil of 1 to 2 ohms resistance. The tangent

galvanometer, though more accurate than *cheap* commercial voltmeters and ammeters, is less convenient, because

(1) The volts or amperes can be obtained from the plain scale of degrees only indirectly by using the proper reduction factors.

(2) The adjustment of a tangent galvanometer is a somewhat difficult matter, and the reading of deflections a much more laborious process than with ammeters, etc. These latter have also the further advantage that they require no levelling, and can be placed facing in any direction, as they are unaffected by the earth's magnetic field.

The Adjustments of a Tangent Galvanometer.—These are explained under Expt. 106 below.

THE USE OF THE LOW RESISTANCE COIL, OR THE TANGENT GALVANOMETER USED AS AMMETER

Absolute Measurement of Current with the Low Resistance or Ammeter Coil of a Tangent Galvanometer.—The formula deduced on page 437, *i. e.*

$$C = \frac{rH}{2\pi n} \tan\theta$$

can be used for measuring a current in absolute electromagnetic units, if H the horizontal component of earth's magnetic force, r the radius of the galvanometer coil, and n the number of turns are known or measured. This, generally, presents no difficulty. The low resistance or 2-turn coil (the *ammeter* coil) is generally used for the measurement of current.

EXPERIMENT 106.—**To measure with a tangent galvanometer the maximum current that a given Daniell cell can supply.**

Apparatus : Daniell cell, tangent galvanometer, *thick* connecting wires (thick *flexible* wire is more easily gripped by terminals than ordinary thick wire). Metre stick, beam compass or large callipers for measuring the diameter of the coil.

Theory of the experiment.—The current supplied by the given cell will be a maximum when the external resistance is practically zero. This current can be obtained and measured by connecting the cell *with short and thick connecting wires* to the 2-turn coil of the tangent galvanometer. The resistance of this coil is negligible, and it may, therefore, be called the ammeter coil of the galvanometer. A commutator should not be used for this experiment, as it is likely to introduce appreciable resistance due to additional contacts and connecting wires.

Adjusting the Galvanometer.—To measure the deflection with accuracy, the galvanometer must be properly adjusted. To do this proceed as follows :

(*i*) Examine the needle and pointer to see if they are sensibly at right angles. This is generally supposed to be the case; but if the angle differs considerably from 90°, the needle must be taken out and the angle adjusted to 90° with a set square by gently twisting the pointer.

(*ii*) Level the instrument till the needle swings freely. Observe if the needle always comes to rest in the same position. If this is not the case, or if the vibrations are dull and sluggish, the needle has probably lost much of its magnetism.* As this is a serious defect. another instrument must be tried.

(*iii*) Look down into the instrument from above, and place your eye so that it is in the same plane with one face of the coil and its image in the mirror. With the eye so adjusted, turn round the *compass box* on its pillar (not the *coil*), till the reading of the coil face on the circular scale is the same on both sides When this is the case, the 0-0 diameter is perpendicular to the plane of the coil.

(*iv*) Now turn round the coil (with the compass box) till the pointer reads 0°, at least on one side. The coil will now be in the meridian or in the same plane as the magnetic needle, ***provided the pointer has been fixed at right angles to the needle.***

---

*The needle should be remagnetised to saturation by being held momentarily so as to bridge the poles of a powerful electromagnet

Performance of the experiment.—**Do not calculate a zero correction.** This will be applied automatically later on. When the galvanometer has been adjusted, connect the cell to the 2-turn coil with as short a piece of *thick twin flexible* wire, as will just suffice. If ordinary thick insulated wire is employed, the two pieces used should be twisted together like the flexible wire, as otherwise the magnetic field of the strong current in the connecting wires may affect the needle considerably. When twisted, the opposite currents in the two wires cancel one another. *This precaution is always necessary with the tangent galvanometer,* ESPECIALLY WHEN THE 2-TURN COIL IS BEING USED.

Tap the galvanometer base *gently*, to make sure that friction is not keeping the needle away from its correct deflected position. Do not tap the compass box. A needle that requires violent tapping should be considered unreliable. Place your eye immediately above the galvanometer, so that one end of the pointer just covers its own image in the mirror, and read the deflection to tenths of a degree. Read similarly the other end of the pointer. Disconnect the wires from the cell terminals. Read the pointer to make sure that the coil has not moved from the meridian. Reconnect so as to pass a current in a direction *opposite* to the first. Read both ends of the pointer as before. The mean of the four readings may be taken as the deflection produced by the maximum current the cell can give. This mean (θ) is automatically corrected for small zero errors, errors of eccentricity, etc. (See deflection magnetometer, page 351).

The current is $\frac{rH}{2\pi n} \tan \theta$.

Obtain from the laboratory stock book or by measurement with beam compass or large callipers with suitably curved legs, the radius of the 2-turn coil. If no particulars are known, take the mean of the internal and external radii of the ring. $H = ·32$, $n = 2$. C can, therefore, be calculated.

Enter observations thus:

$H = ·32$; $n = 2$.

DEFLECTION θ			Radius of the coil r	Reduction factor $\frac{rH}{2\pi n}$	Current in C.G.S. units	Current in amperes
Current one way	Current the other way	Mean θ				
46°·9 44°.	45°·8 44°·5	$\frac{181°·8}{4}$ $=45°·45$	$\frac{15·22}{2}$ cm. $=7·61$ cm.	$\frac{7·61 \times ·32}{4\pi}$ log ..... log......... .... antilog $\bar{1}·2872$ $=·1937$	·1937 *tan* 45°·45 $=·1961$ Use table of logarithmic tangents	1 961 say 1 96

As the E.M.F. of the Daniell cell used is 1·08 volts, calculate its internal resistance from the formula

$$b = \frac{1·08 \text{ volts}}{\text{maximum current in amperes}}$$

EXPERIMENT 107.—**To prepare a curve of corrections for the given ammeter with the help of a tangent galvanometer.**

Apparatus: Ammeter of range 0--5 amperes, tangent galvanometer, commutator, thick German silver wire (No. 20 or 22 *bare*), twin flexible wire (the two members *differently* coloured), two accumulators.

[NOTE.—The experiment consists in placing the ammeter and the galvanometer in series, and comparing the readings of the ammeter with the current values deduced from the deflections of the tangent galvanometer whose reduction factor should be known. The best plan is to use an accurate value of the reduction factor of the two-turn coil (about 1·75 amperes) obtained by a B. Sc. student by the electrolysis of copper sulphate. The reduction factor $\left(=\frac{rH}{2\pi n}\right)$ can be calculated, if r is accurately known. The value of r supplied by the maker is best.]

For complete directions for adjustment of a tangent galvanometer see Expt. 106.

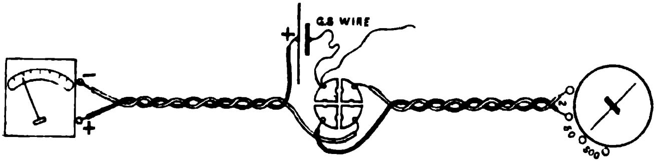


Fig 199 —The tangent galvanometer used for calibrating an ammeter.

Connect the positive pole of the battery (2 volts to start with) to the proper ammeter terminal (marked +). To the other pole of the battery attach the German silver wire. Carry the other end of the German silver wire and a wire from the second ammeter terminal to a "corresponding pair" of terminals (see page 384) on the commutator. Connect the two-turn coil of the tangent galvanometer to the second "corresponding pair." If connected as described above (Fig. 199), the current will be reversed only in the galvanometer, and neither the ammeter nor the galvanometer disturbed when the German silver wire is shortened or lengthened for varying the current in the circuit. A suitable rheostat may raplace the German silver wire. Place the ammeter and galvanometer well away from one another (*minimum* distance 5 ft., on account of the ammeter containing a powerful magnet, which, if nearer, will completely vitiate the galvanometer readings), and from the rest of the apparatus. The two leads of each should be twisted together as shown (use twin flexible wire).

Interpose sufficient resistance wire (*a resistance box*, if used, *will burn out*) to obtain a reading of about 1 ampere. Tap the galvanometer base gently, and read both ends of the pointer accurately. Tap and read also the ammeter carefully. Reverse the current, tap, and read again both ammeter and galvanometer.

Obtain similar readings for currents of about 2, 3, 4 and 5 amperes. The deflections will vary from about 25° to 70°. For the larger currents two accumulators may be necessary.

Observations:

Reduction factor of coil used $\left(= k = \frac{rH}{2\pi n}\right)$ = 1·75 amp. Log k = ·2430.

Ammeter reading (amperes) A	Deflection			Log $\tan\theta$	Log C $=\log k\tan\theta$ $=\log k+\log\tan\theta$	C	Ammeter correction $C-A$
	Current one way	Current the other way	Mean θ				
...							
2·95 Mean 2·945	58°·6	59°·4	59°·0	·2351	·2351 ·2430 ·4781	3·007	+·06 retain two figures only
2·94	59°·2	58°·8					
...							
...							

Plot a "curve of corrections" with A as abscissæ and C-A as ordinates. With the help of the curve so drawn, calculate the correction and the true value of the current for an ammeter reading of 4·31 amperes.

CHAPTER XL– (*contd.*)

THE USE OF THE HIGH RESISTANCE COIL, OR THE TANGENT GALVANOMETER USED AS VOLTMETER

Comparison of E.M.F.s by means of the High Resistance or Voltmeter Coil of a Tangent Galvanometer.—When two cells of E. M. F.s E_1 and E_2 are connected in turn to the high resistance coil of a tangent galvanometer, the total circuit resistance may be regarded the same in the two cases, as the internal resistances of the two cells, even if different, are in both cases negligible as compared with the resistance of the galvanometer. The currents in the two cases ($k \tan \theta_1$ and $k \tan \theta_2$) are, therefore, proportional to E_1 and E_2, so that $\frac{E_1}{E_2} = \frac{\tan \theta_1}{\tan \theta_2}$. The high resistance coil may, therefore, be properly called the voltmeter coil.

NOTE.—In the absence of a high resistance coil, electromotive forces can be compared by placing in series with the 50-turn coil a resistance *large* compared with the internal resistances of the cells to be compared. A resistance *above* 100 ohms is essential for an accuracy of 2 to 3%. If, with such a resistance in series, the deflections are small, or if the cells are of very high resistance, other methods have to be employed (see page 450).

EXPERIMENT 108.—**To compare the E. M. F.s of two cells of low internal resistance by means of a tangent galvanometer.**

Apparatus: Leclanche and Daniell cells, tangent galvanometer, two-way key, reversing switch or commutator, connecting wires.

Theory of the Experiment.—We know that the E. M. F. of a cell can be directly read off in volts on the scale of a voltmeter (*high* resistance galvanometer). The 500-turn coil of a tangent galvanometer has a comparatively *high* resistance, and when this coil is used, the tangent galvanometer is practically a *voltmeter*, with this difference, that while the conveniently graduated scale of a voltmeter allows of direct readings in volts of the potential difference between the terminals, this P. D. cannot be so read in the case of a tangent galvanometer, but is *proportional to the tangent of the angle of deflection.* Consequently, if θ_1 and θ_2 are the deflections produced by two cells of E.M.F.s E_1 and E_2 (using the 500-turn or the *voltmeter* coil), we have $\frac{E_1}{E_2} = \frac{\tan\theta_1}{\tan\theta_2}$.

Performance of the experiment.—Level and adjust the galvanometer (*Full details under Expt 106*). Connect up the two cells to be compared, a two-way key, resistance box, commutator and the 500-turn coil of the tangent galvanometer as shown in Fig. 200. Connect the commutator so that the current can be re-

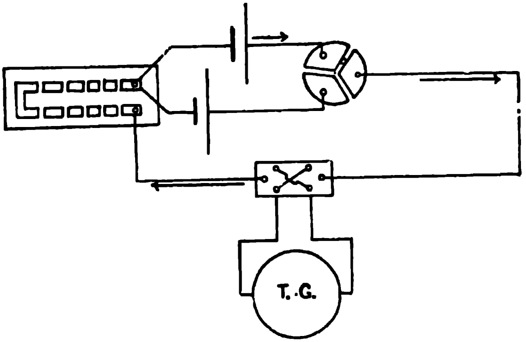


Fig. 200—The tangent galvanometer used as voltmeter (with the high resistance coil) for comparing the electromotive forces of two cells.

versed in the galvanometer only. With a voltmeter a resistance box is unnecessary, but as with a tangent galvanometer (as also in a deflection magnetometer) results based on deflections above 70° or below 25° are decidedly inaccurate, the resistance box has to be included in the circuit for regulating the current. Deflections above 60° should, as a rule, be avoided. The best results are obtained with deflections in the neighbourhood of 45°* (say 35° to 55°). Unplug a suitable resistance so that the deflections are, as far as possible, within this range for *both* the cells.

Start with the Leclanche cell. Unplug a suitable resistance so as to obtain a deflection of about 50° or 55°. Tap the *base* of the galvanometer (not the compass box) *gently*, as usual, to be sure that friction is not keeping the needle away from its correct deflected position. Read *both* ends of the pointer, placing your eye so that the pointer covers its own image in the mirror. Reverse the current by means of the commutator. Tap gently, and read both ends of the pointer again. The four readings of deflection, taken as described above, are intended to correct automatically for small errors, due to the eccentricity of the needle, the departure of the coil from the meridian, and so on. *Separate zero readings are unnecessary.*

Now place the Daniell cell in the circuit, everything else remaining unchanged. The resistance from the resistance box should remain the same as for the Leclanche. Tap, and read both ends. Reverse the current, tap, and read both ends of the pointer again.

---

* Suppose you are liable to read a deflection θ as $\theta \pm \phi$, *i. e.*, with a *small* average experimental error ϕ. This is equivalent to a difference in current values of $\frac{d}{d\theta}(k \tan\theta) \times \phi$ or $\phi\, k \sec^2\theta$. Compared with the actual current $k \tan\theta$, this difference represents a percentage error of $\frac{\phi k \sec^2\theta}{k \tan\theta} \times 100 = \frac{200\phi}{\sin 2\theta}$, which is *least* when $2\theta = 90°$, or $\theta = 45°$.

Enter observations thus:

Galvanometer coil used: 500-turn (*high* resistance or *voltmeter*) coil.

Resistance from box : 100 ohms.

Deflection for LECLANCHE Electromotive Force E_1			Deflection for DANIELL Electromotive Force E_2			$\frac{E_1}{E_2} = \frac{\tan\theta_1}{\tan\vartheta_2}$
Current one way	Current the other way	Mean θ_1	Current one way	Current the other way	Mean ϑ_2	
53°·1 53°·7	52°·9 52°·2	$\frac{211°·9}{4}$ $=52°·97$ say 53°·0	44°·5 43°·9	43°·1 43°·8	$\frac{175°·3}{4}$ $=43°·82$ say 43°·8	$\frac{\tan 53°·0}{\tan 43°·8} =$ log *tan* 53°·0 = ·1229 log *tan* 43°·8 = $\bar{1}$·9818 $\frac{E_1}{E_2}$ = antilog ·1411 = 1·384 or 1·38

E. M. F. of Daniell (*assumed*) = 1·08 volts.
E. M. F. of Leclanche (*calculated*) = 1·08 × 1·38 volts
= 1·49 volts

Use table of logarithmic tangents (appendix).

The Equal Deflection Method.—In this method, the cells whose electromotive forces (E_1, E_2) are to be compared, are connected as in the above experiment, and the resistances (R_1, R_2) adjusted till each cell gives the *same* deflection. $E_1/E_2 = R_1/R_2$. Only one particular end of the pointer need be read in both cases.

CHAPTER XL—(*contd.*)

MISCELLANEOUS MEASUREMENTS WITH THE 50-TURN COIL OF A TANGENT GALVANOMETER

The Sum and Difference Method of Comparing E. M. F.s with a Tangent Galvanometer.—If a tangent galvanometer is not provided with a high resistance coil, E. M. F.s of cells may be compared by interposing in series a resistance high enough (200 ohms about) to make the internal resistances of the cells negligible. If with such a resistance in series, the galvanometer gives only a *small* deflection, or if the resistances of the cells are so high that they cannot be considered negligible as compared with the high resistance placed in series (with such cells even the high resistance coil method described in the previous experiment may be faulty), the **sum and difference method** should be used. In this method, all the readings are taken with both the cells simultaneously in the circuit, *so that the circuit resistance is always the same, and the currents are proportional to the E. M. F.s in the circuits, irrespective of galvanometer or battery resistance*:

EXPERIMENT 109.—**To compare with a tangent galvanometer of any resistance, high or low, the electromotive forces of two cells of any internal resistance, high or low, using the sum and difference method.**

Apparatus : Bunsen and Daniell cells, tangent galvanometer, resistance box, two commutators, connecting wires.

Connect up the apparatus as shown (Fig. 201) using the 50-turn coil (see note at end) of the galvanometer, and interposing the resistance box (not shown in the figure) in any part of the circuit. The commutator K_2 is used for sending a current from A (supposed of higher E. M. F.) in *either* direction through B. The commutator K_1 reverses this resultant current in the galvanometer as usual. For one setting of K_2, the cells are *in series*, and the E. M. F. in the circuit E_1+E_2. For the alternative position of K_2, the cells are *in opposition* (*not in parallel*, as the student often supposes), and the effective E. M. F. is E_1-E_2.

Set K_2 so as to arrange the cells in series (*the sum* position). A large deflection may result. Reduce this to about 60° (not less, or the deflection for the *difference* position may become unduly small) by means of the resistance box. Tap the base *gently*, and read both ends of the pointer. K_2 remaining undisturbed, put K_1 in the alternative position, so as to reverse the current in the galvanometer. Tap and read again as usual.

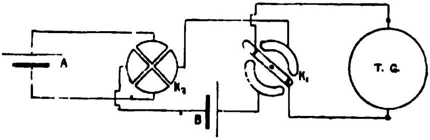


Fig. 201. Two cells and a tangent galvanometer connected up for comparing electromotive forces by the *sum* and *difference* method. *Two* commutators are a great convenience.

Reset K_2 in the alternative position so as to arrange the cells in opposition (the *difference* position). Tap and read. Without disturbing K_2, reverse the current in the galvanometer with K_1. Tap and read again as usual.

If θ_1 and θ_2 are the mean deflections in the *sum* and *difference* positions, we have, as the circuit resistance is identical in the two cases,

$$\frac{E_1+E_2}{E_1-E_2}=\frac{\tan\theta_1}{\tan\theta_2}$$

or

$$\frac{E_1}{E_2}=\frac{\tan\theta_1+\tan\theta_2}{\tan\theta_1-\tan\theta_2}.$$

Enter observations thus :

Cells tried : Bunsen and Daniell.

Galvanometer coil employed : 50-turn coil (resistance 1 5 ohms).

Resistance from box : 40 ohms.

Deflection for CELLS IN SERIES (*sum position*) E. M. F., $E_1 + E_2$			Deflection for CELLS IN OPPOSITION (*difference position*) E. M. F., $E_1 - E_2$			$\frac{\tan\theta_1 + \tan\theta_2}{\tan\theta_1 - \tan\theta_2}$ or $\frac{E_1}{E_2}$
Current one way	Current the other way	Mean θ_1	Current one way	Current the other way	Mean θ_2t	
......	......	$\frac{......}{4}$ =......	...	......	$\frac{......}{4}$ = ...	$\frac{......+......}{......-......}$ $=\frac{......}{......}$
.....	......	*tan* θ_1 =......	.....	......	*tan* θ_2 =......	=...... log......=...... log .....=...... antilog ......

Use ordinary table of tangents, logarithms to be used for the final division only.

The 50-turn (low resistance) coil is used merely to illustrate how a low resistance galvanometer *can* be used for the comparison of electromotive forces by the sum and difference method. It is preferable, however, to use, even with this method, the high resistance coil, as this reduces the currents the cells have to supply, and difficulties due to the polarisation of the cells are thus reduced to a minimum. In fact, the *high* resistance coil or circuit should alone be employed, when electromotive forces are compared by deflection methods.

The Increased Deflection Method of comparing Electromotive Forces.—This method, like the sum and difference method, is designed to eliminate the effects of battery and galvanometer resistance, so that electromotive forces can be accurately compared, even when the resistances of the cells are high. Suppose a cell of E. M. F. E_1 and any resistance B gives a deflection θ_1, when connected to the galvanometer through an extra resistance R_1 from the box, and a deflection θ_2 through resistance R_2 from the box. If G be the galvanometer resistance, we have.

$$\frac{E_1}{B+G+R_1} = k\ tan\ \theta_1\ or\ \frac{E_1}{k\ tan\ \vartheta_1} = B+G+R_1$$

$$and, \quad \frac{E_1}{B+G+R_2} = k\ tan\ \theta_2\ or\ \frac{E_1}{k\ tan\ \vartheta_2} = B+G+R_2$$

By subtraction, $E_1\left(\frac{1}{k\ tan\ \vartheta_1} - \frac{1}{k\ tan\ \vartheta_2}\right) = R_1 - R_2 \ldots\ldots\ \ldots$I.

If the other cell of E. M F. E_2 and any resistance whetever gives the *same* deflections ϑ_1 and ϑ_2 through extra resistances S_1 and S_2, we have

$$E_2\left(\frac{1}{k\ tan\ \theta_1} - \frac{1}{k\ tan\ \theta_2}\right) = S_1 - S_2 \ \ldots\ldots\ \text{II}$$

Dividing I by II, we have

$$\frac{E_1}{E_2} = \frac{R_1 - R_2}{S_1 - S_2}.$$

To apply this method, connect the cells as in Fig 200. Adjust the box resistance till, with one of the cells, the reading of *any one end* of the pointer is about 35°. Next decrease the resistance till the reading of *the same end* is about 55°. Substitute the second cell, and determine the resistances in the box when *the same end of the pointer as used before, gives the same two deflections*. Apply the formula.

The only objection to the method is that it is generally not possible with an ordinary resistance box to adjust so that the deflections with the second cell are *exactly* the same as with the first cell.

Comparison of Resistances with the Tangent Galvanometer.—Resistance can be compared by deflection methods with any *low* resistance galvanometer of moderate sensitivity. The 50-turn coil of the galvanometer is well suited for such work. A convenient and simple method of "determining" an unknown resistance (which means comparing it with known resistances from a box) is **the substitution method.** This consists in (*i*) determining the deflection produced by a constant cell (Daniell) through the unknown resistance placed in series in an otherwise low resistance circuit, (*ii*) ***substituting*** for the unknown resistance a known resistance from the resistance box so that the ***same*** deflection is produced. This known resistance evidently equals the unknown resistance. With an ordinary box *fractions* of an ohm cannot be thus estimated. Such fractions can, however, be easily calculated.

EXPERIMENT 110.—**To determine an unknown resistance by the method of substitution.**

Apparatus: Unknown resistance (10 to 50 ohms), two-way key, resistance box, galvanometer, Daniell cell.

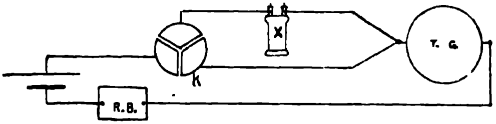


Fig. 202.—Determination of an unknown resistance X by the *substitution* method.

Connect the apparatus as shown (Fig. 202). Use the 50-turn coil. X is the unknown resistance. Provide, by means of the 2-way key, an alternative path for the current along a plain copper wire. *No commutator is required.*

Level the instrument carefully, and adjust as usual. Place X in the circuit. All box plugs must be ***in*** unless the deflection is considerably *more* than 45°. In that case insert resistance till the deflection is nearly 45°. Note the resistance employed. Tap gently, and read *any* ONE END *of the pointer*.

Next put X out of the circuit. Use the alternative path, ***increasing*** the box resistance till the reading (use the *same pointer end* as used for X) is, ***as nearly as possible***, the same as with X. Tap gently, as usual, before deciding. ***The increase*** in the box resistance equals X. Fractions of an ohm cannot be obtained with an ordinary box. If these are required, read the deflection for each of the two resistances differing by ***one*** ohm, between which the desired resistance seems to lie.

Record observations thus:

Galvanometer coil used : 50-turns (resistance 1·5 ohms).
Deflection of one end (A) of pointer = 47°·2
Resistance in circuit = X + 9 ohms from box.
Deflection of end A, using copper wise path = 47° 1
Resistance in circuit = 42 ohms
∴ X = 42 - 9 = 33 ohms.

Continue for greater accuracy :
Deflection of end A, = 47° 5
When box resistance = 41 ohms.
The proper box resistance lies between

$$\left.\begin{array}{ll}\text{Ohms} & 41 \text{ and } 42 \\ \text{Deflections} & 47^\circ\cdot5,\ 47^\circ\cdot1\end{array}\right\} \begin{array}{c}\text{difference for one ohm} \\ 0^\circ\,4\end{array} \left.\right\} \begin{array}{l}0^\circ\cdot1 \text{ corresponds to} \\ \frac{\cdot1}{\cdot4} \text{ or } \cdot25 \text{ ohm.}\end{array}$$

∴ Proper box resistance = 42 − ·25 or 41·75 ohms.
∴ X = 41·75 − 9
= 32·75 say
32·8 ohms.

Record answers to the following :

Q. Can you use the 2-turn coil for this experiment ? Is the objection, if any, connected with the *resistance* of the coil ?

Q. Can you use the 500-turn coil ? What is the nature of the objection, if any ?

EXPERIMENT 110*a*.—**To compare two resistances by means of the tangent galvanometer.**

Apparatus : Daniell cell, resistances to be compared, two-way key, commutator, resistance box.

Unknown resistances less than 100 ohms. These are best compared by determining each by substitution as in experiment 110.

Unknown resistances 100—500 ohms. These can be compared without a resistance box, with an accuracy of about 2%, if on connecting a Daniell cell to the 50-turn coil through the two resistances in turn, measurable deflections (not less than 25°) can be obained. If θ_1 and θ_2 are the deflections with the two resistances R_1 and R_2 separately in series with the same cell (E.M.F., E) and the same coil, we have

$$\frac{E/(R_1+B+G)}{E/(R_2+B+G)}=\frac{\tan\theta_1}{\tan\theta_2}$$

$$\text{or } \frac{R_1+B+G}{R_2+B+G}=\frac{\tan\theta_2}{\tan\theta_1}$$

B = Battery resistance (about 1 ohm).
G = Resistance of 50-turn coil (1 to 2 ohms).

As R_1 and R_2 are each above 100 ohms, while $B+G$ is between 2 and 3 ohms, we have, approximately,

$$\frac{R_1}{R_2}=\frac{\tan\theta_2}{\tan\theta_1}.$$

If the deflections obtained with the 50-turn coil are very small, the method becomes inapplicable. *The high resistance coil (500-turns) should in no case be employed, because even if the value of its resistance is known, the ratio* $\frac{R_1}{R_2}$ *cannot be obtained without a third set of readings with a known resistance in series.*

Connect the two resistances by means of a two-way key, so that either one can be placed in series with the cell and the galvanometer (50-turn coil) by placing the plug in the proper hole. To do so connect as in Fig. 200, substituting the two resistances for the two cells and omitting the resistance box.

Obtain four readings with each resistance in series (two with current one way, and two after reversing the current with the commutator). Calculate mean θ_1 and θ_2. $\frac{R_1}{R_2}=\frac{\tan\theta_2}{\tan\theta_1}$.

Observations:

Tabulate these exactly as for comparison of E. M. F.s, page 448, but with small necessary changes (substitute R_1 and R_2 for E_1 and E_2).

Record answers to the following :

Q. How will you compare the two resistances, used in the above experiment, by means of the slide-wire bridge, no resistance box being supplied ? Of the two methods, which is the more accurate ? Give reasons.

CHAPTER XLI

MEASUREMENT OF ELECTRIC POWER

Units of Work and Power.—The C. G. S. unit of work is the **erg,** defined as *the work* done in overcoming **a force of one dyne** through **one centimetre**. As this is inconveniently small for practical purposes, the **joule,** equal to 10^7 ergs, is frequently employed.

An agent that can perform work at the rate of **one joule per second** *is said to have a power of one* **Watt.**

The student will do well to refer, at this stage, to the table on page 176 of the section on mechanics, paying special attention to the units of *work* and power.

Measurement of Electric Power—Electrical *work* is measured by the product of *quantity* of charge and the difference of potential overcome, exactly as gravitational work is measured by the product of a mass and the height through which that mass is raised. If proper units are employed.

Electrical Work = Difference of Potential × Charge

and Gravitational Work = Difference of Leval × Mass.

As electric *power* is the *rate* at which electrical work is done, it varies as the product of differenee of potential and *current* (charge delivered *per second*). Extending the analogy used above, we can say

Electric Power of a current	=	**Difference of Potential**	×	**Electric Current**
(watts)	=	(volts)	×	(amperes)
Gravitational Power of a waterfall	=	**Difference of Level**	×	**Water Current**
(ft. lbs. per sec).	=	("Head" in ft.)	×	(lbs. per second)

The **Volt** and the **Ampere** are so related that *if the current through a conductor is* **one ampere** *and the difference of potential at its ends* **one volt,** *the rate at which work* (thermal, chemical or mechanical) *is done in it* is **one watt.** Or

Volts × Amperes = Watts.

The practical unit of power on the F. P. S. system is the **horsepower,** related to the **watt** as follows :

One Horsepower = 746 Watts, or

very nearly, $\frac{4}{3}$ **Horsepower=one Kilowatt (1000 watts).**

EXPERIMENT 111.—**To determine the watts consumed by a given lamp, and to calculate the power of a steam-electric plant for lighting a factory with 1500 such lamps.**

Apparatus : Electric lamp (4 volts, 12 watts) voltmeter (5 volts) and ammeter (5 amp.) Battery (2 accumulators), connecting wires, etc.

Place the lamp and the ammeter in series with the battery, and connect the voltmeter in parallel to the lamp terminals (Fig. 184). The connections are exactly the same as those used for determining the resistance of the lamp. The lamp should be correctly lighted. A suitable rheostat in the battery circuit is convenient. Adjust the current till the voltmeter indicates the voltage marked on the lamp.

Read the *volts* and the *amperes.* Their product gives the *watts* consumed. The *power* of the dynamo required is easily calculated.

Observations:

Voltmeter reading = 4·00 volts

Ammeter reading = 3·12 amperes

Power consumed by one lamp = 4·00 × 3·12

or 12·48 watts

Output of dynamo necessary for 1500 lamps = 12·48 × 1500 watts

= 12·48 × 1·5

or 18·72 kilowatts

Efficiency of the dynamo (given) = 80%

Power of steam engine required = $18{\cdot}72 \times \frac{100}{80} \times \frac{4}{3}$ H P.

= 31·2 H. P.

CHAPTER XLII

THE INDUCTION COIL

Induced Currents.—Whenever the number of magnetic lines of force threading a coil of wire undergoes a *change*, **temporary electric currents are induced** in that coil. These currents cease as soon as the number of magnetic lines of forces becomes *constant*, no matter what the constant number may be. The induced current, produced when the lines of force through the coil are *increasing*, is *opposite* to the induced current produced when the lines of force are *decreasing.*

The Induction Coil.—Induced currents of *high electromotive force* can be produced by bringing about rapid *changes* in the number of lines of force threading a coil of very many turns of wire. In **the induction coil,** this is effected by the rapid magnetisation and demagnetisation of a straight electromagnet which forms the axis of the coil. The electromagnet brings about its own magnetisation and demagnetisation by its action on an armature, which makes and breaks the circuit of the magnetising current, exactly as in the electric bell. For a very short time after the instant of *make*, the magnetism of the electromagnet, and hence the number of magnetic lines of force threading the coil, is on the *increase*, and a *temporary induced current* is produced in the coil. An *opposite* current is produced for a short time after the *break* when the magnetism is disappearing, and the number of lines of force is on the *decrease.*

EXPERIMENT 112.—**To study the construction and mode of action of the induction coil.**

Apparatus: Small induction coil (¼″ spark), 4-volt battery, connecting wires, etc. One or two small vacuum tubes.

The construction of the Induction coil.—Properly speaking, the induction coil should be described as consisting of the three essentials for the production of induced currents, *i. e.*, *the coil* proper in which the induced currents are produced, *the electromagnet* for generating the necessary magnetic lines of force, and *the interrupter*, for bringing about the necessary changes in the magnetic lines of force, by interruptions of the magnetising current. Accordingly, examine the following parts:

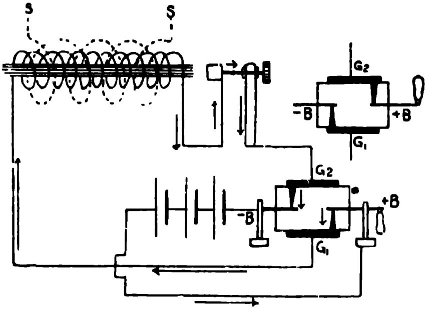


Fig. 203.—Diagrammatic representation of Ruhmkorff's induction coil with spring interrupter and Ruhmkorff's commutator.

(*i*) THE COIL PROPER generally called **the secondary coil.** This consists of a mile or two of *very fine wire* built into a coil of numerous turns (*SS* Fig. 203).

Accessories. **The spark discharger** with the two arms supported on ebonite pillars. The spark gap is adjustable in length. In small coils, the discharger is absent, the coil ending merely in terminals for connecting wires.

(*ii*) THE ELECTROMAGNET, forming the axis of the secondary coil. **The electromagnet** consists of a bundle of straight soft iron wire inside a magnetising coil (called the primary coil) of 50 to 100 turns of *thick copper wire* **The battery** forms a necessary item in connection with the electromagnet.

Accessories. These consists of—

(*a*) **A commutator,** often of the Ruhmkorff type. A turn of the handle through 180° reverses the battery current. Both positions of the commutator are illustrated. Draw a complete diagram, similar to Fig. 203, but with the commutator in the reversed position, and indicate clearly by means of arrows, the path of the current through the various parts.

(*b*) **A condenser,** generally housed under the base board. This is of use in accelerating the demagnetisation of the iron wires after the circuit is broken, so that the induced current at *break* becomes intensified and pronounced, while the induced current at *make* remains somewhat inconspicuous. This results in the secondary terminals acquiring a definite polarity, which is necessary for X-ray work.

(*iii*) THE INTERRUPTER. This consists of a small block of soft iron supported at the end of a brass spring, between the bundle of iron wires on one side, and the contact screw on its pillar on the other. The contact can be regulated and set for duration and pressure, by means of screws and lock-nuts near the base.

The use of the Induction coil.—It is of importance to arrange the wires from the secondary coil ***so that there is no chance of their coming into contact with the body of the experimenter.*** Carelessness may lead to smart (perhaps dangerous) shocks. Turn the commutator and see that the springs press well on the brass cheeks on the sides of the ebonite cylinder. Scrape with sandpaper, if necessary. Withdraw the contact screw, and clean the platinum contact at its end ***very carefully*** by gentle friction on a ***wooden*** surface. Clean similarly the platinum contact mounted behind the armature. Connect up the battery. Replace the screw so as to complete rhe circuit, and adjust the distance between the armature and the iron wires as also the pressure of the contact, by means of the screws (if provided) near the base of the interrupter, till the armature vibrates steadily, and a stream of sparks is obtained with the spark knobs ¼″ to ½″ apart. Fix up firmly the spring and screw when correctly adjusted, by means of the lock-nuts provided on each.

Experiments.—A piece of paper held between the knobs will burn. For this, use a *short fat* spark.

Put the commutator in the "off" position, and connect the electrodes of a vacuum tube to the secondary terminals. Turn on the current. The vacuum tube will light up.

Note-book record: Draw a diagram and prepare a list of the various parts, giving the function of each. Describe briefly the experiments performed.

APPENDIX

PHYSICAL TABLES

I

USEFUL DATA

For log π, log π^2, etc., see special entries under log table.

1 in. = 2·54 cm. 1 cm. = 0·3937 in.
1 metre = 3·281 ft. = 39·37 in.
1 mile = 5280 ft. = 1·609 kilometre.
1 radian = 57·3 degrees.
1 sq. in. = 6·451 sq cm. 1 cub. in. = 16·39 c.c.
1 gallon = ·1604 cub. ft.=10 lbs. (water).
1 oz. = 28·35 grammes.
1 lb. = 7000 grains = 453·6 grammes.
1 gramme = 15·43 grains.
1 cub. ft. of water weighs 62·3 lbs.
Volts × amperes = watts.
1 watt = 10^7 ergs per second.
1 Horse-power = 550 ft. lbs. per second.
= 746 watts
= nearly $\frac{3}{4}$ kilowatt.
1 therm or calorie= $4{\cdot}2 \times 10^7$ ergs.
Surface of sphere = $4\pi\ (\text{radius})^2$.
Volume of sphere = $\frac{4}{3}\pi\ (\text{radius})^3$.

II

DENSITY AND SPECIFIC VOLUME OF WATER FROM 20°C TO 41°C. (*See page 100*).

III. —HUMIDITY TABLES FOR WET AND DRY BULB HYGROMETER

For Supplement to zero column see table III a.

Figures in ordinary type give the aqueous *pressure in inches of mercury.* The italics below give *percentage humidity.*

Wet Bulb °F	Dry Bulb – Wet Bulb °F															
	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
50	·36	·33	·31	·28	·25	·23	·20	·18	·15	·12	·10	·07	·04	·02	...	...
	100	*86*	*74*	*63*	*53*	*44*	*36*	*29*	*23*	*18*	*13*	*9*	*5*	*2*	...	...
52	·39	·36	·33	·31	·28	·26	·23	·20	·18	·15	·12	·10	·07	·04	·02	...
	100	*87*	*75*	*64*	*54*	*46*	*38*	*32*	*26*	*20*	*16*	*12*	*8*	*5*	*2*	...
54	·42	·39	·36	·34	·31	·28	·26	·23	·21	·18	·15	·13	·10	·07	·05	·02
	100	*87*	*76*	*65*	*56*	*48*	*40*	*34*	*28*	*23*	*18*	*14*	*10*	*7*	*4*	*2*
56	·45	·42	·40	·37	·34	·32	·29	·26	·24	·21	·18	·16	·13	·10	·08	·05
	100	*88*	*76*	*66*	*57*	*49*	*42*	*36*	*30*	*25*	*20*	*16*	*13*	*9*	*7*	*4*
58	·48	·46	·43	·40	·38	·35	·32	·30	·27	·24	·22	·19	·16	·14	·11	·08
60	·52	·49	·46	·44	·41	·38	·36	·33	·30	·28	·25	·22	·20	·17	·14	·12
	100	*88*	*78*	*68*	*60*	*52*	*45*	*39*	*34*	*29*	*25*	*21*	*17*	*14*	*11*	*8*
62	·56	·53	·50	·48	·45	·42	·40	·37	·34	·32	·29	·26	·23	·21	·18	·15
	100	*89*	*79*	*69*	*61*	*54*	*47*	*41*	*36*	*31*	*26*	*22*	*19*	*16*	*13*	*10*
64	·60	·57	·54	·52	·49	·46	·44	·41	·38	·35	·33	·30	·27	·25	·22	·19
	100	*89*	*79*	*70*	*62*	*55*	*49*	*43*	*37*	*32*	*28*	*24*	*21*	*18*	*15*	*12*
66	·64	·61	·58	·56	·53	·50	·48	·45	·42	·40	·37	·34	·32	·29	·26	·24
	100	*89*	*80*	*71*	*63*	*56*	*50*	*44*	*39*	*34*	*30*	*26*	*22*	*19*	*16*	*14*
68	·68	·66	·63	·60	·58	·55	·52	·50	·47	·44	·42	·39	·36	·34	·31	·28
	100	*90*	*80*	*72*	*64*	*57*	*51*	*45*	*40*	*36*	*31*	*28*	*24*	*21*	*18*	*16*
70	·73	·71	·68	·65	·62	·60	·57	·54	·52	·49	·46	·44	·41	·38	·36	·33
	100	*90*	*81*	*73*	*65*	*58*	*52*	*47*	*42*	*37*	*33*	*29*	*26*	*23*	*20*	*17*
72	·78	·76	·73	·70	·68	·65	·62	·60	·57	·54	·51	·49	·46	·43	·41	·38
	100	*90*	*81*	*73*	*66*	*59*	*55*	*48*	*43*	*38*	*34*	*31*	*27*	*24*	*21*	*19*
74	·84	·81	·78	·76	·73	·70	·68	·65	·62	·60	·57	·54	·51	·49	·46	·43
	100	*90*	*82*	*74*	*67*	*60*	*54*	*49*	*44*	*40*	*36*	*32*	*28*	*25*	*23*	*20*

Wet Bulb °F	Dry Bulb − Wet Bulb °F															
	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
76	·90	·87	·84	·82	·79	·76	·73	·71	·68	·65	·63	·60	·57	·54	·52	·49
	100	*91*	*82*	*75*	*68*	*61*	*55*	*50*	*45*	*41*	*37*	*33*	*30*	*27*	*24*	*21*
78	·96	·93	·90	·88	·85	·82	·79	·77	·74	·71	·69	·66	·63	·61	·58	·55
	100	*91*	*83*	*75*	*68*	*63*	*56*	*51*	*46*	*42*	*38*	*34*	*31*	*28*	*25*	*23*
80	1·02	1·00	·97	·94	·91	·89	·86	·83	·81	·78	·75	·72	·70	·67	·64	·62
	100	*91*	*83*	*76*	*69*	*63*	*57*	*52*	*47*	*43*	*39*	*36*	*32*	*29*	*26*	*24*
82	1·09	1·06	1·04	1·01	·98	·96	·93	·90	·87	·85	·82	·79	·76	·74	·71	·68
	100	*91*	*84*	*76*	*70*	*64*	*58*	*55*	*48*	*44*	*40*	*37*	*33*	*30*	*28*	*25*
84	1·16	1·14	1·11	1·08	1·05	1·03	1·00	·97	·95	·92	·89	·86	·84	·81	·78	·76
	100	*92*	*84*	*77*	*70*	*64*	*59*	*54*	*49*	*45*	*41*	*38*	*34*	*31*	*29*	*26*
86	1·24	1·21	1·19	1·16	1·13	1·10	1·08	1·05	1·02	1·00	·97	·94	·91	·89	·86	·83
	100	*92*	*84*	*77*	*71*	*65*	*60*	*55*	*50*	*46*	*42*	*39*	*35*	*32*	*30*	*27*
88	1·32	1·30	1·27	1·24	1·21	1·19	1·16	1·13	1·10	1·08	1·05	1·02	1·00	·97	·94	·91
	100	*92*	*85*	*78*	*71*	*66*	*60*	*56*	*51*	*47*	*43*	*40*	*36*	*35*	*31*	*28*
90	1·41	1·38	1·35	1·33	1·30	1·27	1·24	1·22	1·19	1·16	1·13	1·11	1·08	1·05	1·03	1·00
	100	*92*	*85*	*78*	*72*	*66*	*61*	*56*	*52*	*48*	*44*	*41*	*37*	*34*	*32*	*29*
92	1·50	1·47	1·44	1·42	1·39	1·36	1·34	1·3.	1·28	1·25	1·23	1·20	1·17	1·14	1·12	1·09
	100	*92*	*85*	*79*	*72*	*67*	*62*	*57*	*55*	*49*	*45*	*41*	*38*	*35*	*33*	*30*
94	1·60	1·57	1·54	1·51	1·49	1·46	1·43	1·40	1·37	1·35	1·32	1·29	1·26	1·24	1·21	1·18
	100	*92*	*86*	*79*	*73*	*67*	*62*	*58*	*55*	*49*	*46*	*42*	*39*	*36*	*33*	*31*
96	1·70	1·67	1·64	1·61	1·59	1·56	1·53	1·50	1·48	1·45	1·42	1·39	1·37	1·34	1·31	1·28
	100	*93*	*86*	*79*	*73*	*68*	*63*	*58*	*54*	*50*	*46*	*43*	*40*	*37*	*34*	*32*
98	1·80	1·78	1·75	1·72	1·69	1·67	1·64	1·61	1·58	1·56	1·53	1·50	1·47	1·45	1·42	1·39
	100	*93*	*86*	*80*	*74*	*69*	*63*	*59*	*55*	*51*	*47*	*44*	*41*	*38*	*35*	*33*
100	1·92	1·89	1·86	1·83	1·81	1·78	1·75	1·72	1·70	1·67	1·64	1·61	1·59	1·56	1·53	1·50
	100	*93*	*86*	*80*	*74*	*69*	*64*	*60*	*55*	*51*	*48*	*45*	*41*	*39*	*36*	*33*

IIIa.

PRESSURE OF SATURATED WATER VAPOUR FROM 0°F TO 50°F. IN INCHES OF MERCURY

(Supplement to 0 column of Humidity Tables (III).)

Temp. °F	Pressure	Temp. 0 °F	Pressure	Temp. °F	Pressure
0	·045	20	·110	40	·246
2	·049	22	·119	42	·266
4	·054	24	·130	44	·287
6	·059	26	·141	46	·310
8	·065	28	·153	48	·334
10	·071	30	·166	50	·360
12	·078	32	·180		
14	·085	34	·195		
16	·092	36	·211		
18	·101	38	·228		

IV

LATEST (1927) VALUES OF THE HORIZONTAL COMPONENT OF EARTH'S MAGNETIC FIELD AND MAGNETIC DIP

Compiled to three correct significant figures from the values supplied by the Geodetic Branch of the Survey of India.

Station.	"H" in dynes.	DIP in degree.
Srinagar	0·303	51° − 8′ N.
Jammu	0·311	49° − 1′ N.
Lahore	0·315	47° − 57′ N.
Dehra Dun	0·328	45° − 38′ N.
Delhi	0·336	43° − 1′ N.

MATHEMATICAL TABLES

I.—TRIGONOMETRICAL RATIOS OF ANGLES

Degrees	Sine	Tangent	Log Sin	Log Tan
0	0·0000	0·0000	Avoid angles of incidence etc. below 10°.	Avoid deflections etc. below 10°.
1	·0175	·0175		
2	·0349	·0349		
3	·0523	·0524		
4	·0698	·0699		
5	·0872	·0875		
6	·1045	·1051		
7	·1219	·1228		
8	·1392	·1405		
9	·1564	·1584		
10	·1736	·1763	$\bar{1}$ ·2397	$\bar{1}$ ·2463
11	·1908	·1944	,, ·2806	,, ·2887
12	·2079	·2126	,, ·3179	,, ·3275
13	·2250	·2309	,, ·3521	,, ·3634
14	·2419	·2493	,, ·3837	,, ·3968
15	·2588	·2679	,, ·4130	,, ·4281
16	·2756	·2867	,, ·4403	,, ·4575
17	·2924	·3057	,, ·4659	,, ·4853
18	·3090	·3249	,, ·4900	,, ·5118
19	·3256	·3443	,, ·5126	,, ·5370
20	·3420	·3640	,, ·5341	,, ·5611
21	·3584	·3839	,, ·5543	,, ·5842
22	·3746	·4040	,, ·5736	,, ·6064
23	·3907	·4245	,, ·5919	,, ·6279
24	·4067	·4452	,, ·6093	,, ·6486
25	·4226	·4663	,, ·6259	,, ·6687
26	·4384	·4877	,, ·6418	,, ·6882
27	·4540	·5095	,, ·6570	,, ·7072
28	·4695	·5317	,, ·6716	,, ·7257
29	·4848	·5543	,, ·6856	,, ·7438

Degrees	Sine	Tangent	Log Sin	Log Tan
30	·5000	·5774	1 ·6990	$\bar{1}$ ·7614
31	·5150	·6009	„ ·7118	„ ·7788
32	·5299	·6249	„ ·7242	„ ·7958
33	·5446	·6494	„ ·7361	„ ·8125
34	·5592	·6745	„ ·7476	„ ·8290
35	·5736	·7002	„ ·7586	„ ·8452
36	·5878	·7265	„ ·7692	„ ·8613
37	·6018	·7536	„ ·7795	„ ·8771
38	·6157	·7813	„ ·7893	„ ·8928
39	·6293	·8098	„ ·7989	„ ·9084
40	·6428	·8391	„ ·8081	„ ·9238
41	·6561	·8693	„ ·8169	„ ·9392
42	·6691	·9004	„ ·8255	„ ·9544
43	·6820	·9325	„ ·8338	„ ·9697
44	·6947	·9657	„ ·8418	„ ·9848
45	·7071	1·0000	„ ·8495	0·0000
46	·7193	1·0355	„ ·8569	·0152
47	·7314	1·0724	„ ·8641	·0303
48	·7431	1·1106	„ ·8711	·0456
49	·7547	1·1504	„ ·8778	·0608
50	·7660	1·1918	„ ·8843	·0762
51	·7771	1·2349	„ ·8905	·0916
52	·7880	1·2799	„ ·8965	·1072
53	·7986	1·3270	„ ·9023	·1229
54	·8090	1·3764	„ ·9080	·1387
55	·8192	1·4281	„ ·9134	·1548
56	·8290	1·4826	„ ·9186	·1710
57	·8387	1·5399	„ ·9236	·1875
58	·8480	1·6003	„ ·9284	·2042
59	·8572	1·6643	„ ·9331	·2212

Degrees	Sine	Tangent	Log Sin	Log Tan
60	·8660	1·7321	$\bar{1}$ ·9375	·2386
61	·8746	1·8040	,, ·9418	·2562
62	·8829	1·8807	,, ·9459	·2743
63	·8910	1·9626	,, ·9499	·2928
64	·8988	2·0503	,, ·9537	·3118
65	·9063	2·1445	,, ·9573	·3313
66	·9135	2·2460	,, ·9607	·3514
67	·9205	2·3559	,, ·9640	·3721
68	·9272	2·4751	,, ·9672	·3936
69	·9336	2·6051	,, ·9702	·4158
70	·9397	2·7475	,, ·9730	·4389
71	·9455	2·9042	,, ·9757	·4630
72	·9511	3·0777	,, ·9782	·4882
73	·9563	3·2709	,, ·9806	·5147
74	·9613	3·4874	,, ·9828	·5425
75	·9659	3·7321	,, ·9849	·5719
76	·9703	4·0108	,, ·9869	·6032
77	·9744	4·3315	,, ·9887	·6366
78	·9781	4·7046	,, ·9904	·6725
79	·9816	5·1446	,, ·9919	·7113
80	·9848	5·6713	,, ·9934	·7537
81	·9877	Avoid deflections etc. above 70°.	,, ·9946	Avoid deflections etc. above 70°.
82	·9903		,, ·9958	
83	·9925		,, ·9968	
84	·9945		,, ·9976	
85	·9962		,, ·9983	
86	·9976		,, ·9989	
87	·9986		,, ·9994	
88	·9994		,, ·9997	
89	·9998		,, ·9999	
90	1·0000		0·0000	

II.—RECIPROCALS FOR CALCULATION OF FOCAL LENGTHS

[*Numbers in difference columns to be subtracted, not added.*]

[Example: $u=23{\cdot}94$ cm, $v=24{\cdot}65$ cm. $1/f=1/u+1/v=1/23{\cdot}94+1/24{\cdot}65={\cdot}04177+{\cdot}04057={\cdot}08234$. $f=1/{\cdot}08234=1000/82{\cdot}34=1000\times{\cdot}01214=12{\cdot}14$ cm.

Or, looking up main table we find **12·1** as the number against which the reciprocal nearest to ·08234 stands. Find the last figure from the difference column.]

	0	1	2	3	4	5	6	7	8	9	Mean Differences.								
											1	2	3	4	5	6	7	8	9
10	·1000	9901	9804	9709	9615	9524	9434	9346	9259	9174	9	18	27	36	45	55	64	73	82
11	·09091	9009	8929	8850	8772	8696	8621	8547	8475	8403	8	15	23	30	38	45	53	61	68
12	·08333	8264	8197	8130	8065	8000	7937	7874	7813	7752	6	13	19	26	32	38	45	51	58
13	·07692	7634	7576	7519	7463	7407	7353	7299	7246	7194	5	11	16	22	27	33	38	44	49
14	·07143	7092	7042	6993	6944	6897	6849	6803	6757	6711	5	10	14	19	24	29	33	38	43
15	·06667	6623	6579	6536	6494	6452	6410	6369	6329	6289	4	8	13	17	21	25	29	33	38
16	·06250	6211	6173	6135	6098	6061	6024	5988	5952	5917	4	7	11	15	18	22	26	29	33
17	·05882	5848	5814	5780	5747	5714	5682	5650	5618	5587	3	6	10	13	16	20	23	26	29
18	·05556	5525	5495	5464	5435	5405	5376	5348	5319	5291	3	6	9	12	15	17	20	23	26
19	·05263	5236	5208	5181	5155	5128	5102	5076	5051	5025	3	5	8	11	13	16	18	21	24
20	·05000	4975	4950	4926	4902	4878	4854	4831	4808	4785	2	5	7	10	12	14	17	19	21
21	·04762	4739	4717	4695	4673	4651	4630	4608	4587	4566	2	4	7	9	11	13	15	17	20
22	04545	4525	4505	4484	4464	4444	4425	4405	4386	4367	2	4	6	8	10	12	14	16	18
23	·04348	4329	4310	4292	4274	4255	4237	4219	4202	4184	2	4	5	7	9	11	13	14	16
24	·04167	4149	4132	4115	4098	4082	4065	4049	4032	4016	2	3	5	7	8	10	12	13	15
25	·04000	3984	3968	3953	3937	3922	3906	3891	3876	3861	2	3	5	6	8	9	11	12	14
26	·03846	3831	3817	3802	3788	3774	3759	3745	3731	3717	1	3	4	6	7	8	10	11	13
27	·03704	3690	3676	3663	3650	3636	3623	3610	3597	3584	1	3	4	5	7	8	9	11	12
28	·03571	3559	3516	3534	3521	3509	3497	3484	3472	3460	1	2	4	5	6	7	9	10	11
29	·03448	3436	3425	3413	3401	3390	3378	3367	3356	3344	1	2	3	5	6	7	8	9	10
30	·03333	3322	3311	3300	3289	3279	3268	3257	3247	3236	1	2	3	4	5	6	7	9	10
31	·03226	3215	3205	3195	3185	3175	3165	3155	3145	3135	1	2	3	4	5	6	7	8	9
32	·03125	3115	3096	3106	3086	3077	3067	3058	3049	3040	1	2	3	4	5	6	7	8	9
33	·03030	3021	3012	3003	2994	2985	2976	2967	2959	2950	1	2	3	4	4	5	6	7	8
34	·02941	2933	2924	2915	2907	2899	2890	2882	2874	2865	1	2	3	3	4	5	6	7	8
35	·02857	2849	2841	2833	2825	2817	2809	2801	2793	2786	1	2	2	3	4	5	6	6	7
36	·02778	2770	2762	2755	2747	2740	2732	2725	2717	2710	1	2	2	3	4	5	5	6	7
37	·02703	2695	2688	2681	2674	2667	2660	2653	2646	2639	1	1	2	3	4	4	5	6	6
38	·02632	2625	2618	2611	2604	2597	2591	2584	2577	2571	1	1	2	3	3	4	5	5	6
39	·02564	2558	2551	2545	2538	2532	2525	2519	2513	2506	1	1	2	3	3	4	4	5	6
40	·02500	2494	2488	2481	2475	2469	2463	2457	2451	2445	1	1	2	2	3	4	4	5	5
41	·02439	2433	2427	2421	2415	2410	2404	2398	2392	2387	1	1	2	2	3	3	4	5	5
42	·02381	2375	2370	2364	2358	2353	2347	2342	2336	2331	1	1	2	2	3	3	4	4	5
43	·02326	2320	2315	2309	2304	2299	2294	2288	2283	2278	1	1	2	2	3	3	4	4	5
44	·02273	2268	2262	2257	2252	2247	2242	2237	2232	2227	1	1	2	2	3	3	4	4	5
45	·02222	2217	2212	2208	2203	2198	2193	2188	2183	2179	0	1	1	2	2	3	3	4	4
46	·02174	2169	2165	2160	2155	2151	2146	2141	2137	2132	0	1	1	2	2	3	3	4	4
47	·02128	2123	2119	2114	2110	2105	2101	2096	2092	2088	0	1	1	2	2	3	3	4	4
48	·02083	2079	2075	2070	2066	2062	2058	2053	2049	2045	0	1	1	2	2	3	3	3	4
49	·02041	2037	2033	2028	2024	2020	2016	2012	2008	2004	0	1	1	2	2	2	3	3	4
50	·02000	1996	1992	1988	1984	1980	1976	1972	1969	1965	0	1	1	2	2	2	3	3	4
51	·01961	1957	1953	1949	1946	1942	1938	1934	1931	1927	0	1	1	2	2	2	3	3	3
52	·01923	1919	1916	1912	1908	1905	1901	1898	1894	1890	0	1	1	1	2	2	3	3	3
53	·01887	1883	1880	1876	1873	1869	1866	1862	1859	1855	0	1	1	1	2	2	2	3	3
54	·01852	1848	1845	1842	1838	1835	1832	1828	1825	1821	0	1	1	1	2	2	2	3	3

RECIPROCALS FOR CALCULATION OF FOCAL LENGTHS

[*Numbers in difference columns to be subtracted, not added.*]

	0	1	2	3	4	5	6	7	8	9	Mean Differences.								
											1	2	3	4	5	6	7	8	9
55	·01818	1815	1812	1808	1805	1802	1799	1795	1792	1789	0	1	1	1	2	2	2	3	3
56	·01786	1783	1779	1776	1773	1770	1767	1764	1761	1757	0	1	1	1	2	2	2	3	3
57	·01754	1751	1748	1745	1742	1739	1736	1733	1730	1727	0	1	1	1	1	2	2	2	3
58	·01724	1721	1718	1715	1712	1709	1706	1704	1701	1698	0	1	1	1	1	2	2	2	3
59	·01695	1692	1689	1686	1684	1681	1678	1675	1672	1669	0	1	1	1	1	2	2	2	3
60	·01667	1664	1661	1658	1656	1653	1650	1647	1645	1642	0	1	1	1	1	2	2	2	3
61	·01639	1637	1634	1631	1629	1626	1623	1621	1618	1616	0	1	1	1	1	2	2	2	2
62	·01613	1610	1608	1605	1603	1600	1597	1595	1592	1590	0	1	1	1	1	2	2	2	2
63	·01587	1585	1582	1580	1577	1575	1572	1570	1567	1565	0	0	1	1	1	1	2	2	2
64	·01562	1560	1558	1555	1553	1550	1548	1546	1543	1541	0	0	1	1	1	1	2	2	2
65	·01538	1536	1534	1531	1529	1527	1524	1522	1520	1517	0	0	1	1	1	1	2	2	2
66	·01515	1513	1511	1508	1506	1504	1502	1499	1497	1495	0	0	1	1	1	1	2	2	2
67	·01493	1490	1488	1486	1484	1481	1479	1477	1475	1473	0	0	1	1	1	1	2	2	2
68	·01471	1468	1466	1464	1462	1460	1458	1456	1453	1451	0	0	1	1	1	1	2	2	2
69	·01449	1447	1445	1443	1441	1439	1437	1435	1433	1431	0	0	1	1	1	1	2	2	2
70	·01409	1427	1425	1422	1420	1418	1416	1414	1412	1410	0	0	1	1	1	1	1	2	2
71	·01408	1406	1404	1403	1401	1399	1397	1395	1393	1391	0	0	1	1	1	1	1	2	2
72	·01389	1387	1385	1383	1381	1379	1377	1376	1374	1372	0	0	1	1	1	1	1	2	2
73	·01370	1368	1366	1364	1362	1361	1359	1357	1355	1353	0	0	1	1	1	1	1	2	2
74	·01351	1350	1348	1346	1344	1342	1340	1339	1337	1335	0	0	1	1	1	1	1	1	2
75	·01333	1332	1330	1328	1326	1325	1323	1321	1319	1318	0	0	1	1	1	1	1	1	2
76	·01316	1314	1312	1311	1309	1307	1305	1304	1302	1300	0	0	1	1	1	1	1	1	2
77	·01299	1297	1295	1294	1292	1290	1289	1287	1285	1284	0	0	0	1	1	1	1	1	1
78	·01282	1280	1279	1277	1276	1274	1272	1271	1269	1267	0	0	0	1	1	1	1	1	1
79	·01266	1264	1263	1261	1259	1258	1256	1255	1253	1252	0	0	0	1	1	1	1	1	1
80	·01250	1248	1247	1245	1244	1242	1241	1239	1238	1236	0	0	0	1	1	1	1	1	1
81	·01235	1233	1232	1230	1229	1227	1225	1224	1222	1221	0	0	0	1	1	1	1	1	1
82	·01220	1218	1217	1215	1214	1212	1211	1209	1208	1206	0	0	0	1	1	1	1	1	1
83	·01205	1203	1202	1200	1199	1198	1196	1195	1193	1192	0	0	0	1	1	1	1	1	1
84	·01190	1189	1188	1186	1185	1183	1182	1181	1179	1178	0	0	0	1	1	1	1	1	1
85	·01176	1175	1174	1172	1171	1170	1168	1167	1166	1164	0	0	0	1	1	1	1	1	1
86	·01163	1161	160	1159	1157	1156	1155	1153	1152	1151	0	0	0	1	1	1	1	1	1
87	·01149	1148	1147	1145	1144	1143	1142	1140	1139	1138	0	0	0	1	1	1	1	1	1
88	·01136	1135	1134	1133	1131	1130	1129	1127	1126	1125	0	0	0	1	1	1	1	1	1
89	·01124	1122	1121	1120	1119	1117	1116	1115	1114	1112	0	0	0	1	1	1	1	1	1
90	·01111	1110	1109	1107	1106	1105	1104	1103	1101	1100	0	0	0	1	1	1	1	1	1
91	·01099	1098	1096	1095	1094	1093	1092	1090	1089	1088	0	0	0	0	1	1	1	1	1
92	·01087	1086	1085	1083	1082	1081	1080	1079	1078	1076	0	0	0	0	1	1	1	1	1
93	·01075	1074	1073	1072	1071	1070	1068	1067	1066	1065	0	0	0	0	1	1	1	1	1
94	·01064	1063	1062	1060	1059	1058	1057	1056	1055	1054	0	0	0	0	1	1	1	1	1
95	·01053	1052	1050	1049	1048	1047	1046	1045	1044	1043	0	0	0	0	1	1	1	1	1
96	·01042	1041	1039	1038	1037	1036	1035	1034	1033	1032	0	0	0	0	1	1	1	1	1
97	·01031	1030	1029	1028	1027	1026	1025	1024	1022	1021	0	0	0	0	1	1	1	1	1
98	·01020	1019	1018	1017	1016	1015	1014	1013	1012	1011	0	0	0	0	1	1	1	1	1
99	·01010	1009	1008	1007	1006	1005	1004	1003	1002	1001	0	0	0	0	0	1	1	1	1

III.—LOGARITHMS

(For calculation of products, quotients, powers and roots. Formulæ used:

$\log(ab) = \log a + \log b$; $\log \frac{a}{b} = \log a - \log b$; $\log a^b = b \times \log a$; $\log \sqrt[b]{a} = \frac{1}{b} \log a$).

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25	29
14	1461	1492	1523	1553	1584						3	6	9	12	15	19	22	25	28
						1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875						3	6	9	11	14	17	20	23	26
						1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20	23
						2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6484	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8851	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
35	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

$\mathrm{Log}\frac{\pi}{6}=\bar{1}\cdot7190$; $\log\pi=\cdot4972$; $\log\frac{4}{3}\pi=\cdot6221$; $\log 2\pi=\cdot7982$; $\log 4\pi=1\cdot0993$;
$\log\pi^2=\cdot9943$; $\log 4\pi^2=1\cdot5964$; $\log\sqrt{2}=1505$; $\log\sqrt{3}=\cdot2386$; $\log$ 'g' $=2\cdot9910$.

IV—ANTILOGARITHMS

To be used in connection with logarithms.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
24	1738	1742	1746	7150	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3590	3606	3614	3622	1	2	2	3	4	5	6	7	7
56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8051	8072	8091	8110	2	4	6	7	9	11	13	15	17
91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20
1	0	·1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

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